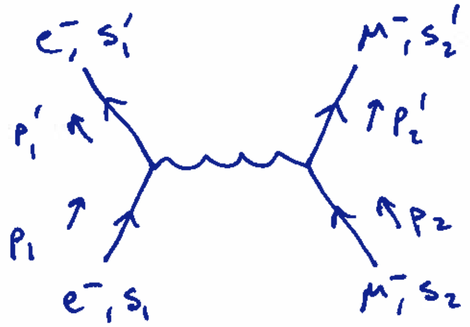
Crossing symmetry $\boxed{\text{P+S 5.3}}$

We've been considering $e^+e^- \to \mu^+\mu^-$

Let's now flip this diagram onto its side and consider instead $e^-\mu^- \to e^-\mu^-$ (electron-muon Møller scattering)

$\boxed{1.}$ Diagram is



$\boxed{2.}$ $i\mathcal{M} = \bar{u}^{s_1'}(\bar{p}_1')(-ig\gamma^\mu)u^{s_1}(\bar{p}_1) \cdot \frac{-ig_{\mu\nu}}{(p_1-p_1')^2+i\epsilon} \cdot \bar{u}^{s_2'}(\bar{p}_2')(-ie\gamma^\nu)u^{s_2}(\bar{p}_2)$

$\boxed{3.}$ $\overline{|\mathcal{M}|^2} = \frac{1}{4}\sum_{\text{spins}} |\mathcal{M}|^2$

$$= \frac{1}{4}\frac{g^4}{((p_1-p_1')^2+i\epsilon)^2} \mathrm{Tr}\left[(\not{p}_1' + m_e)\gamma^\mu(\not{p}_1+m_e)\gamma^\nu\right] \times \mathrm{Tr}\left[(\not{p}_2' + m_\mu)\gamma_\mu(\not{p}_2+m_\mu)\gamma_\nu\right]$$

Recall that our expression for $e^+e^- \to \mu^+\mu^-$ was

$$\overline{|\mathcal{M}|^2} = \frac{1}{4}\frac{g^4}{(p+p')^4} \mathrm{Tr}\left[(\not{p}+m_e)\gamma^\mu(\not{p}'-m_e)\gamma^\nu\right] \times \mathrm{Tr}\left[(\not{k}+m_\mu)\gamma_\mu(\not{k}'-m_\mu)\gamma_\nu\right]$$

So if we make the substitutions

$p \to p_1$
$p' \to -p_1'$
$k \to p_2$
$k' \to -p_2$

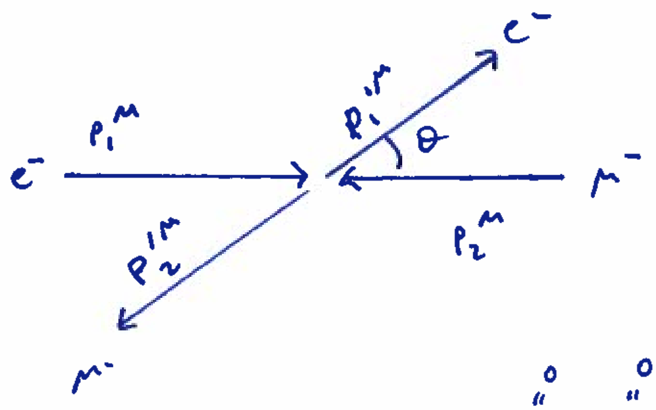
and the fact that
$\mathrm{Tr}\left[(\not{k}+m_\mu)\gamma_\mu(\not{k}'-m_\mu)\gamma_\nu\right]$
$= \mathrm{Tr}\left[(\not{k}+m_\mu)\gamma_\nu(\not{k}'-m_\mu)\gamma_\mu\right]$

Then we can use the old results!

So we immediately get

$$\overline{|M|^2} = \frac{8g^4}{(p_1-p_1')^4}\Big[(p_1\cdot p_2')(p_1'\cdot p_2) + (p_1\cdot p_2)(p_1'\cdot p_2') - m_e^2(p_2\cdot p_2') - m_\mu^2\, p_1\cdot p_1' + 2m_e^2 m_\mu^2\Big]$$

$\boxed{14.}$ We work in the CM frame, with $\frac{m_e}{E} \ll 1$



$$p_1^\mu = (|\bar{k}|, |\bar{k}|\hat{z})$$
$$p_2^\mu = (E, -|\bar{k}|\hat{z})$$
$$p_1'^\mu = (|\bar{k}|, \bar{k})$$
$$p_2'^\mu = (E, -\bar{k})$$

