# Scattering and interacting theories

## Cross-sections and decay rates

Schwartz 5.1

So far we have been looking at the basic building blocks of quantum field theory - operator-valued functions of spacetime that transform under irreducible representations of the Lorentz group. Now we want to connect this to physics - things we can measure in (very large) labs.

To do this, we will relate $\underbrace{\text{cross-sections and decay rates}}^{\text{experimental measurables}}$ to S-matrix elements via the LSZ reduction

S-matrix elements ← calculable in QFT

LSZ reduction ← a math thing

Let's start by looking at the experimental end of things.

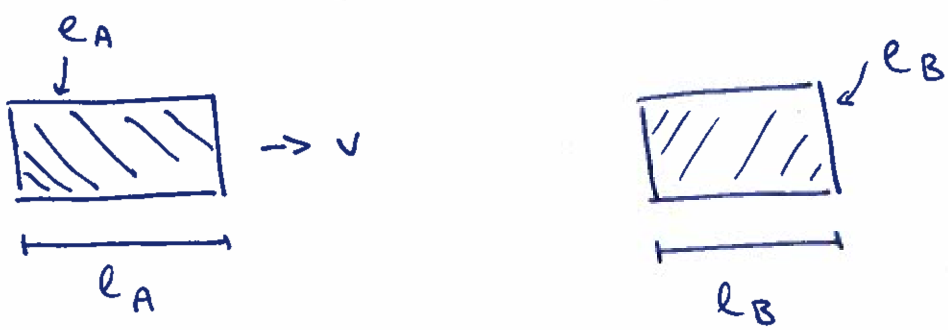
Consider two beams of particles, or, equivalently, a beam and a target. Let the target contain particles of type A with density $\rho_A$ and the beam contain particles of type B with density $\rho_B$ ← $\rho_i \equiv \frac{\#\text{ of particles of type } i}{\text{volume}}$

We will assume that the particles are in bunches of length $\ell_A$ and $\ell_B$ and that they have a cross-sectional area $A$.

↑ Awkward notation, but different from particle type A!

Then the cross-section is defined as

$$\boxed{\sigma \equiv \frac{\text{Number of scattering events}}{\rho_A \ell_A \, \rho_B \ell_B \, A}}$$




Note:

- definition is symmetric between A and B
- $\sigma$ has units of area (so $\hbar = c = 1 \Rightarrow [\sigma] = M^{-2}$)
- usually $\rho_A$ and $\rho_B$ are not constant

  $\Rightarrow$ # of events $= \sigma \, \ell_A \ell_B \int d^2\bar{x} \; \rho_A(\bar{x}) \rho_B(\bar{x})$

  but we often approximate them as constant

  # of events $= \dfrac{\sigma N_A N_B}{A}$ ← $N_A, N_B$ = total # of each type

- it is usual to introduce the differential cross-section

  $\dfrac{d\sigma}{d^3\bar{p}_1 \cdots d^3\bar{p}_n}$, which is the quantity that "when integrated over any small $d^3\bar{p}_1 \cdots d^3\bar{p}_n$, gives the cross-section for scattering into that region of final-state momentum space"

  these are not independent! remember 4-momentum conservation

We can also introduce the decay rate

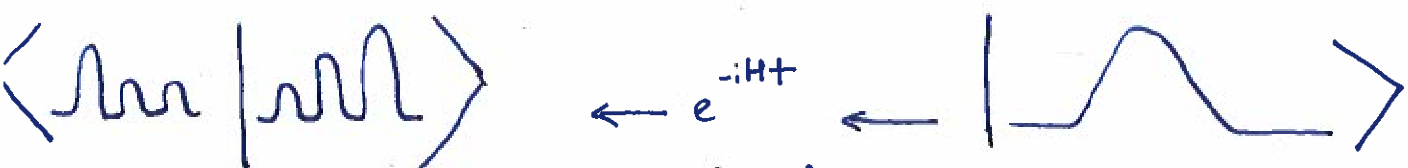
$$\boxed{\Gamma \equiv \frac{\text{number of decays per unit time}}{\text{number of particles present}}}$$

