Four-vector notation - a reminder

a tangent vector

Vectors : $x^{\mu} = (x^0, x^1, x^2, x^3)$

Co-vectors : $x_{\mu} = (x_0, x_1, x_2, x_3) = (x_0, -x^1, -x^2, -x^3)$

"Lorentz index"

a one-form

related via metric tensor $g^{\mu\nu}$

generally I use convention $g^{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$ Minkowski spacetime

$x^{\mu} = g^{\mu\nu} x_{\nu}$

"Einstein summation convention"

- $x^{\alpha} y_{\alpha} = \sum_{\alpha=0}^{3} x^{\alpha} y_{\alpha}$
- repeated indices are summed over
- repeated indices must come in upstairs-downstairs pairs
- free indices must match in a valid equation

All this goes out of the window in Euclidean spacetime ü

$x_{\alpha} y_{\alpha}$ is not allowed →

Dot product : $x^{\mu} x_{\mu} = x^{\mu} x^{\nu} g_{\mu\nu}$

$= (x^0)^2 - (\bar{x})^2$ ← $\bar{x} = (x^1, x^2, x^3)$ (spatial) three-vector

$= x^2$

Lorentz invariant

Derivatives : $\partial^{\mu} \equiv \frac{\partial}{\partial x_{\mu}}$ and $\partial_{\mu} \equiv \frac{\partial}{\partial x^{\mu}}$

Note indices!

$\boxed{\text{Q: what is } \partial^{\mu} x_{\nu} \text{ ?}}$

$$\Box = \nabla^2 = \partial^2 \equiv \partial^{\mu} \partial_{\mu}$$

distinguished from $\bar{\nabla}^2$!

Useful four-vectors

From relativity:

- spacetime $x^\mu = (x^0, x^1, x^2, x^3)$
- momentum $p^\mu = (E, \bar{p}) = (E, p^1, p^2, p^3)$

$$p^2 = p^\mu p_\mu$$
$$= E^2 - \bar{p}^2 = m_0^2$$

- velocity $p^\mu = m_0 u^\mu$

$$u^2 = 1$$

From electromagnetism

- vector potential $A^\mu = (\phi, \bar{A})$
- field strength tensor $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$

antisymmetric in $\mu \leftrightarrow \nu$

$$= \begin{pmatrix} 0 & -E^x & -E^y & -E^z \\ E^x & 0 & -B^z & B^y \\ E^y & B^z & 0 & -B^x \\ E^z & -B^y & B^x & 0 \end{pmatrix}$$