$E^2 = |\bar{k}|^2 + m_\mu^2$
$\bar{k}\cdot\hat{z} = |\bar{k}|\cos\theta$
$E + k = E_{cm}$

Then $(p_1 - p_1')^2 = \overset{0}{p_1^2} + \overset{0}{p_1'^2} - 2p_1\cdot p_1'$
$$= -2(|\bar{k}|^2 - |\bar{k}|^2\cos\theta)$$
$$p_1\cdot p_1' = |\bar{k}|^2(1-\cos\theta)$$
$$p_1\cdot p_2' = E|\bar{k}| + |\bar{k}|^2\cos\theta = |\bar{k}|(E + |\bar{k}|\cos\theta) = p_1'\cdot p_2$$
$$p_1\cdot p_2 = E|\bar{k}| + |\bar{k}|^2 = |\bar{k}|(E + |\bar{k}|) = p_1'\cdot p_2'$$

So we have

$$\overline{|M|^2} = \frac{8g^4}{4|\bar{k}|^4(1-\cos\theta)^2}\Big[|\bar{k}|^2(E+|\bar{k}|\cos\theta)^2 + |\bar{k}|^2(E+|\bar{k}|)^2 - m_\mu^2|\bar{k}|^2(1-\cos\theta)\Big]$$
$$= 2g^4\frac{1}{|\bar{k}|^2(1-\cos\theta)^2}\Big[E^2 + 2E|\bar{k}|\cos\theta + |\bar{k}|^2\cos^2\theta + E^2 + 2E|\bar{k}| + |\bar{k}|^2 - m_\mu^2(1-\cos\theta)\Big]$$
$$= 2g^4\frac{1}{|\bar{k}|^2(1-\cos\theta)^2}\Big[2E^2 + 2E|\bar{k}|(1+\cos\theta) + |\bar{k}|^2(\cos^2\theta+1) + m_\mu^2(\cos^2\theta-1)\Big]$$

$\boxed{5.}$ Our expression for the cross-section is pretty simple when one particle is massless

$$\left(\frac{d\sigma}{d\Omega}\right)_{cm} = \frac{1}{2E_A 2E_B |v_A - v_B|} \frac{|\vec{p}_1|}{(2\pi)^2 4E_{cm}} |\mathcal{M}|^2 \rightarrow \left(\frac{d\sigma}{d\Omega}\right)_{cm} = \frac{|\mathcal{M}|^2}{64\pi^2 \underbrace{(E+k)^2}_{E_{cm}}}$$

Thus

$$\left(\frac{d\sigma}{d\Omega}\right)_{cm} = \frac{\alpha^2}{2k^2 E_{cm}^2 (1-\cos\theta)^2} \left(E_{cm}^2 + (E + k\cos\theta)^2 - m_\mu^2 (1-\cos\theta)\right)$$

N.B.

$$\frac{|\vec{p}_1|}{2E_A 2E_B |v_A - v_B| (2\pi)^2 4E_{cm}} = \frac{|\vec{k}|}{16(2\pi)^2 E|\vec{k}|\left|\frac{\vec{k}}{k} + \frac{\vec{k}}{E}\right|(E+k)} = \frac{1}{16(2\pi)^2 E\left|1+\frac{k}{E}\right|(E+k)} = \frac{1}{64\pi^2 E_{cm}^2}$$

And if $E \gg m_\mu$ we obtain

$$\left(\frac{d\sigma}{d\Omega}\right)_{cm} = \frac{\alpha^2}{2E_{cm}^2 (1-\cos\theta)^2} \left(4 + (1+\cos\theta)^2\right)$$

For small angles

$$\left(\frac{d\sigma}{d\Omega}\right)_{cm} \propto \frac{1}{\theta^4}$$

$\Rightarrow$ this diverges for back-to-back scattering - this is a feature of Coulomb scattering and arises from the photon going "onshell"

The trick we used to replace the momenta, rather than doing all the trace algebra again, is our first example of crossing symmetry:

amplitude for process with particle of momentum p in initial state	$=$	amplitude for process with antiparticle of momentum $-p$ in final state