Note:

- particle lifetime $\tau = \frac{1}{\Gamma}$ $\left(\text{or } \tau = \frac{1}{\sum_i \Gamma_i} \text{ for multiple channels}\right)$
- particle half-life $= \tau \ln 2$
  
  ↑ not widely used in particle physics

## S-matrix

The intuitive picture we have of experimental scattering processes is the following




3. calculate overlap of resulting state with wavepackets representing desired final states

$_{out}\langle p_1 \cdots p_n | k_A k_B \rangle_{in}$

2. evolve forward in time via unitary evolution operator

1. set up wavepacket(s) in distant past

$|k_A k_B\rangle_{in}$

↪ 4. Determine probability amplitude for creating that final state $\quad P = |{}_{out}\langle p_1 \cdots p_n | k_A k_B \rangle_{in}|^2$

5. Compute the scattering cross-section from that probability amplitude

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So how do we relate this to QFT?

In general, in quantum mechanics what we calculate are probabilities, given as the modulus squared of the inner products of states in a Hilbert space.

For example:

$|\langle f, t_f | i, t_i \rangle|^2$ is the probability of obtaining final state $f$ at time $t_f$, given an initial state $i$ at earlier time $t_i < t_f$.

This notation corresponds to the Schrödinger picture of quantum mechanics, but it is much more useful to work in the Heisenberg picture.

Q: what is the difference?

In particle physics, we typically use collider experiments to smash stuff together. The natural basis for our initial and final states is therefore the momentum basis. In particular, ← pun intended we make the approximation that our momentum eigenstates are in the asymptotic past and future - in this case the time-evolution operator is called the S-matrix

$$\langle f | S | i \rangle_{\text{Heisenberg}} = \langle f, \infty | i, -\infty \rangle$$

The (badly named) S matrix encodes all the information we need to describe our quantum system. To make things tractable, we will work under the assumptions that all interactions are confined to a finite time interval, so that our <u>asymptotic states</u> can be treated as free particles (i.e. without interactions).

Our primary task will be to relate experimental observables, such as cross-sections and decay rates, to the S-matrix and then relate that to something we can calculate in QFT.

Note: The S-matrix is not the only thing we can calculate via QFT, but we focus on it here, because it is most relevant to particle/nuclear physics.

## Cross-sections and the S-matrix

Practically speaking, it is highly unlikely to collide more than two particles at once, so we focus on the case where our initial state $|i\rangle$ is a two-particle state:

$$\underbrace{p_1 + p_2}_{\text{two particle momenta}} \longrightarrow \underbrace{\{p_j\}}_{\text{arbitrary number of particle momenta}} \qquad \leftarrow \text{"}2 \to n\text{" process}$$

Flux of incoming particles:

$$\Phi = \frac{|\bar{v}|}{V}$$ ← rest frame of one of the particles

$$\Phi = \frac{|\bar{v}_1 - \bar{v}_2|}{V}$$ ← centre-of-mass frame

flux = $\frac{\text{particle velocity}}{\text{volume}}$ = number density × velocity

The differential cross-section can be written as

$$d\sigma = \frac{1}{T}\frac{1}{\Phi}\,dP \quad \Rightarrow \quad d\sigma = \frac{V}{T|\bar{v}_1 - \bar{v}_2|}\,dP$$

← probability of scattering

time   flux

Now the probability of scattering is given by the modulus squared of the probability of turning an initial state into a final state, normalised by the states themselves

note $\langle f|f\rangle = \langle i|i\rangle = 1$ is not Lorentz invariant!

$$dP = \frac{|\langle f|S|i\rangle|^2}{\langle f|f\rangle\langle i|i\rangle}\,d\Pi$$

region of final state momenta at which we are looking

We can figure out $d\Pi$ by considering a finite box, in which case the allowable momenta are discrete $p_i = \frac{2\pi}{L} i$. The number of allowable states in the box is $i = \left(\frac{L}{2\pi}\right)^3 p_i$ and in the limit of $L \to \infty$ this becomes

$$N = \int \left(\frac{L}{2\pi}\right)^3 dp \quad \text{or} \quad dN = \frac{V}{(2\pi)^3}\,dp$$

