General Physics 1–Honors (PHYS 101H):
Problem Set 2–Solutions

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Overview

The written Problem Sets will help you gain experience with how to present your solutions to university-level physics problems. This will be necessary for your midterm and final exams, as well as future courses thoughout your undergraduate career. Present your solutions legibly and as logically as you can. What this means in practice is the following:

1. Write down what quantities you know.
2. Write down the relevant equations.
3. When carrying out manipulations or substituting values into equations, try to write each equality on separate lines. For example:

$$\begin{aligned} x(t_0) &= \frac{1}{2}at_0^2 + v_0t_0 + x_0 \\ &= \frac{1}{2} \cdot 9.8 \cdot 1.0^2 + 1.5 \cdot 1.0 + 0.7 \\ &= \boxed{7.1\,\mathrm{m}}. \end{aligned}$$

4. Circle or underline your final answers to identify them clearly (see the equation above).

I will post solutions that will also provide one possible model for how to present solutions.

Some hints:

1. Only substitute values at the end of your calculation and try to carry out all manipulations symbolically.
2. Double check the order of magnitude of your answer.
3. Double check the units of your answer.
4. Double check the number of significant figures of your answer.

This Problem Set, in particular, will provide practice in applying the kinematic equations to projectile motion and free fall problems in one and two dimensions. You will also practice applying a Taylor series. This Problem Set is worth 50 points; there are three questions in this Problem Set.

Instructions

Read these instructions carefully. You must submit your Problem Set as a **single PDF** file (it is best to use an app like Adobe Scan to make your solutions legible), with the file name `lastname_hwXX.pdf` (replace `lastname` with your last name and `XX` with the problem set number). If you do not submit your Problem Set according to these instructions, you will be deducted five points.

Question 1 **15pts**

A hot-air balloon rises from ground level at a constant velocity of 5.3 m/s. Three minutes after liftoff, a sandbag is dropped accidentally from the balloon. (Assume the upward direction is positive.)

(a) Calculate the time it takes for the sandbag to reach the ground (in s).

(b) Calculate the velocity (in m/s) of the sandbag when it hits the ground. (Indicate the direction with the sign of your answer.)

(c) What assumptions did you make in this calculation? Suppose that we actually recreated this question in an actual experiment and measured the actual time it took to reach the ground. Do you think that your result in part (a) would be greater or smaller than the actual time taken in our real experiment, and why?

Solution 1

(a) We don't have quite enough information to solve this problem immediately, so we must calculate one more quantity. In this case, we can calculate the distance from which the sandbag is dropped, because we know the upward velocity and time travelled for the balloon itself. This is given by

$$y = v_0 t = 5.3 \cdot 180 = 954.0\,\text{m}.$$

Now we have the distance travelled by the sandbag, its acceleration, and its initial velocity. This means we can use

$$y = \frac{at^2}{2} + v_0 t + y_0,$$

or (with $a = -g$)

$$\frac{-gt^2}{2} + v_0 t + y_0 - y = 0.$$

This is a quadratic equation for the time, and the solutions are

$$\begin{aligned} t &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= -\frac{v_0}{2(-g/2)} \pm \frac{1}{2(-g/2)}\sqrt{v_0^2 - 4(-g/2)(y_0 - y)} \\ &= \frac{v_0}{g} \mp \frac{1}{g}\sqrt{v_0^2 + 2g(y_0 - y)}. \end{aligned}$$

Putting in some numbers, we have

$$t = \frac{5.3}{9.8} \mp \frac{1}{9.8}\sqrt{5.3^2 + 2 \cdot 9.8(954 - 0)} = -13.4\,\text{s} \qquad \text{or} \qquad 14.5\,\text{s}.$$

The negative solution is unphysical, so our result is

$$\boxed{t = 14.5\,\text{s}}.$$

(b) Now that we have the time taken, we can use

$$\begin{aligned} v_f &= v_0 + at \\ &= 5.3 + (-9.8) \cdot 14.5 \\ &= \boxed{-137\,\text{m/s}}. \end{aligned}$$

(c) We assumed that we can neglect air resistance [**This assumption in particular required to be mentioned, although others could be included.**]. The actual time would be **longer** than calculated in part (a), because air resistance would decrease the speed of the sandbag (although, probably not by much).

In addition, we also assumed constant acceleration due to gravity. As we will see later in the course, the acceleration due to gravity does vary with distance from the Earth's surface, but that variation is likely to be negligible at the precision with which we treat this problem (three significant figures).

Question 2 **10pts**

If a ball is dropped from rest at height h, and if the drag force from the air takes the form $F_d = bv$, then it can be shown that the ball's height as a function of time equals

$$y(t) = h - \frac{mg}{b}\left(t - \frac{m}{b}\left(1 - e^{-bt/m}\right)\right)$$

Expand the exponential as a Taylor Series to find an approximate expression for $y(t)$ in the limit where t is very small.