↑ amplitude = S-matrix element

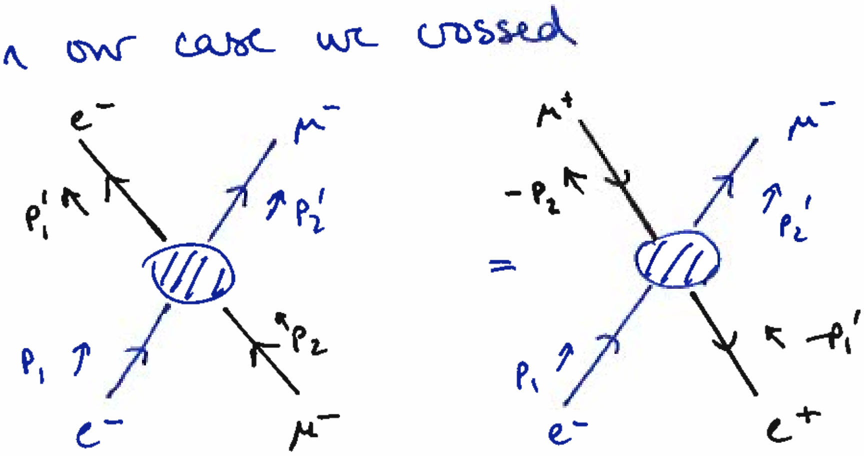
↑ strictly this means "analytic continuation of"

Schematically, we can write this as

$$i\mathcal{M}(\phi(p)\, X \to X') = i\mathcal{M}(X \to \bar{\phi}(-p)\, X')$$

We do, however, have to be careful when the external particles have spin, but we don't need the details here. See P+S p. 155 for the details.

In our case we crossed



Crossing symmetry is particularly manifest with the Mandelstam variables

$$s = (p+p')^2 \qquad t = (k-p)^2 \qquad u = (k'-p)^2$$
$$[= (k+k')^2] \qquad [= (k'-p')^2] \qquad [= (k-p')^2]$$

For equal masses, we have

$$s = (p+p')^2 = (2E)^2 = E_{cm}^2$$

$$t = (k-p)^2 = -2|\bar{p}|^2(1-\cos\theta)$$

$$u = (k'-p)^2 = -2|\bar{p}|^2(1+\cos\theta)$$

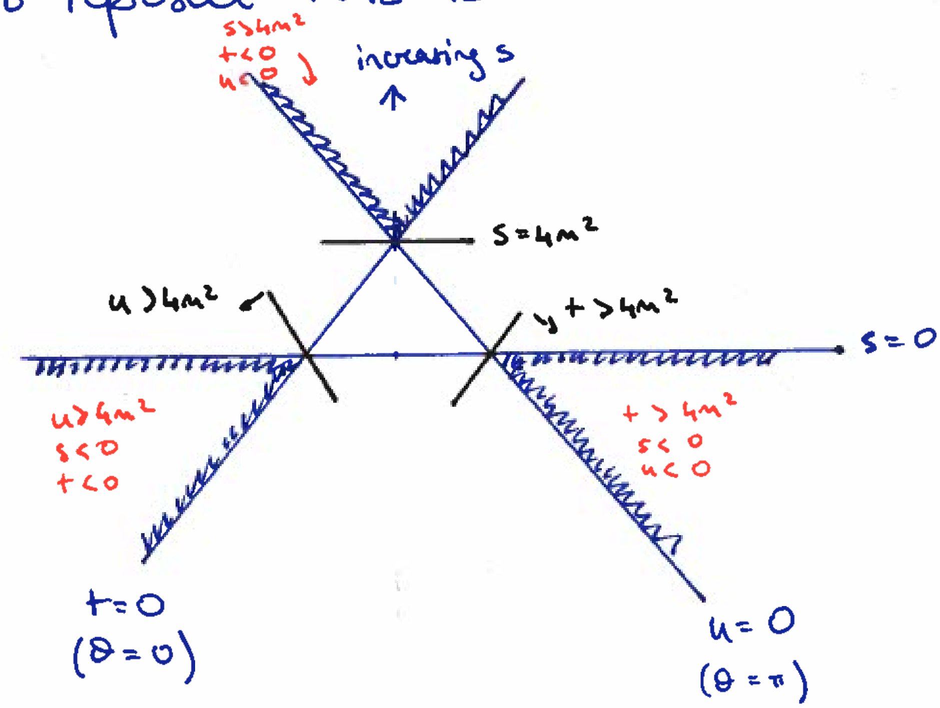
For physical scattering $s>0,\ t<0$

There are only two independent variables, since

$$s+t+u = 4m^2$$

We have traded (E_{cm}, θ) for (s,t), which are Lorentz invariants.