The differential phase space available is therefore

$$d\Pi = \prod_j \frac{V}{(2\pi)^3} dp_j$$

↑ product of final state particles $j$

To obtain $\langle f|f\rangle$ and $\langle i|i\rangle$ we use the state normalisation

$$a_k^+|0\rangle = \frac{1}{\sqrt{2E_k}}|k\rangle$$

and

$$[a_k, a_q^+] = (2\pi)^3 \delta^{(3)}(\bar{k}-\bar{q})$$

to obtain

$$\langle k|k\rangle = (2\pi)^3 (2E_k)\, \delta^{(3)}(0)$$

Infinite! But

N.B. $\delta^{(3)}(\bar{p}) = \frac{1}{(2\pi)^3}\int d^3\bar{x}\, e^{i\bar{p}\cdot\bar{x}}$

Put this in a box so that

$$\delta^{(3)}(0) = \frac{1}{(2\pi)^3}\int_V d^3x = \frac{V}{(2\pi)^3}$$

$$\langle k|k\rangle = (2\pi)^3(2E_k)\frac{V}{(2\pi)^3}$$

$$= 2E_k V$$

Thus:

$$\langle i|i\rangle = (2E_1V)(2E_2V)$$

$$\langle f|f\rangle = \prod_j (2E_jV)$$

The final piece of our expression is the S-matrix itself.

We are not interested in the case where no scattering happens (or the free theory, in which no interactions occur) so generally write

$$S = \mathbb{1} + iT$$

the "transfer matrix"

We further define

$$\langle f | T | i \rangle = (2\pi)^4 \delta^{(4)}\left(\sum_i p_i^\mu - \sum_f p_f^\mu\right) \mathcal{M}$$

usually referred to as a (or the) "matrix element" of a process

to account for the requirement of momentum conservation in our process. Assuming that $|f\rangle \neq |i\rangle$, we have

$$|\langle f | S | i \rangle|^2 = \delta^{(4)}(0)\, \delta^{(4)}\left(\sum p\right) (2\pi)^8 |\langle f | M | i \rangle|^2$$

$$= \delta^{(4)}\left(\sum_i p\right) TV (2\pi)^4 |M|^2$$

$|M|^2 \equiv |\langle f | M | i \rangle|^2$

one delta function enforces momentum conservation, one is "left over" and gives $\delta^{(4)}(0) = \frac{TV}{(2\pi)^4}$

Now we can put this all together

$$dP = \frac{\delta^{(4)}(\sum p)\, TV\, (2\pi)^4}{2E_1 V\; 2E_2 V} \frac{1}{\prod_j (2E_j V)} |M|^2 \prod_j \frac{V}{(2\pi)^3} dp_j$$

$$= \frac{T}{V} \frac{1}{2E_1\, 2E_2} |M|^2\, d\Pi_{LIPS}$$

"Lorentz-invariant phase space"

$$d\Pi_{LIPS} \equiv (2\pi)^4 \delta\left(\sum p\right) \prod_j \frac{d^3 \bar{p}_j}{(2\pi)^3} \frac{1}{2E_{p_j}}$$

This leads us to

$$d\Pi_{LIPS} = \frac{1}{16\pi^2} d\Omega_f \int_{m_3+m_4-E_{cm}}^{\infty} d(E_3+E_4-E_{cm}) \frac{p_f^2}{\frac{p_f E_{cm}}{E_3 E_4}} \frac{\overbrace{\delta(E_3+E_4-E_{cm})}^{\text{called } x \text{ in Schwartz}}}{E_3 E_4}$$

$$= \frac{d\Omega_f}{16\pi^2} \frac{p_f}{E_{cm}} \theta(E_{cm}-m_3-m_4)$$

Heaviside step function $\theta(x) = \begin{cases} 1 & x > 0 \\ 0 & \leq 0 \end{cases}$

