# General Physics I–Honors: PHYS 101H (Fall 2023)
# Quiz 1

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## Instructions

In this quiz you will apply your understanding of dimensional analysis and problem solving techniques in physics. Read the following instructions carefully.

**DO NOT TURN OVER THIS SHEET UNTIL INSTRUCTED.**

Please write your name on the quiz.

You have ten minutes to attempt all three multiple-choice questions in this quiz. Indicate your answer clearly by **circling** the correct option.

You may use electronic calculators.

You may **not** use:

- any formula sheets or notes;
- electronic devices, including phones, tablets and laptops (unless previously arranged);
- textbooks or other reference resources;
- course notes or slides.

You may (or may not) find the following table of Taylor series helpful:

$$\begin{aligned}
\frac{1}{1+x} &= \sum_{n=0}^{\infty} x^n && = 1 - x + x^2 - x^3 + \dots \\
\ln(1+x) &= \sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^n}{n} && = x - \frac{x^2}{2} + \frac{x^3}{3} + \dots \\
e^x &= \sum_{n=0}^{\infty} \frac{x^n}{n!} && = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \dots \\
\sin x &= \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!} && = x - \frac{x^3}{6} + \frac{x^5}{120} + \dots \\
\cos x &= \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!} && = 1 - \frac{x^2}{2} + \frac{x^4}{24} + \dots \\
\arctan x &= \sum_{n=0} (-1)^n \frac{x^{2n+1}}{2n+1} && = x - \frac{x^3}{3} + \frac{x^5}{5} + \dots \\
(1+x)^m &= \sum_{n=0}^{\infty} \begin{pmatrix} m \\ n \end{pmatrix} x^n && = 1 + mx + \frac{m(m+1)}{2} x^2 + \dots
\end{aligned}$$

**Question 1** **4pts**

The Planck length is a quantity that characterizes the distance at which both quantum effects and gravitational effects must be considered. In other words, the Planck length tells us the distance at which we must have a quantum theory of gravity to be able to correctly describe the physics that happens at those distances (we have several candidate theories, but none that have been experimentally verified, yet). The Planck length depends on three fundamental physical constants. Each of these constants is associated with one of the three theories that form the pillars of modern physics. These are: Planck's constant, $\hbar = 1.05 \times 10^{-34}$ kg m$^2$/s, which characterizes quantum mechanics; the gravitational constant, $G = 6.67 \times 10^{-11}$ m$^3$ /(kg s$^2$), which characterizes general relativity, the theory of gravity; and the speed of light, $c = 3.0 \times 10^8$ m/s, which is associated with special relativity. Which combination of these fundamental constants gives the correct expression for the Planck length?

(a) $\sqrt{\hbar G/c^2}$; (b) $\sqrt{\hbar G/c^3}$; (c) $\sqrt{\hbar G/c^4}$.

**Solution 1**

There are two ways to solve this, but both start by recognising that any "length" must have units of metres. From there, you can either use the method we applied in class last week of solving an expression of the form

$$[\mathrm{m}] = [\hbar]^{a_1}[G]^{a_2}[c]^{a_3}$$

for $a_1$, $a_2$, and $a_3$, or you can compute the units associated with each option and see which one has units length. Let's practice applying the first method, which means we need to solve

$$[\mathrm{m}] = [\mathrm{kg\,m^2/s}]^{a_1}[\mathrm{m^3/(kg\,s^2)}]^{a_2}[\mathrm{m/s}]^{a_3}$$

to find $a_1$, $a_2$, and $a_3$.