One way to represent this is the Mandelstam plane



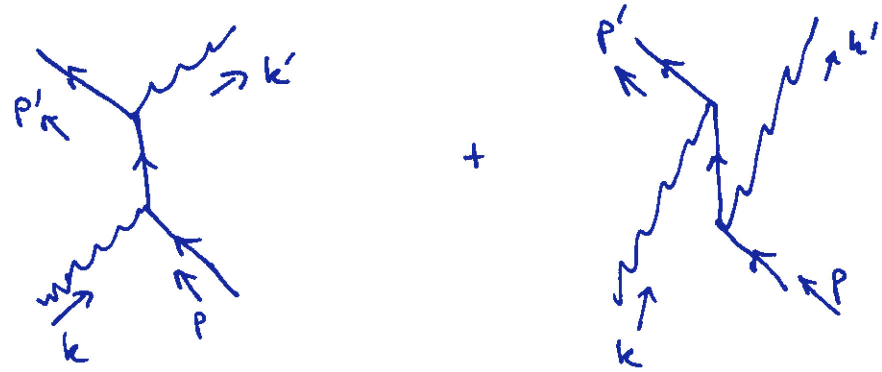
Our scattering examples in QED have focussed on processes with external fermions, but one of the processes we wrote down at the beginning of this section was Compton scattering, which involves external photons.

So let's see what complications they bring.

Compton scattering $\boxed{\text{P+S } 5.5}$

Recall this is $e^{\pm}\gamma \to e^{\pm}\gamma$

$\boxed{1.}$



(a) (b)

$\boxed{2.}$ $iM = iM_a + iM_b$

where

$$iM_a = (-ig)^2 \epsilon^*_\mu(\bar{k}', \lambda') \epsilon_\nu(\bar{k}, \lambda) \bar{u}^{s'}(\bar{p}')\gamma^\mu \frac{i(\not{p} + \not{k} + m)}{(p+k)^2 - m^2 + i\epsilon} \gamma^\nu u^s(\bar{p})$$

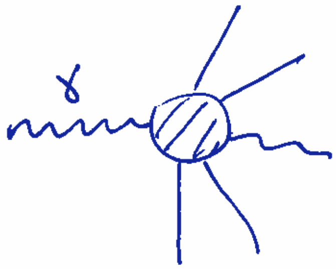
$$iM_b = (-ig)^2 \epsilon^*_\mu(\bar{k}', \lambda') \epsilon_\nu(\bar{k}, \lambda) \bar{u}^{s'}(\bar{p}')\gamma^\nu \frac{i(\not{p} - \not{k}' + m)}{(p-k')^2 - m^2 + i\epsilon} \gamma^\mu u^s(\bar{p})$$

It's clear that when we calculate $|M|^2$ we will need to deal with sums and squares over polarisation vectors ...

Before we figure out what to do with these, we first consider a seeming tangent – the "<u>Ward Identity</u>"

like the Bourne Identity but more physics-y and exciting

Consider a diagram with an external photon



this must have a part where the photon couples to a fermion

1. If fermion is external $\Rightarrow$ $\propto \epsilon_\mu(\bar{k},\lambda)\,\bar{u}(\bar{p}')\,\gamma^\mu u(\bar{p})$

2. If fermion is internal $\Rightarrow$ $\propto \epsilon_\mu(\bar{k},\lambda)\,(\not{p}'+m)\gamma^\mu(\not{p}+m)$

$= \sum_s u^s(\bar{p})\bar{u}^s(\bar{p})$

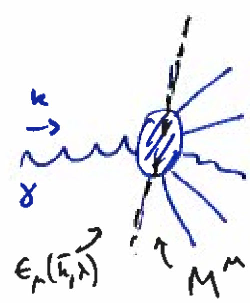
still get $\epsilon_\mu(\bar{k},\lambda)\,\bar{u}'\gamma^\mu u$

Now momentum conservation means $k^\mu = p'^\mu - p^\mu$

So replacing ϵ_μ by k_μ gives

$$\begin{aligned} k_\mu \bar{u}(\bar{p}')\gamma^\mu u(\bar{p}) &= (p'-p)_\mu \bar{u}(\bar{p}')\gamma^\mu u(\bar{p}) \\ &= \bar{u}(\bar{p}')(\not{p}'-\not{p})u(\bar{p}) \\ &= 0 \quad ! \end{aligned}$$

This is, in fact, true in general



$$\boxed{\text{If } \; M = \epsilon_\mu(\bar{k},\lambda)M^\mu \quad \text{then} \quad k_\mu M^\mu = 0}$$

field theoretic expression of <u>current conservation</u> implied by <u>gauge invariance</u>

the Ward Identity

I think $\partial_\mu j^\mu = 0$ in FT

The Ward identity helps us understand $\sum_\lambda \epsilon_\mu(\bar{k},\lambda)\epsilon^*_\nu(\bar{k},\lambda)$