Now we put this into our formula for the differential cross-section

$$d\sigma = \frac{1}{(2E_1)(2E_2)|\bar{v}_1-\bar{v}_2|} \frac{d\Omega_f}{16\pi^2} \frac{p_f}{E_{cm}} |M|^2 \theta(E_{cm}-m_3-m_4)$$

$$|\bar{v}_1-\bar{v}_2| = \left| \frac{|\bar{p}_1|}{E_1} + \frac{|\bar{p}_2|}{E_2} \right| = p_i \frac{E_{cm}}{E_1 E_2}$$

$$\boxed{\left(\frac{d\sigma}{d\Omega}\right)_{cm} = \frac{1}{64\pi^2 E_{cm}^2} \frac{p_f}{p_i} |M|^2 \theta(E_{cm}-m_3-m_4)}$$

And if $m_1 = m_2 = m_3 = m_4$ then $p_f = p_i$ and

$$\boxed{\left(\frac{d\sigma}{d\Omega}\right)_{cm,\ \text{masses equal}} = \frac{|M|^2}{64\pi^2 E_{cm}^2}}$$

and substitute this into our expression for the differential cross-section for $2 \to n$ scattering

$$\boxed{d\sigma = \frac{1}{(2E_1)(2E_2)|\bar{v}_1 - \bar{v}_2|} |M|^2 d\Pi_{LIPS}}$$

$$\boxed{d\Pi_{LIPS} = (2\pi)^4 \delta\left(\sum_i p_i^\mu - \sum_f p_f^\mu\right) \prod_j \frac{d^3 p_j}{(2\pi)^3} \frac{1}{2E_{p_j}}}$$

An alternative derivation appears in P+S : page 102 to 106

For decay rates, which we treat as a $1 \to n$ process, one can follow the same steps to obtain

$$\boxed{d\Gamma = \frac{1}{2E_1} |M|^2 d\Pi_{LIPS}}$$

The special case of $2 \to 2$ scattering can be simplified further (especially if all the masses are equal).

We have

$$p_1 + p_2 \longrightarrow p_3 + p_4$$

and in the centre-of-mass frame this means

$$\bar{p}_1 = -\bar{p}_2, \quad \bar{p}_3 = -\bar{p}_4, \qquad E_{cm} = E_1 + E_2 = E_3 + E_4$$

$$\Rightarrow d\Pi_{LIPS} = (2\pi)^4 \delta^{(4)}(\Sigma p) \frac{d^3 p_3}{(2\pi)^3} \frac{1}{2E_3} \frac{d^3 p_4}{(2\pi)^4} \frac{1}{2E_4}$$

We now integrate over $p_4^\mu$. The $\delta^{(3)}(p_3^\mu + p_4^\mu - p_1^\mu - p_2^\mu)$ will constrain $\bar{p}_3 = -\bar{p}_4$, because $\bar{p}_1 + \bar{p}_2 = 0$ (by definition of the centre-of-mass frame). This leaves us with

$$d\Pi_{LIPS} = \frac{(2\pi)^4}{(2\pi)^3} \int \frac{d^3 p_3}{(2\pi)^3} \frac{1}{2E_3} \frac{1}{2E_4} \delta(E_3 + E_4 - E_1 - E_2)$$

with $\bar{p}_3 = -\bar{p}_4$ implicit

$$= \frac{1}{16\pi^2} \int \frac{d\Omega_f \, p_f^2 \, dp_f}{E_3 E_4} \delta(E_3 + E_4 - E_{cm})$$

