Quantum Mechanics II: PHYS 314 (Spring 2021)
Problem Set 8–Due Thursday April 22.

Overview

In this Problem Set you will apply time-independent perturbation theory (degenerate and nondegenerate) to one-dimensional quantum mechanical systems.

Question 1 [Griffiths 7.1] 10pts

Suppose we put a delta-function bump in the center of the infinite square well:

$$H' = \alpha\delta(x - a/2),$$

where α is a constant.

(a) Find the first-order correction to the allowed energies. Explain why the energies are not perturbed for even n.

(b) Find the first three nonzero terms in the expansion

$$\psi_n^1 = \sum_{m\neq n} \frac{\langle\psi_m^0|H'|\psi_n^0\rangle}{E_n^0 - E_m^0}\psi_m^0$$

of the correction to the ground state, ψ_1^1.

Question 2 [Griffiths 7.6] 15pts

Consider a charged particle in the one-dimensional harmonic oscillator potential. Suppose we turn on a weak electric field (E), so that the potential energy is shifted by an amount $H' = -qEx$.

(a) Show that there is no first-order change in the energy levels, and calculate the second-order correction. *Hint:* See Problem 3.39.

(b) The Schrödinger equation can be solved directly in this case, by a change of variables, $x' = x - qE/(m\omega^2)$. Find the exact energies, and show that they are consistent with the perturbation theory approximation.

Question 3 [Griffiths 7.9] **25pts**

Consider a particle of mass m that is free to move in a one-dimensional region of length L that closes on itself (for instance, a bead that slides frictionlessly on a circular wire of circumference L, as in Problem 2.46).

(a) Show that the stationary states can be written in the form

$$\psi_n(x) = \frac{1}{\sqrt{L}} e^{2\pi i n x/L}, \qquad (-L/2 < x < L/2),$$

where $n = 0, \pm 1, \pm 2, \ldots$, and the allowed energies are

$$E_n = \frac{2}{m}\left(\frac{n\pi\hbar}{L}\right)^2.$$

Notice that—with the exception of the ground state ($n = 0$)—these are all doubly degenerate.

(b) Now suppose we introduce the perturbation

$$H' = -V_0 e^{-x^2/a^2},$$

where $a \ll L$. (This puts a little "dimple" in the potential at $x = 0$, as though we bent the wire slightly to make a "trap".) Find the first-order correction to E_n, using

$$E^1_{\pm} = \frac{1}{2}\left[W_{aa} + W_{bb} \pm \sqrt{(W_{aa} - W_{bb})^2 + 4|W_{ab}|^2}\right].$$

Hint: To evaluate the integrals, exploit the fact that $a \ll L$ to extend the limits from $\pm L/2$ to $\pm\infty$; after all, H' is essentially zero outside $-a < x < a$.

(c) What are the "good" linear combinations of ψ_n and ψ_{-n}, for this problem?

($\textit{Hint:}$ Use $\alpha W_{aa} + \beta W_{ab} = \alpha E^1$.)

Show that with these states you get the first-order correction using

$$E^1_n = \left\langle \psi^0_n | H' | \psi^0_n \right\rangle.$$

(d) Find a Hermitian operator A that fits the requirements of the theorem, and show that the simultaneous eigenstates of H^0 and A are precisely the ones you used in (c).