General Physics I–Honors: PHYS 101H (Fall 2023)
Quiz 2

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Instructions

In this quiz you will apply your understanding of dimensional analysis and problem solving techniques in physics. Read the following instructions carefully.

DO NOT TURN OVER THIS SHEET UNTIL INSTRUCTED.

Please write your name on the quiz.

You have ten minutes to attempt all three questions in this quiz. Two questions are open response questions and the third question is a multiple choice question. For the open response questions, write your answer, using complete sentences, in the space provided. For the multiple choice question, indicate your answer clearly by **circling** the correct option.

You may use electronic calculators, but you will not need one.

You may **not** use:

- any formula sheets or notes;
- electronic devices, including phones, tablets and laptops (unless previously arranged);
- textbooks or other reference resources;
- course notes or slides.

You may (or may not) find the following table of Taylor series helpful:

$$
\begin{aligned}
\frac{1}{1+x} &= \sum_{n=0}^{\infty} x^n &&= 1 - x + x^2 - x^3 + \dots \\
\ln(1+x) &= \sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^n}{n} &&= x - \frac{x^2}{2} + \frac{x^3}{3} + \dots \\
e^x &= \sum_{n=0}^{\infty} \frac{x^n}{n!} &&= 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \dots \\
\sin x &= \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!} &&= x - \frac{x^3}{6} + \frac{x^5}{120} + \dots \\
\cos x &= \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!} &&= 1 - \frac{x^2}{2} + \frac{x^4}{24} + \dots \\
\arctan x &= \sum_{n=0} (-1)^n \frac{x^{2n+1}}{2n+1} &&= x - \frac{x^3}{3} + \frac{x^5}{5} + \dots \\
(1+x)^m &= \sum_{n=0}^{\infty} \begin{pmatrix} m \\ n \end{pmatrix} x^n &&= 1 + mx + \frac{m(m+1)}{2} x^2 + \dots
\end{aligned}
$$

Question 1 **4pts**

Write down the three key equations (one each for position, velocity, and acceleration) for a particle undergoing **freefall in one dimension**.

Solution 1

I define the positive y direction to be pointed vertically up, with the origin $y = 0$ at the Earth's surface. Then the three equations that describe freefall are:

$$\begin{aligned} a &= -g, \\ v &= -gt + v_0, \\ y &= -\frac{gt^2}{2} + v_0 t + y_0. \end{aligned}$$

Here a, v, and y are the acceleration, velocity, and displacement above the Earth, g is the acceleration due to gravity. The initial position and velocity are y_0 and v_0, respectively, and t is the time elapsed.

Question 2 **4pts**

Explain the difference between the **dot** (or scalar) product and the **cross** (or vector) product of two different vectors. Your answer should be expressed using complete sentences and may include equations.

Solution 2

Let $\vec{a}$ and $\vec{b}$ be two two-dimensional vectors. In a Cartesian coordinate frame with unit vectors $\hat{x}$ and $\hat{y}$, we can denote their components by $\vec{a} = a_x\hat{x} + a_y\hat{y}$ and $\vec{b} = b_x\hat{x} + b_y\hat{y}$.

The key difference between the dot and cross products of $\vec{a}$ and $\vec{b}$ is that the result of the dot product is a scalar, whereas the result of the cross product can be thought of as a vector. The dot product (denoted by '$\cdot$') is zero when the two vectors are perpendicular and has a maximum value when the two vectors are parallel. In contrast, the cross product (denoted by '$\times$') is zero when the two vectors are parallel and has a maximum magnitude when the two vectors are perpendicular.

The dot product of $\vec{a}$ and $\vec{b}$ is defined by

$$\vec{a} \cdot \vec{b} = ab\cos\theta = a_x b_x + a_y b_y.$$

Here I represent the magnitude of $\vec{a}$ by a and the magnitude of $\vec{b}$ by b. The angle between $\vec{a}$ and $\vec{b}$ is θ.

The cross product of $\vec{a}$ and $\vec{b}$ has magnitude

$$|\vec{a} \times \vec{b}| = ab\sin\theta.$$

Question 3 **2pts**

Draw two labelled diagrams (velocity vs. time, and acceleration vs. time) representing the motion of a particle in one dimension if the particle has velocity given by $v = 3t\,\mathrm{m/s}$, where t is the time elapsed, in seconds. Assume that the initial velocity, at time $t = 0$, is $v_0 = 0\,\mathrm{m/s}$.

Solution 3

We are given the velocity, but not the acceleration, so our first job is to calculate that. We use

$$a = \frac{\mathrm{d}v}{\mathrm{d}t} = \frac{\mathrm{d}}{\mathrm{d}t}(3t) = 3\,\mathrm{m/s^2}.$$

Now we can draw our diagrams. The velocity is a linear, increasing function with slope equal to three and the acceleration is a constant function, equal to three. Note that "labelled" diagrams means that I include labels for both axes in both diagrams, with units, and I show the origin and some illustrative or relevant numerical values.