$d^3\bar{p}_3 = p_3^2 dp_3 \, d(\cos\theta_3) d\phi_3$

We want to get rid of the final delta fn, $= p_f^2 dp_f \, d\Omega_f$

so we use

$$\delta(f(x)) = \sum_{x_0 \in \text{zeroes of } f} \frac{1}{|f'(x_0)|} \delta(x - x_0)$$

to change from $E_3$ to $p_3$. We need $\left| \frac{\partial (E_3 + E_4)}{\partial p_3} \right|^{-1}$,

$$\frac{\partial E_3}{\partial p_3} = \frac{p_3}{E_3} \qquad \frac{\partial E_4}{\partial p_{\cancel{3}4}} = \frac{p_4}{E_4} = \frac{p_3}{E_4}$$

since $E_3^2 = p_3^2 + m^2$ $\qquad$ $\bar{p}_3^2 = \bar{p}_4^2$

$$\Rightarrow \left| \frac{\partial (E_3 + E_4)}{\partial p_3} \right| = \frac{p_3}{E_3} + \frac{p_3}{E_4} = \frac{p_3 (E_3 + E_4)}{E_3 E_4} = \frac{p_f E_{cm}}{E_3 E_4}$$

[Schwartz 6.1]

# Lehman-Symanzik-Zimmerman reduction formula P+S 4.5

We have related our experimental observables - cross-sections and decay rates - to S-matrix elements.

We now go further and relate these S-matrix elements to vacuum expectation values, which we can calculate in QFT. Here we go...

Recall that we are only interested in nontrivial scattering, so we will take $|f\rangle \neq |i\rangle$. Then

$$\langle f|S-\mathbb{1}|i\rangle = \langle f|S|i\rangle = i\,(2\pi)^4\,\delta^{(4)}(\Sigma p)\,\mathcal{M}$$

or T

The asymptotic states are

remember, we are considering $2 \to n$ processes

$$|i\rangle = \sqrt{2E_1}\sqrt{2E_2}\;a^\dagger_{p_1}(-\infty)\,a^\dagger_{p_2}(-\infty)\,|\Omega\rangle$$
$$|f\rangle = \sqrt{2E_3}\cdots\sqrt{2E_n}\;a_{p_3}(\infty)\cdots a_{p_n}(\infty)\,|\Omega\rangle$$

vacuum = no particles

This means

$$\langle f|S|i\rangle = 2^{n/2}\sqrt{E_1E_2E_3\cdots E_n}\,\langle\Omega|a_{p_3}(\infty)\cdots a_{p_n}(\infty)\,a^\dagger_{p_1}(-\infty)\,a^\dagger_{p_2}(-\infty)|\Omega\rangle$$

Now we need to turn this into something useful.

We write our fields as

$$\phi(x) = \int\frac{d^3\bar{p}}{(2\pi)^3}\,\frac{1}{\sqrt{2E_p}}\left(a_p(t)\,e^{-ipx} + a^\dagger_p(t)\,e^{ipx}\right)$$

In an interacting theory, the time evolution of the operators will mean that creation and annihilation operators will be different at different times.

$$a_{\bar{p}}(t) = e^{iH(t-t_0)} a_p(t_0)\, e^{-iH(t-t_0)}$$

But, to derive the LSZ reduction formula, we do not need to worry about this. All we need are operators that satisfy

$$\langle \Omega | \phi(\bar{x}, t = \pm\infty) | p \rangle = c\, e^{i\bar{p}\cdot\bar{x}}$$

for free fields, we can directly use the form of $\phi(x)$ acting on $|0\rangle$ to show this

$$\langle \bar{p} | \phi(x) | 0 \rangle = \int \frac{d^3\bar{k}}{(2\pi)^3} \sqrt{\frac{\omega_p}{\omega_k}} \times \left( e^{i\bar{k}\cdot\bar{x}} \langle 0 | a_p a_k | 0 \rangle + e^{-i\bar{k}\cdot\bar{x}} \langle 0 | a_p a_k^\dagger | 0 \rangle \right) = e^{-i\bar{p}\cdot\bar{x}}$$

Before we derive the LSZ reduction formula, we need to prove the intermediate result

$$i\int d^4x\, e^{ip\cdot x} (\partial^2 + m^2)\phi(x) = \sqrt{2\omega_{\bar{p}}}\, \left( a_{\bar{p}}(\infty) - a_{\bar{p}}(-\infty) \right)$$