For external (on-shell) photons, there are only two polarisations

eg, choose $k^\mu = (k,0,0,k)$ $\quad k = |\bar{k}|$

so then $\epsilon^\mu(\bar{k},\lambda=\pm 1) = \mp\frac{1}{\sqrt{2}}(0,1,\pm i,0)$

recall this appeared in our earlier scattering example

We have

$$\sum_\lambda \epsilon^\mu(\bar{k},\lambda)\epsilon^{\nu *}(\bar{k},\lambda) M_\mu M^*_\nu = |M^1|^2 + |M^2|^2$$

But recall

$$k_\mu M^\mu = 0 \Rightarrow k^0 M^0 - k^3 M^3 = 0 \Rightarrow k M^0 = k M^3 \Rightarrow M^3 = M^0$$

Then we can add this to the right hand side

$$\sum_\lambda \epsilon^\mu(\bar{k},\lambda)\epsilon^{\nu *}(\bar{k},\lambda) M_\mu M^*_\nu = |M^1|^2 + |M^2|^2 + |M^3|^2 - |M^0|^2$$
$$= -g^{\mu\nu} M_\mu M^*_\nu$$

So we can use

$$\boxed{\sum_\lambda \epsilon^\mu(\bar{k},\lambda)\epsilon^{\nu *}(\bar{k},\lambda) \rightarrow -g^{\mu\nu}}$$

N.B. Provided we contract this sum with the rest of an invariant matrix element in QED, or other gauge invariant theory

Now let's see how we can use this in our expression for the Compton amplitude

Let's now take our contributions in turn

$$iM_a = \frac{-ig^2}{2p\cdot k} \epsilon^*_\mu(\bar{k}',\lambda')\epsilon_\nu(\bar{k},\lambda)\,\bar{u}^{s'}(\bar{p}')\gamma^\mu(\not{p}+\not{k}+m)\gamma^\nu u^s(\bar{p})$$

$\downarrow A^* = (A^T)^+$

$$\Rightarrow (iM_a)^* = \frac{ig^2}{2p\cdot k}\epsilon_\rho(\bar{k}',\lambda')\epsilon^*_\sigma(\bar{k},\lambda)\, u^{s+}(\bar{p})\gamma^{\sigma+}(\not{p}+\not{k}+m)^+\gamma^{\rho+}\gamma^{0+}u^{s'}(\bar{p}')$$

$$\underbrace{u^{s+}\gamma^0}_{\bar{u}^s}\underbrace{\gamma^0\gamma^{\sigma+}\gamma^0}_{\gamma^\sigma}\underbrace{\gamma^0(\not{p}+\not{k}+m)^+\gamma^0}_{\not{p}+\not{k}+m}\underbrace{\gamma^0\gamma^{\rho+}\gamma^0}_{\gamma^\rho}u^{s'}$$

$$= \frac{ig^2}{2p\cdot k}\epsilon_\rho(\bar{k}',\lambda')\epsilon^*_\sigma(\bar{k},\lambda)\,\bar{u}^s(\bar{p})\gamma^\sigma(\not{p}+\not{k}+m)\gamma^\rho u^{s'}(\bar{p}')$$

Then

$$\overline{|M_a|^2} = \frac{1}{4}\sum_{\substack{s,s'\\ \lambda,\lambda'}}|M_a|^2 \qquad -g_{\nu\sigma} \qquad -g_{\rho\mu}$$

$$= \frac{1}{4}\left(\frac{g^2}{2p\cdot k}\right)^2\left(\sum_\lambda \epsilon_\nu\epsilon^*_\sigma\right)\left(\sum_{\lambda'}\epsilon'_\rho\epsilon'^*_\mu\right)$$

$$\times \mathrm{Tr}\Big[\underbrace{\Big(\sum_s u^s\bar{u}^s\Big)}_{\not{p}+m}\gamma^\sigma(\not{p}+\not{k}+m)\gamma^\rho\underbrace{\Big(\sum_{s'}u^{s'}\bar{u}^{s'}\Big)}_{\not{p}'+m}\gamma^\mu(\not{p}+\not{k}+m)\gamma^\nu$$

$$= \frac{g^4}{16(p\cdot k)^2}\mathrm{Tr}\left[(\not{p}+m)\gamma^\nu(\not{p}+\not{k}+m)\gamma^\mu(\not{p}'+m)\gamma_\mu(\not{p}+\not{k}+m)\gamma_\nu\right]$$

