Quantum Field Theory I: PHYS 721
Problem Set 4

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Overview

The questions in this problem set first reinforce the close relationship between symmetries and conservation laws with a flashback to the free theory, and then familiarise you with deriving Feynman rules for a new example of an interacting theory.

Question 1

(a) Use the expressions for the Hamiltonian and the momentum operator

$$H_0 = \int \frac{\mathrm{d}^3\vec{k}}{(2\pi)^3} E_k a^\dagger(\vec{k})a(\vec{k}),$$
$$P^i = \int \frac{\mathrm{d}^3\vec{k}}{(2\pi)^3} k^i a^\dagger(\vec{k})a(\vec{k})$$

for a free, real scalar field, $\phi(x)$, to show that the four-vector (H, P^i) generates spacetime translations

$$\phi(x) = e^{i(Ht-\vec{P}\cdot\vec{x})}\phi(0)e^{-i(Ht-\vec{P}\cdot\vec{x})}.$$

This is another example of the phenomenon we have seen previously: the conserved charge due to a symmetry generates the corresponding symmetry transformation on the fields.

[Hint: You will need to consider expressions of the form $e^{iH_0t}a(\vec{k})e^{-iH_0t}$.]

(b) Use this result to show that

$$\langle 0|\phi(x)\phi(y)|0\rangle = \langle 0|\phi(x-y)\phi(0)|0\rangle.$$

Question 2

In our mini-lectures we derived the Feynman rules for $\lambda\phi^4$ theory, which ended up being pretty simple. In fact, it is usually possible to just read off the Feynman rules directly from the Lagrangian.

Consider the following Lagrangian,

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2 - \frac{\lambda}{3!}\phi^3,$$

which defines the so-called $\lambda\phi^3$ theory (very imaginatively named, as you can see).

(a) What is the mass dimension of the coupling λ in four spacetime dimensions? Is this coupling relevant, marginal, or irrelevant in four spacetime dimensions? In how many spacetime dimensions is λ dimensionless?

(b) Write out what you expect the relevant position-space Feynman rules to be. You do not need to provide a detailed derivation via Wick's theorem.

(c) Draw all the connected diagrams contributing to the:

(i) The two-point function

$$\langle 0|T\{\phi(x)\phi(y)\}|0\rangle$$

up to order λ^4.

(ii) The three-point function

$$\langle 0|T\{\phi(x)\phi(y)\phi(z)\}|0\rangle$$

up to order λ^2.

(iii) The four-point function

$$\langle 0|T\{\phi(w)\phi(x)\phi(y)\phi(z)\}|0\rangle$$

up to order λ^2.