Starting from the left hand side

$$i\int d^4x\, e^{ip\cdot x}(\partial^2 + m^2)\phi(x) = i\int d^4x\, e^{ip\cdot x} (\partial_t^2 - \bar{\nabla}^2 + m^2)\phi(x)$$

integrate by parts to give $(\bar{\nabla}^2 e^{ip\cdot x})$

$$= i\int d^4x\, e^{ip\cdot x} (\partial_t^2 + \bar{p}^2 + m^2)\phi(x)$$

$$= i\int d^4x\, e^{ip\cdot x} (\partial_t^2 + \omega_{\bar{p}}^2)\phi(x)$$

Next we note

$$\partial_t\left(e^{ip\cdot x}\left(i\partial_t + \omega_{\bar p}\right)\phi(x)\right) = \left(\partial_t e^{ip\cdot x}\right)\left(i\partial_t + \omega_{\bar p}\right)\phi(x)$$

$$\qquad = -e^{ip\cdot x}\omega_{\bar p}\partial_t \qquad + e^{ip\cdot x}\left(i\partial_t^2 + \omega_{\bar p}\partial_t\right)\phi(x)$$

$$= i\omega_{\bar p} e^{ip\cdot x}\left(i\partial_t + \omega_{\bar p}\right)\phi(x) + e^{ip\cdot x}\left(i\partial_t^2 + \omega_{\bar p}\partial_t\right)\phi(x)$$

$$= i e^{ip\cdot x}\left(\partial_t^2 + \omega_{\bar p}^2\right)\phi(x)$$

which we can plug into our expression to obtain

$$i\int d^4x\, e^{ip\cdot x}\left(\partial^2 + m^2\right)\phi(x) = \int d^4x\, \partial_t\left(e^{ip\cdot x}\left(i\partial_t + \omega_{\bar p}\right)\phi(x)\right)$$

$$= \int dt\, \partial_t\left[e^{i\omega_{\bar p}t}\int d^3\bar x\, e^{-i\bar p\cdot\bar x}\left(i\partial_t + \omega_{\bar p}\right)\phi(x)\right]$$

Now we assume that our operators at $t = \pm\infty$ are time independent, so our time integral (over a total derivative in time) is just related to the fields at the boundaries at $t = \pm\infty$. Moreover, we plug in our solution for $\phi(x)$ to find the $\bar x$ integral, which is

$$\int d^3\bar x\, e^{-i\bar p\cdot\bar x}\left(i\partial_t + \omega_{\bar p}\right)\phi(x)$$

$$= \int d^3\bar x\, e^{-i\bar p\cdot\bar x}\left(i\partial_t + \omega_{\bar p}\right)\int\frac{d^3\bar k}{(2\pi)^3}\frac{1}{\sqrt{2\omega_{\bar k}}}\left(a_{\bar k}(t)e^{-ik\cdot x} + a_{\bar k}^\dagger(t)e^{ik\cdot x}\right)$$

$$= \int\frac{d^3\bar k}{(2\pi)^3}\int d^3\bar x\left[\left(\frac{\omega_{\bar k}+\omega_{\bar p}}{\sqrt{2\omega_{\bar k}}}\right)a_{\bar k}(t)e^{-ik\cdot x}e^{-i\bar p\cdot\bar x} + \left(\frac{-\omega_{\bar k}+\omega_{\bar p}}{\sqrt{2\omega_{\bar k}}}\right)a_{\bar k}^\dagger(t)e^{ik\cdot x}e^{-i\bar p\cdot\bar x}\right]$$

assumed $\partial_t a_{\bar k} = 0$, which is only true at $t = \pm\infty$

The spatial integral gives delta functions

$$\int d^3\bar{x}\; e^{-i\bar{p}\cdot\bar{x}} \left(i\partial_t + \omega_{\bar{p}}\right)\phi(x)$$

