

Quantum Mechanics II: PHYS 314 (Spring 2021)
Problem Set 7–Due Thursday April 15.

Overview

In this Problem Set you will explore variational methods and apply them to several physical systems, such as the Yukawa potential and a model of molecular spectra.

Question 1 [Griffiths 8.21]

20pts

If the photon had a nonzero mass ($m_\gamma \neq 0$), the Coulomb potential would be replaced by the **Yukawa potential**,

$$V(\mathbf{r}) = -\frac{e^2}{4\pi\epsilon_0} \frac{e^{-\mu r}}{r},$$

where $\mu = m_\gamma c/\hbar$. With a trial wavefunction of your own devising, estimate the binding energy of a “hydrogen” atom with this potential. Assume $\mu a \ll 1$, and give your answer correct to order $(\mu a)^2$.

Question 2

15pts

A particle of mass m and positive charge q , moving in one dimension, is subject to a uniform electric field $E(x) = -E_0$ for $x > 0$ and $E(x) = E_0$ for $x < 0$. Consider a trial wave function $\psi(x) \propto e^{-\alpha|x|}$ and estimate the ground-state energy by minimizing the expectation value of the energy.

Question 3

15pts

Molecular spectra calculations often involve the following potential:

$$U(x) = U_0 \left(e^{-2\alpha x} - 2e^{-\alpha x} \right).$$

With the trial wavefunction of your own devising, estimate the ground state energy of such a molecule, assuming $U_0 = \alpha^2 \hbar^2 / (2m)$. To receive full credit please explain why you chose your particular trial wavefunction. As long as your choice of trial wavefunction is reasonable, your grade will not depend on the closeness of your estimate to the actual answer.