Equating units of length gives us:

$$1 = 2a_1 + 3a_2 + a_3.$$

Equating units of mass gives us:

$$0 = a_1 - a_2,$$

which immediately tells us that $a_2 = a_1$. Equating units of time tells us:

$$0 = -a_1 - 2a_2 - a_3.$$

Let's eliminate $a_2$ from our expressions by applying $a_2 = a_1$ to our first and third equations. This leads us to

$$1 = 5a_1 + a_3,$$
$$0 - 3a_1 - a_3.$$

Adding these two equations together gives

$$1 = 2a_1,$$

and therefore we have $a_1 = 1/2$. We know that $a_2 = a_1$, so we immediately obtain $a_2 = 1/2$ as well. That leaves us with $a_3$, which we can find from the equation

$$0 = -3a_1 - a_3 = -3 \cdot \frac{1}{2} - a_3.$$

Thus, $a_3 = -3/2$.

Putting this all together, we find that to create a quantity with units of length, we need the combination $\hbar^{1/2} G^{1/2} c^{-3/2}$. This is answer $\boxed{b}$.

In this case, it probably would have been quicker to check each answer in turn…

**Question 2** **3pts**

The equation for the period of a simple pendulum undergoing oscillations is usually given as

$$T_0 = 2\pi \sqrt{\frac{\ell}{g}},$$

where $\ell$ is the length of the pendulum and $g$ is the acceleration due to gravity. In fact, this equation is only valid when the pendulum is swinging with a small angle, $\theta \lesssim 6°$. For larger angles, a different expression is valid. Which of the following expressions could be the correct equation for the period, $T$, of a simple pendulum for angles of oscillation up to $\theta \approx 90°$?

(a) $T = 2\pi \sqrt{\frac{\ell}{g}} \cdot \frac{2}{1+\cos(\theta/2)}$; (b) $T = 2\pi \sqrt{\frac{\ell}{g}} \cdot \frac{2}{1+\sin(\theta/2)}$; (c) $T = 2\pi \sqrt{\frac{\ell}{g}} \cdot \frac{2}{1+\tan(\theta/2)}$.

**Solution 2**

To answer this question, we need to notice that the small angle case (for which the period is given by $T_0$) is a **special** or **limiting case** of the more general formulae quoted in answers (a), (b), (c). So we need to make sure that when the angle is very small, that is, when $\theta \ll 90°$ or $\theta \approx 0°$, the full expression **reduces** to $T_0$.

We can do that by plugging in $\theta = 0$ into each of the trigonometric functions, $\cos(\theta/2)$, $\sin(\theta/2)$, and $\tan(\theta/2)$. We find

$$\cos(0) = 1, \qquad \sin(0) = 0, \qquad \text{and} \qquad \tan(0) = 0.$$

Thus, when $\theta = 0$, answer (a) reduces to $T = T_0$, whereas answers (b) and (c) reduce to $T = 2T_0$. Thus answer $\boxed{(a)}$ must be correct!

If you want expressions for when $\theta$ is small, but not exactly zero, you can use the table of Taylor series on the front page, but that is not really necessary to obtain the answer in this case.

**Question 3** **3pts**

Which of the following options is a good estimate for the number of heartbeats in an adult human's lifetime?

(a) $2 \times 10^6$; (b) $2 \times 10^9$; (c) $2 \times 10^{12}$.

**Solution 3**

The key to answering this question is estimating the average adult human's lifetime (frist in years, then in seconds) and estimating that there is one heartbeat per second. A good estimate is something like 70 years. The number of seconds in one (non-leap) year is[1]

$$60 \cdot 60 \cdot 24 \cdot 365 = 31536000.$$

This means the number of seconds in 70 years is

$$70 \cdot 31536000 = 2207520000 \approx 2 \times 10^9.$$

This is answer $\boxed{b}$.

An important point to note here is that it doesn't matter too much whether you chose 70 years as the average lifetime or 50 years or 100 years. Any of these (very reasonable) choices is much closer to answer (b) than to the other answers. That is because answer (a) is actually about 23 days and answer (c) is about 63,000 years. Neither would qualify as **good** estimates of the average adult human lifetime.

[1] You can immediately write down the number of minutes in a year if you know the soundtrack of the musical Rent well enough. Google "Seasons of love (song)" if this comment does not mean anything to you.