Quantum Field Theory I: PHYS 721
Problem Set 5

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Overview

This homework is due on **Wednesday October 7**.

You must submit your homework as a PDF, using LaTeX. You should include:

- Title;
- Author information;
- Abstract;
- Introduction;
- And Conclusion;

as well as ensuring you have answered all the questions. This homework will be graded out of 40 points. Ten of those points are for clarity of presentation. You should use complete sentences throughout. This mini-project will: give you greater familiarity with LaTeX; help you solidify your understanding of scattering in scalar field theory; and give you practice communicating your work to professional peers.

Your abstract should be three to five sentences, explaining what you have done, how, and what your central result is.

Your introduction will consist of your solution to the first part of the homework (i.e. part (a)).

Your conclusion should summarise what you've done, how, and what your central result was. It is standard to include a suggestion for future work in your conclusion, so you should incorporate your answer to the final part of the homework (i.e. part (g)) as part of your conclusion.

You can submit your write-up on Wednesday September 30 for comments on your presentation, and the abstract, introduction, and conclusion. I will not comment on the actual mathematical solutions you hand in, if any, at that stage.

There are some very nice overviews of how to write well for a physics audience here:

- https://www.southwestern.edu/live/files/4179-guide-for-writing-in-physicspdf
- https://cdn.journals.aps.org/files/rmpguapa.pdf
- https://www.geneseo.edu/ mclean/Dept/JournalArticle.pdf

and you can find many others online. The second of these has useful hints for speakers whose first language is not English. The last reference aims to familiarise you with how to write a scientific article, but also has some tips for writing well. Perhaps the most important points are

1. Use the active voice where possible.
2. Be succinct, but do not omit relevant details.
3. Write so that the reader has an easy time, not you. For example, try not to use abbreviations if you can avoid them.
4. Be concrete where possible.
5. Try not to start a sentence with a mathematical symbol.
6. Punctuate mathematical equations as if they were phrases or other parts of a sentence.
7. Practise, practise, practise!

Mini-project

(a) Write an overview of scattering in (real) scalar field theory. Your summary should address the main steps that are required to go from the Lagrangian for a (real) scalar field to experimental observables. You should motivate each of these steps. As an example, part of your introduction should address why we use the interaction picture to study interacting theories. This overview will form the introduction to your write up.

This introduction should balance completeness with brevity. In other words, you should include enough detail to show you understand the complete picture, but not so much detail that the reader is swamped. You may include a diagram, flow chart, or other pictorial representation, if you choose, but you should also summarise that diagram (or flow chart or other pictorial representation) in words as well.

(b) Study Section 3.2 of David Tong's notes "A First Look at Scattering" (you can find them here (PDF): http://www.damtp.cam.ac.uk/user/tong/qft/qft.pdf). This section address scattering at leading order ('tree-level') in scalar Yukawa theory, which is defined by the Lagrangian density

$$\mathcal{L} = \partial_\mu \psi^* \partial^\mu \psi - M^2 \psi^* \psi + \frac{1}{2}\partial_\mu \phi \partial^\mu \phi - \frac{1}{2} m^2 \phi^2 - g\psi^* \psi \phi,$$

where ψ is a complex scalar field and ϕ is a real scalar field. We will assume that $g \ll M, m$. This is equation 3.7 of the notes. The fields ψ and ϕ are scalar analogues of nucleons and mesons (such as pions), respectively.

Decompose this Hamiltonian into a free Hamiltonian, H_0, and an interaction Hamiltonian, H_I. Write down:

1. explicit expressions for both H_0 and H_I;
2. expressions for the fields in terms of creation and annihilation operators;
3. the Feynman rules in position space.

(c) Now consider the example of $\psi\psi \to \psi\psi$ scattering discussed in Section 3.3.3 of David Tong's notes. This is the scalar analogue of nucleon-nucleon scattering. Write down an expression for the time-ordered operator that represents this process and use Wick's theorem to represent this operator in terms of normal-ordered products and contracted fields, up to and including $\mathcal{O}(g^4)$.

(d) Using the logic of Section 3.3.3, derive an expression for the scattering amplitude for $\psi\psi \to \psi\psi$ scattering up to $\mathcal{O}(g^4)$. You do not need to evaluate the remaining momentum integrals over the undetermined "loop momenta".

(e) Read the first example in Section 3.5 of David Tong's notes. Draw all fully connected Feynman diagrams that contribute to this process (up to and including $\mathcal{O}(g^4)$) and identify which terms that appeared in part (c) correspond to which Feynman diagrams. You do not need to derive all symmetry factors from first principles.

(f) Use your position space Feynman rules to write down an expression for each diagram. Comment on how the diagrams relate to the expression you derived in part (d).

(g) What aspects of this calculation do you think will change in a more realistic model, in which the "nucleons" are treated as fermions (but the mesons are still scalars)? Incorporate your answer as part of your conclusion, in the form of a suggestion for future avenues of research that includes a description of what parts of the calculation will be simple extensions and what might change more drastically or require considerable work.