To simplify this trace we note

$$\gamma^\mu(\not{p}'+m)\gamma_\mu = -2(\not{p}'-2m)$$

$$\gamma_\nu(\not{p}+m)\gamma^\nu = -2(\not{p}-2m) \quad \leftarrow \text{used cyclic property of trace to get LHS}$$

$$\Rightarrow \mathrm{Tr}[\;] = 4\,\mathrm{Tr}\big[(\not{p}-2m)(\underbrace{\not{p}+\not{k}}_{\not{a}}+m)(\not{p}'-2m)(\not{p}+\not{k}+m)\big]$$

$$= 4\,\mathrm{Tr}\big[(\not{p}\not{a}-2m\not{a}+m\not{p}-2m^2)(\not{p}'\not{a}-2m\not{a}+m\not{p}'-2m^2)\big]$$

$$= 4\,\mathrm{Tr}\big[\not{p}\not{a}\not{p}'\not{a}-2m^2\not{p}'\not{a}+4m^2a^2-2m^2\not{p}\not{a}-2m^2\not{a}\not{p}'+m^2\not{p}\not{p}'-2m\not{p}\not{a}+4m^4\big]$$

$$= 4\Big\{4\left(a\cdot p\, a\cdot p' - p\cdot p' a^2 + a\cdot p\, a\cdot p'\right) - 8m^2 a\cdot p' + 16m^2 a^2$$
$$- 8m^2 a\cdot p - 8m^2 a\cdot p' + 4m^2 p\cdot p' - 8m^2 a\cdot p + 16m^4\Big\}$$
$$= 16\left\{2a\cdot p\, a\cdot p' - p\cdot p' a^2 - 4m^2 a\cdot p - 4m^2 a\cdot p' + 4m^2 a^2 + m^2 p\cdot p' + 4m^4\right\}$$
$$= 16\left\{2a\cdot p\, a\cdot p' - p\cdot p' a^2 + 4m^2\left[a^2 + m^2 - a\cdot p - a\cdot p'\right] + m^2 p\cdot p'\right\}$$

We can simplify these expressions by using the kinematics

$$p^2 = p'^2 = m^2$$
$$k^2 = k'^2 = 0 \quad \leftarrow \text{massless photons}$$
$$p + k = p' + k' \quad \leftarrow \text{momentum conservation}$$
$$\Rightarrow (p+k)^2 = (p'+k')^2 \quad \text{or} \quad (p-p')^2 = (k-k')^2$$
$$m^2 - p\cdot p' = -k\cdot k'$$

To find $k\cdot k'$ we use

$$p\cdot(p + k - k') = m^2 + p\cdot k - p\cdot k' \quad [= p\cdot p']$$
$$\Rightarrow k\cdot k' = p\cdot p' - m^2 = p\cdot k - p\cdot k'$$

Then we need

$$a\cdot p = (p+k)\cdot p = m^2 + p\cdot k$$
$$a\cdot p' = (p+k)\cdot p' = (p+k)\cdot(p+k-k') = m^2 + 2p\cdot k - k\cdot k'$$
$$= m^2 + p\cdot k + p\cdot k'$$
$$a\cdot p + a\cdot p' = 2m^2 + 2p\cdot k + p\cdot k'$$
$$a^2 = (p+k)^2 = m^2 + 2p\cdot k$$
$$p\cdot p' = m^2 + p\cdot k - p\cdot k'$$

Plugging these in we obtain

$$\mathrm{Tr}[\,] = 32\left(m^4 + m^2 p\cdot k + p\cdot k\, p\cdot k'\right)$$

$$\Rightarrow \overline{|\mathcal{M}_a|^2} = 2g^4\left(\left(\frac{m^2}{p\cdot k}\right)^2 + \frac{m^2}{p\cdot k} + \frac{p\cdot k'}{p\cdot k}\right)$$

Now, the whole matrix element is $i\mathcal{M} = i\mathcal{M}_a + i\mathcal{M}_b$