Solution 2

The formula for the Taylor expansion, in x, of an arbitrary function $f(x)$ around $x = a$ is

$$f(x) = f'(a) + f'(a)(x-a) + \frac{f''(a)}{2}(x-a)^2 + \ldots$$

Here the prime indicates a derivative of $f(x)$ with respect to x. So, for example, $f''(x)$ is the second derivative of $f(x)$ with respect to x, and $f''(a)$ is the second derivative of $f(x)$ evaluated at $x = a$.

We could calculate the derivatives of the whole function $y(t)$, but it is easiest just to look up the expansion of the exponential function, which is the only nontrivial part of the $y(t)$ function. We have

$$\exp(x) \sim 1 + x + \frac{x^2}{2} + \ldots$$

In our case, $x = (-bt)/m$, and our function becomes

$$\begin{aligned}
y(t) &\sim h - \frac{mg}{b}\left\{t - \frac{m}{b}\left[1 - \left(1 + \left(\frac{-bt}{m}\right) + \frac{1}{2}\left(\frac{-bt}{m}\right)^2 + \ldots\right)\right]\right\} \\
&= h - \frac{mg}{b}\left\{t - \frac{m}{b}\left[\frac{bt}{m} - \frac{b^2t^2}{2m^2} + \ldots\right]\right\} \\
&= h - \frac{mg}{b}\left\{t - t + \frac{bt^2}{2m} + \ldots\right\} \\
&= \boxed{h - \frac{gt^2}{2} + \ldots}
\end{aligned}$$

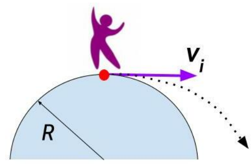

Figure 1: Person kicking a pebble on a boulder for Problem 3. I don't know why they are doing this.

This is exactly form you would expect from the one dimensional kinematics equation, $y = at^2/2 + v_0 t + y_0$, for freefall ($a = -g$) from rest ($v_0 = 0$) from postion $y_0 = 0$! In other words, for very short times, the friction does not really have an effect on the kinematics of the fall.

Question 3 **25pts**

A person standing on top of a hemispherical boulder of radius R kicks a pebble (see Figure 1), initially at rest on top of the boulder, with an initial horizontal velocity $\overline{v}_i$.

(a) What is the minimum value of the initial speed, v_i, that ensures the pebble does not touch the boulder after it is kicked?

(b) With this initial speed (i.e. your answer to part a), where does the pebble hit the ground?

Solution 3

(a) Let's start by setting up our coordinate system (our reference frame). We define the horizontal and vertical components as x and y respectively, and assume that the motion occurs in a two-dimensional plane. For the pebble not to hit the surface of the boulder, we must have

$$x^2 + y^2 > R^2,$$

for $t > 0$.

The acceleration is given by

$$\mathbf{a} = -g\hat{y},$$

and the initial velocity is

$$\mathbf{v}_i = v_i\hat{x}.$$

Thus, the equation of motion in the x direction is

$$x = v_i t,$$

which tells us that $t = x/v_i$, and the equation of motion in the y direction is

$$y = -\frac{gt^2}{2} + R.$$

Substituting our value of t into this equation for y, we obtain

$$y = -\frac{g}{2}\left(\frac{x}{v_i}\right)^2 + R,$$

or

$$x^2 = \frac{2v_i^2}{g}(R-y).$$

Now we apply our constraint condition, so

$$x^2 + y^2 = \frac{2v_i^2}{g}(R-y) + y^2 > R^2,$$

or

$$\frac{2v_i^2}{g}(R-y) > R^2 - y^2 = (R+y)(R-y).$$

Assuming $y \neq R$, we can cancel the factor of $(R-y)$ on both sides to find

$$\frac{2v_i^2}{g} > R + y,$$

or

$$v_i^2 > \frac{g(R+y)}{2} \qquad \Rightarrow \qquad v_i > \sqrt{\frac{g(R+y)}{2}}.$$

For all $t > 0$, the maximum height is the initial height, so $y < R$. This means the maximum value that the initial velocity can take is at $y = R$. So

$$v_i > \sqrt{\frac{2gR}{2}}$$

or

$$\boxed{v_i > \sqrt{gR}}.$$

(b) With this initial speed in the y direction, we can use the kinematic equation in the vertical direction to find the distance (x) until the pebble hits the ground ($y = 0$):

$$y = 0 = -\frac{g}{2}\left(\frac{x}{v_i}\right)^2 + R.$$

We rearrange this as

$$\frac{g}{2}\left(\frac{x}{v_i}\right)^2 = R,$$

and substitute our value of $v_i = \sqrt{gR}$ to give

$$\frac{g}{2}\left(\frac{x}{\sqrt{gR}}\right)^2 = R,$$

or

$$\frac{gx^2}{2gR} = R.$$

This we solve to find

$$x = \pm\sqrt{2}R.$$

The negative solution is the solution in the opposite direction, so we take the positive solution. This means the pebble lands at

$$\boxed{x = \sqrt{2}R}.$$

An alternative acceptable answer is the distance **beyond** the boulder, which is

$$\boxed{x = (\sqrt{2} - 1)R}.$$