Quantum Mechanics II: PHYS 314 (Spring 2021)
Final exam–Due 11:59 am on Monday May 17.

Overview

In this exam you will apply your knowledge of time-independent and time-dependent approximation methods to a novel quantum system, working in one and two dimensions.

There are three questions, for a total of **100 points**. **Answer all three questions**. You are welcome to use the textbook and other internet resources, but you should cite your sources appropriately. You may ask clarifying questions via slack, but you cannot discuss this exam with other students. During this exam, you are expected to abide by the Honor Code. Please return your solutions to me as pdf files, with your **name in the title** of the file, by **11:59 am on Monday May 17**.

Question 1 — 40pts

Consider the one-dimensional quantum system defined by the following potential

$$V(x) = \begin{cases} \epsilon E x & 0 \leq x \leq L \\ \\ \infty & \text{otherwise.} \end{cases} \tag{1}$$

Here ϵ and E are real-valued constants.

(a) Use the variational method to find an upper bound on the ground state energy of this system. Justify your choice of trial wavefunction. You do not need to include an optimisation parameter in your trial wavefunction.

(b) Now assume that ϵ is sufficiently small that we can apply (time-independent) perturbation theory.

(i) What are the wavefunctions and energy levels of the unperturbed system in this case? Is this unperturbed system nondegenerate or degenerate?

(ii) Find the first-order correction to the energy levels of this system.

(iii) Find the second-order correction to the ground state energy of this system.

Hint: You may find the following result useful

$$\sum_{n=2}^{\infty} \frac{n^2(1+(-1)^n)^2}{(1-n^2)^5} = \frac{\pi^2(\pi^2-15)}{768}.$$

Question 2 **20pts**

Consider now the two-dimensional quantum system defined by the following potential

$$V(x,y) = \begin{cases} \epsilon E(x+y) & 0 \leq (x,y) \leq L \\ \\ \infty & \text{otherwise} \end{cases}, \tag{2}$$

where ϵ is a small, real-valued parameter and E is a real-valued constant.

(a) Write down the energy eigenstates and their corresponding energies for the unperturbed system.

(b) Find the first order correction to the energy level of the first excited state for this system.

Question 3 **40pts**

Consider a one-dimensional infinite square well with the time-dependent potential

$$V(x,t) = \begin{cases} \epsilon E(t)x & 0 \leq x \leq L \\ \\ \infty & \text{otherwise.} \end{cases} \tag{3}$$

Here

$$E(t) = \begin{cases} E_0 t & t \geq 0 \\ \\ 0 & t < 0. \end{cases},$$

with E_0 a real-valued constant. At time $t = 0$, the system is in the ground state of the unperturbed potential.

Using first-order time-dependent perturbation theory, calculate

(a) The transition probability from the ground state of the system to the first excited state for time $t > 0$.

(b) The total probability of finding the system in either the ground state, or the first excited state, at time $t > 0$. Comment on your result.