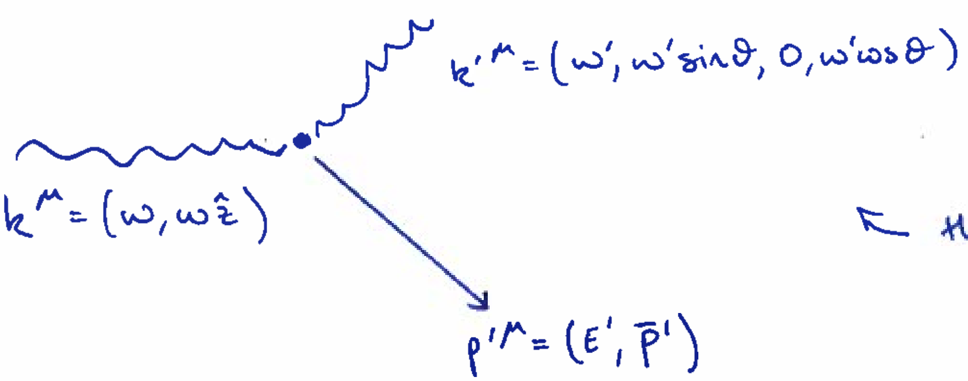
$$\Rightarrow |\mathcal{M}|^2 = (i\mathcal{M}_a + i\mathcal{M}_b)(i\mathcal{M}_a + i\mathcal{M}_b)^*$$
$$= |\mathcal{M}_a|^2 + |\mathcal{M}_b|^2 + 2\,\mathrm{Re}(\mathcal{M}_a\mathcal{M}_b)$$

Repeating this analysis for these second and third terms gives

$$\overline{|\mathcal{M}|^2} = \frac{1}{4}\sum_{ss'}\sum_{\lambda\lambda'}|\mathcal{M}|^2$$
$$= 2g^4\left[\frac{p\cdot k}{p\cdot k'} + \frac{p\cdot k'}{p\cdot k} + 2m^2\left(\frac{1}{p\cdot k} - \frac{1}{p\cdot k'}\right) + m^4\left(\frac{1}{p\cdot k} - \frac{1}{p\cdot k'}\right)^2\right]$$

for more details, but organised slightly differently, see P+S 5.5

$\boxed{4.}$ In contrast to other processes that we've studied, a good frame to study this process in is one in which the initial electron is at rest.



then $p\cdot k = m\omega$
$p\cdot k' = m\omega'$

To solve for ω', we can use brute force

First we note cons. of mom:

$$\omega + m = \omega' + E'$$
$$p_x' + \omega' \sin\theta = 0$$
$$p_y' = 0$$
$$p_z + \omega' \cos\theta = \omega$$

$$E'^2 = |\bar{p}'|^2 + m^2$$
$$= \omega'^2 \sin^2\theta + (\omega - \omega'\cos\theta)^2 + m^2$$
$$= \omega^2 + \omega'^2 - 2\omega\omega'\cos\theta + m^2$$

and

$$E'^2 = (\omega + m - \omega')^2$$
$$= \omega'^2 + (\omega+m)^2 - 2\omega'(\omega+m)$$

So equating these gives

$$-2\omega\omega' - 2\omega' m + 2\omega m = -2\omega\omega'\cos\theta$$
$$2\omega m = 2\omega'(\omega + m - \omega\cos\theta)$$
$$\Rightarrow \omega' = \frac{\omega}{1 + \frac{\omega}{m}(1-\cos\theta)}$$

Or we can use the trick from P+S p. 162

$$m^2 = (p')^2 = (p + k - k')^2 = p^2 + 2p\cdot(k-k') - 2k\cdot k'$$
$$= m^2 + 2m(\omega - \omega') - 2\omega\omega'(1-\cos\theta)$$

$$\Rightarrow \frac{1}{\omega'} - \frac{1}{\omega} = \frac{1}{m}(1-\cos\theta)$$

← Compton's formula for the shift in photon wavelength

$\boxed{5.}$ Finally we want to find the cross-section

$$d\sigma = \frac{1}{2E_A 2E_B |v_B - v_A|}\left(\prod_f \frac{d^3\bar{p}_f}{(2\pi)^3}\frac{1}{2E_f}\right)|\mathcal{M}(p_A, p_B \to \{p_f\})|^2 (2\pi)^4 \delta^{(4)}(p_A + p_B - \sum_f p_f)$$

↑ Eq 4.79 P+S

We write this as

$$d\sigma = \frac{1}{2\omega\, 2m\, |1-0|} \int \frac{d^3\bar{k}'}{(2\pi)^3} \int \frac{d^3\bar{p}'}{(2\pi)^3} \frac{1}{2\omega'} \frac{1}{2E'} (2\pi)^3 \delta^{(3)}(\underset{\uparrow\, 0}{\bar{p}} + \underset{\uparrow\, \omega\hat{z}}{\bar{k}} - \bar{p}' - \bar{k}')$$

$$\times (2\pi)\,\delta(m+\omega-\omega'-E') \qquad \Rightarrow \bar{p}' = \omega\hat{z}' - \bar{k}'$$

$$\times \left| M(p_A, p_B \to \{p_f\}) \right|^2$$

$$= \frac{1}{4m\omega} \int \frac{\omega'^2 d\omega'}{(2\pi)^3} \int d\Omega\, |M|^2 (2\pi)\, \delta\left(m+\omega-\omega' - \sqrt{m^2+\omega^2-\omega'^2-2\omega\omega'\cos\theta_k}\right)$$

$$= \frac{1}{16m\omega} \frac{(2\pi)}{(2\pi)^3} \int_0^{2\pi} d\phi \int_{-1}^{1} d(\cos\theta)\, |M|^2 \int \frac{d\omega'\, \omega'^2}{\omega' E'} \delta(f(\omega'))$$

