Quantum Field Theory I: PHYS 721
Problem Set 7

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The questions in this problem set introduce you to the Lorentz-invariant "Mandelstam variables" for two-to-two scattering.

Question 1

Consider a scattering process $A(p_1)A(p_2) \to A(q_1)A(q_2)$, involving only a single species of particle, A, with mass, m.

The entire process can be considered to occur in a plane, so we only need one angle and one energy variable to describe the process. It is simplest to use the centre-of-momentum frame, in which $\vec{p}_1 + \vec{p}_2 = \vec{q}_1 + \vec{q}_2 = 0$. Define the direction of $\vec{p}_1$ to be the z-axis and the direction of $\vec{q}_1$ to be at an angle θ_{CM} with respect to the z-axis.

We can define three Lorentz-invariant kinematic variables, which are given by

$$s \equiv (p_1 + p_2)^2, \qquad t \equiv (q_1 - p_1)^2, \qquad \text{and} \qquad u \equiv (q_2 - p_1)^2.$$

These are the Lorentz invariant Mandelstam variables.

(a) Show that these variables are related by the condition

$$s + t + u = 4m^2,$$

so that there are only two independent variables.

(b) Show that these Mandelstam variables can be related to the centre-of-momentum frame variables by

$$s = E_{\text{CM}}^2,$$
$$t = 2|\vec{p}_{\text{CM}}|^2(\cos\theta_{\text{CM}} - 1),$$
$$u = -2|\vec{p}_{\text{CM}}|^2(1 + \cos\theta_{\text{CM}}),$$

where

$$|\vec{p}_{\text{CM}}| = \sqrt{\frac{E_{\text{CM}}^2}{4} - m^2}$$

is the momentum of any of the particles in the centre-of-momentum frame.

(c) What are the ranges of the Mandelstam variables for physical scattering?

(d) How do you think the relation between the Mandelstam variables changes if the particles have different masses?

Question 2

Recall the scalar Yukawa theory of Homework 5:

$$\mathcal{L} = \partial_\mu \psi^* \partial^\mu \psi - M^2 \psi^* \psi + \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \frac{1}{2} m^2 \phi^2 - g \psi^* \psi \phi.$$

Here ψ is a complex scalar field and ϕ is a real scalar field.

Draw the leading nontrivial (i.e. do not count the case of no scattering) Feynman diagrams for the following processes, and express the invariant matrix element for each process in terms of Mandelstam variables:

1. Nucleon-nucleon scattering, $\psi\psi \to \psi\psi$;
2. Nucleon-antinucleon scattering, $\psi\psi^* \to \psi\psi^*$;
3. Meson pair production, $\psi\psi^* \to \phi\phi$;
4. Nucleon-meson scattering, $\psi\phi \to \psi\phi$.