$$= \int \frac{d^3\bar{k}}{(2\pi)^3}\left[\left(\frac{\omega_{\bar{k}} + \omega_{\bar{p}}}{\sqrt{2\omega_{\bar{k}}}}\right) a_{\bar{k}}(t)\, e^{-i\omega_k t} \quad (2\pi)^3 \delta^{(3)}(\bar{p}-\bar{k})\right.$$

$$\left. + \left(\frac{-\omega_{\bar{k}} + \omega_{\bar{p}}}{\sqrt{2\omega_{\bar{k}}}}\right) a^\dagger_{\bar{k}}(t)\, e^{i\omega_k t} \quad (2\pi)^3\, \delta^{(3)}(\bar{p}+\bar{k})\right]$$

both delta functions enforce $\omega_{\bar{p}} = \omega_{\bar{k}}$

$$= \sqrt{2\omega_{\bar{p}}}\; a_{\bar{p}}(t)\, e^{-i\omega_p t}$$

We have nearly proved our intermediate result!

$$i\int d^4x\, e^{ip\cdot x}\left(\partial^2 + m^2\right)\phi(x) = \int dt\; \partial_t\left[e^{i\omega_p t}\left(\sqrt{2\omega_{\bar{p}}}\, a_{\bar{p}}(t)\, e^{-i\omega_p t}\right)\right]$$

$$= \sqrt{2\omega_{\bar{p}}}\left(a_{\bar{p}}(\infty) - a_{\bar{p}}(-\infty)\right)$$

Complex conjugating, we can also show

$$-i\int d^4x\; e^{-ip\cdot x}\left(\partial^2 + m^2\right)\phi(x) = \sqrt{2\omega_p}\left(a^\dagger_{\bar{p}}(\infty) - a^\dagger_{\bar{p}}(-\infty)\right)$$

Let's return to what we wanted to show and note that the right-hand side is <u>time ordered</u> ← as we saw in the Feynman propagator

$$\langle f|S|i\rangle = \sqrt{2^n\omega_1\cdots\omega_n}\;\langle\Omega|a_{p_3}(\infty)\cdots a_{\bar{p}_n}(\infty)\, a^\dagger_{p_1}(-\infty)\, a_{\bar{p}_2}(-\infty)|\Omega\rangle$$

$$= \sqrt{2^n\omega_1\cdots\omega_n}\;\langle\Omega|T\Big\{\left[a_{\bar{p}_3}(\infty) - a_{\bar{p}_3}(-\infty)\right]\cdots\left[a_{\bar{p}_n}(\infty) - a_{\bar{p}_n}(-\infty)\right]$$

$$\times\left[a^\dagger_{\bar{p}_1}(-\infty) - a^\dagger_{\bar{p}_1}(\infty)\right]\left[a^\dagger_{\bar{p}_2}(-\infty) - a^\dagger_{\bar{p}_2}(\infty)\right]\Big\}|\Omega\rangle$$

time ordering causes extra terms to drop out

Using our intermediate result, we end up with the

LSZ reduction formula

$$\boxed{\begin{aligned}\langle p_3 \cdots p_n | S | p_1 p_2 \rangle = & \left[ i \int d^4 x_1 \, e^{-ip_1 x_1} (\partial_1^2 + m^2) \right] \\ & \times \cdots \times \left[ i \int d^4 x_n \, e^{ip_n x_n} (\partial_n^2 + m^2) \right] \\ & \times \langle \Omega | T\{\phi(x_1)\phi(x_2)\phi(x_3)\cdots\phi(x_n)\} | \Omega \rangle\end{aligned}}$$

Note :

- The LSZ reduction formula tells us how to calculate S-matrix elements (which are needed to calculate scattering observables).
- The LSZ reduction formula isolates the exact initial and final states we want, the asymptotic states.
- The LSZ reduction formula holds for any interacting theory and lets us simplify our calculation to computing time-ordered products.
- The LSZ reduction formula does not rely on knowing the solutions to our theory, and applies to composite particles, like protons and neutrons in QCD, the theory of the strong nuclear force.

"There do not have to be fundamental fields associated with every asymptotic state in the theory to calculate the S-matrix."

Schwartz p. 74