General Physics I–Honors: PHYS 101H (Fall 2023)
Quiz 3

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Instructions

In this quiz you will apply your understanding of dimensional analysis and problem solving techniques in physics. Read the following instructions carefully.

DO NOT TURN OVER THIS SHEET UNTIL INSTRUCTED.

Please write your name on the quiz.

You have ten minutes to attempt all four questions in this quiz. Two questions are open response questions and two questions are multiple choice questions. For the open response questions, write your answer, using complete sentences, in the space provided. For the multiple choice questions, indicate your answer clearly by **circling** the correct option.

You may use electronic calculators, but you will not need one.

You may **not** use:

- any formula sheets or notes;
- electronic devices, including phones, tablets and laptops (unless previously arranged);
- textbooks or other reference resources;
- course notes or slides.

You may (or may not) find the following table of Taylor series helpful:

$$\begin{aligned}
\frac{1}{1+x} &= \sum_{n=0}^{\infty} x^n && = 1 - x + x^2 - x^3 + \dots \\
\ln(1+x) &= \sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^n}{n} && = x - \frac{x^2}{2} + \frac{x^3}{3} + \dots \\
e^x &= \sum_{n=0}^{\infty} \frac{x^n}{n!} && = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \dots \\
\sin x &= \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!} && = x - \frac{x^3}{6} + \frac{x^5}{120} + \dots \\
\cos x &= \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!} && = 1 - \frac{x^2}{2} + \frac{x^4}{24} + \dots \\
\arctan x &= \sum_{n=0} (-1)^n \frac{x^{2n+1}}{2n+1} && = x - \frac{x^3}{3} + \frac{x^5}{5} + \dots \\
(1+x)^m &= \sum_{n=0}^{\infty} \begin{pmatrix} m \\ n \end{pmatrix} x^n && = 1 + mx + \frac{m(m+1)}{2} x^2 + \dots
\end{aligned}$$

Question 1 **2pts**

Which of the following statements about a particle undergoing **uniform circular motion** is true? Circle the correct response.

(a) The speed of the particle changes.

(b) The velocity of the particle changes.

(c) The magnitude of the acceleration of the particle changes.

Solution 1

Uniform circular motion is defined by motion around a circle with constant speed. Therefore (a) cannot be correct. Moreover, to ensure that the speed is constant, the magnitude of the centripetal acceleration must also be constant. Therefore (c) is incorrect. The correct answer is $\boxed{\text{(b)}}$, the direction of the velocity changes, even though its magnitude does not.

Question 2 **4pts**

Explain the difference between the **uniform** and **nonuniform circular motion**. Your answer should be expressed using complete sentences and may include equations. To receive full marks you should mention at least three correct and relevant facts, differences, and/or equations.

Solution 2

Uniform motion is defined by motion with **constant speed** in a circle. An object undergoing uniform circular motion has a constantly **changing velocity**, because the direction of the velocity is always changing, which means that there is a **nonzero acceleration**. This centripetal acceleration, $\vec{a}_c$, has **constant magnitude** and is **directed radially inwards**:

$$\vec{a}_c = -\frac{v^2}{r}\hat{r},$$

where v is the speed of the object and r the radial distance from the centre of the circle defining the motion. The direction of the acceleration vector is therefore always perpendicular to the velocity vector.

In contrast, nonuniform circular motion need not have an acceleration that has constant magnitude and a direction that is always perpendicular to the velocity. In this situation, there maybe a **tangential component** of the acceleration (which is parallel to the velocity, tangent to the path defined by the motion), which leads to a **change in the speed** of the object and there may also be a **radial component** of the acceleration, which leads to a **change in the direction** of the object. In equation form:

$$\vec{a}(t) = \vec{a}_{\mathrm{r}}(t) + \vec{a}_{\mathrm{t}}(t),$$

where $\vec{a}_{\mathrm{t}}$ is the tangential component and $\vec{a}_{\mathrm{r}}$ is the radial component. Depending on the situation, the decomposition into these two components may change with time.

Question 3 **2pts**

Write down the equation relating the velocities observed in two different reference frames that are moving with respect to each other at constant velocity. Be sure to define, in words, all symbols that appear in your equation!

Solution 3

The velocity in frame B, denoted $\vec{v}_{\mathrm{B}}$, is related to the velocity in frame A, denoted $\vec{v}_{\mathrm{A}}$, by

$$\vec{v}_{\mathrm{B}} = \vec{v}_{\mathrm{A}} + \vec{v}_{\mathrm{AB}},$$

where $\vec{v}_{\mathrm{AB}}$ is the relative velocity of the two frames and is assumed to be constant for Galilean transformations.

Question 4 **2pts**

Which of the following statements about Galilean transformations is true? Circle the correct response.

(a) Galilean transformations apply to reference frames moving at very high speeds with respect to each other.

(b) Galilean transformations apply to reference frames accelerating with respect to each other.

(c) Galilean transformations apply to reference frames moving at constant speed with respect to each other.

Solution 4

In our discussion of Galilean transformations, we always assumed that the references frames were initial, meaning they did not accelerate with respect to each other (we saw that noninertial frames lead to strange effects, like fictitious forces). Galilean transformations apply to slow-moving objects. Fast-moving objects are described instead by Lorentz transformations, which underpin Special Relativity. Therefore the correct answer is $\boxed{\text{(c)}}$.