Quantum Mechanics II: PHYS 314 (Spring 2021)
Problem Set 1—Due Thursday, February 11.

Overview

In this Problem Set you will revise material you covered in Quantum Mechanics I, including reminding yourself of the properties of the infinite square well, how to calculate expectation values, and reviewing properties of eigenfunctions.

Question 1 [Griffiths 2.4] 20pts

Calculate $\langle x \rangle$, $\langle x^2 \rangle$, $\langle p \rangle$, $\langle p^2 \rangle$, σ_x and σ_p for the n^{th} stationary state of the infinite square well. Check that the uncertainty principle is satisfied. Which state comes closest to the uncertainty limit?

Question 2 [Griffiths 3.7] 15pts

(a) Suppose that $f(x)$ and $g(x)$ are two eigenfunctions of an operator $\widehat{Q}$, with the same eigenvalue q. Show that any linear combination of f and g is itself an eigenfunction of $\widehat{Q}$, with eigenvalue q.

(b) Check that $f(x) = \exp(x)$ and $g(x) = \exp(-x)$ are eigenfunctions of the operator $\mathrm{d}^2/\mathrm{d}x^2$, with the same eigenvalue. Construct two linear combintations of f and g that are *orthogonal* eigenfunctions on the interval $(-1, 1)$.

Question 3 [Griffiths 3.12] 15pts

Find $\Phi(p, t)$, defined through the inverse Fourier transform of the wavefunction [Equation 3.54],

$$\Phi(p,t) = \frac{1}{\sqrt{2\pi\hbar}} \int_{-\infty}^{\infty} \mathrm{d}x \, e^{-ipx/\hbar} \Psi(x,t),$$

for the *free particle*, in terms of the function $\phi(k)$ introduced in Equation 2.101, which is

$$\Psi(x,t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \mathrm{d}k \, \phi(k) e^{i(kx - \hbar^2 k^2 t/(2m))}.$$

Show that for the free particle $|\Phi(p,t)|^2$ is independent of time.