To evaluate the $\delta(f(\omega'))$ we use

$$\delta(f(\omega')) = \frac{1}{\left|\frac{\partial f}{\partial \omega'}\right|_{\omega'=0}} \delta(\omega' - \omega_0') \qquad \text{where } \omega_0' = \frac{\omega}{1+\frac{\omega}{m}(1-\cos\theta)} \text{ is solution of } f(\omega')=0$$

$$= \frac{1}{1 + \frac{\omega' - \omega\cos\theta}{E'}}$$

$$\frac{d}{d\omega'} f(\omega') = \frac{d}{d\omega'}\left(m+\omega-\omega' - \sqrt{m^2+\omega^2-\omega'^2-2\omega\omega'\cos\theta}\right)$$

$$= -1 - \frac{1}{2}\frac{2\omega' - 2\omega\cos\theta}{\sqrt{m^2+\omega^2-\omega'^2-2\omega\omega'\cos\theta}}$$

$$= -\left(1 + \frac{\omega' - \omega\cos\theta}{E'}\right)$$

So

$$d\sigma = \frac{1}{16m\omega} \frac{1}{2\pi} \int_{-1}^{1} d(\cos\theta) \frac{\omega'}{E'} \frac{1}{1+\frac{\omega'-\omega\cos\theta}{E'}}$$

$$\times 2g^4 \Big[\underbrace{\frac{m\omega}{m\omega'} + \frac{m\omega'}{m\omega} + 2m^2\left(\frac{1}{m\omega} - \frac{1}{m\omega'}\right) + m^4\left(\frac{1}{m\omega} - \frac{1}{m\omega'}\right)^2}_{\frac{\omega'}{\omega} + \frac{\omega}{\omega'} - \sin^2\theta} \Big]$$

$$\omega' = \frac{\omega}{1+\frac{\omega}{m}(1-\cos\theta)}$$

We end up with

$$\frac{d\sigma}{d\cos\vartheta} = \frac{\pi\alpha^2}{m^2}\left(\frac{\omega'}{\omega}\right)^2\left[\frac{\omega'}{\omega}+\frac{\omega}{\omega'}-\sin^2\vartheta\right]$$

the spin-averaged "Klein-Nishina formula"

Limits

- $\omega \ll m$; $\omega' \simeq \omega$ ← low energy / long wavelength limit

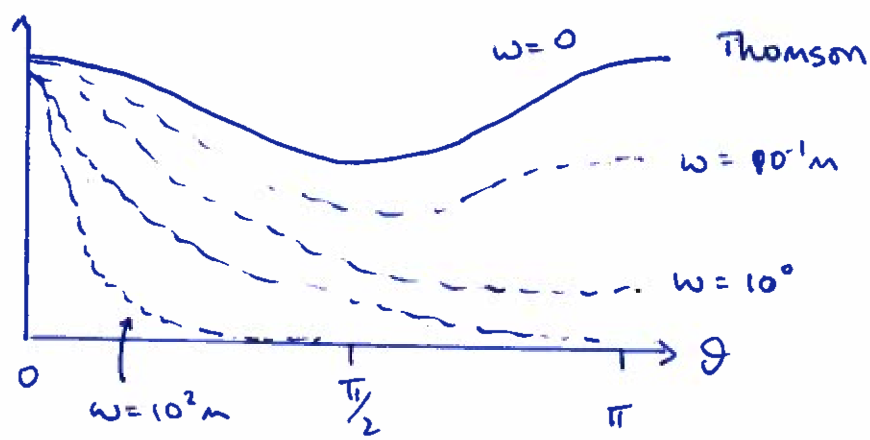
$$\frac{d\sigma}{d\cos\vartheta} \longrightarrow \frac{\pi\alpha^2}{m^2}\left(1+\cos^2\vartheta\right)$$

"Thomson scattering"
classical photon-electron scattering cross-section

- $\omega \gg m$

very forward peaked

See P+S p. 164-167



Note

We can obtain pair production of photons by crossing symmetry

See P+S p. 168-169