General Physics I–Honors: PHYS 101H (Fall 2023)
Quiz 6–solutions

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Instructions

In this quiz you will apply your understanding of rocket motion, centre of mass, moment of inertia and angular motion. Read the following instructions carefully.

DO NOT TURN OVER THIS SHEET UNTIL INSTRUCTED.

Please write your name on the quiz.

You have ten minutes to attempt all three questions in this quiz.

You may use electronic calculators.

You may **not** use:

- any formula sheets or notes;
- electronic devices, including phones, tablets and laptops (unless previously arranged);
- textbooks or other reference resources;
- course notes or slides.

Question 1 **3pts**

Find the final speed for a rocket that accelerates from rest by expelling 10% of its initial mass as gas at a speed of $v_0 = 50$ m/s.

Solution 1

We apply the rocket equation:

$$v_f - v_i = v_e \log\left(\frac{m_i}{m_f}\right) = 50 \cdot \log\left(\frac{m_i}{0.9m_i}\right) = 50 \cdot \log\left(\frac{1}{0.9}\right) = \boxed{5.3\,\text{m/s}}.$$

Question 2 **4pts**

Find the centre of mass and moment of inertia (about an axis through the centre of mass) of a system of three particles, each with equal mass m, at positions $(1,2)$, $(-1,0)$, and $(0,-2)$ in the (x,y) plane.

Solution 2

The centre of mass can be found by symmetry, or by explicitly adding up the masses times their position and is

$$\vec{r}_{\text{CM}} = \sum_{i=1}^{3} m\vec{r}_i = m \cdot (1-1+0) \cdot \hat{x} + m \cdot (2+0-2) \cdot \hat{y} = \boxed{(0,0)}.$$

The moment of inertia is

$$I_{\text{CM}} = \sum_{i=1}^{3} mr_i^2 = m \cdot (1^2 + 2^2) + m \cdot ((-1)^2 + 0) + m \cdot (0 + (-2)^2) = \boxed{10m}.$$

Note that here we used $r_i^2 = x_i^2 + y_i^2$ for each mass.

Question 3 **3pts**

Explain the relationships between torque, angular momentum and rotational motion, using full sentences. You should use equations where necessary.

Solution 3

Angular momentum and torque are related by the rotational equivalent of Newton's second law: torque measures the turning ability of an applied force and angular momentum measures the corresponding change in (rotational) momentum. In particular, torque is given by

$$\vec{\tau} = \vec{r} \times \vec{F},$$

where $\vec{F}$ is the applied force at displacement $\vec{r}$ from the axis of rotation, and angular momentum is given by

$$\vec{L} = \vec{r} \times \vec{p},$$

where $\vec{p}$ is the linear momentum of the object. The relationship between these two quantities is

$$\vec{\tau} = \frac{\mathrm{d}\vec{L}}{\mathrm{d}t}.$$

One consequence of this equation is that in the absence of external torques, the angular momentum is conserved. Conservation of angular momentum can be used, for example, to explain why ice skaters speed up when they bring in their arms during a rotation. This is because the angular momentum is related to the angular frequency by

$$\vec{L} = I\vec{\omega},$$

where I is the moment of inertia around the axis of rotation. The kinetic energy associated with this rotation around the centre of mass is

$$E_K^{\mathrm{rot}} = \frac{I\omega^2}{2}.$$

Note that torque and angular momentum are maximised when the applied force and momentum, respectively, are perpendicular to the displacement from the axis of rotation.