General Physics 1–Honors (PHYS 101H): Practice Exam 2–Solutions November 2023

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Overview and instructions

In this midterm you will apply your understanding of conservation of energy, elastic and inelastic collisions, and rotational motion.

Read the following instructions carefully.

There are **five questions**, for a total of **100 points**. **Attempt all questions**. The exam will finish at 11:45 am. Please write your name **on every sheet of paper you submit**. It is helpful if you include page numbers at the bottom of each page, too.

You may use:

- an electronic calculator;
- your own formula sheet, written or printed on two sides of letter paper.

You may **not** use:

- electronic devices (except a calculator), including phones, tablets and laptops (unless previously arranged);
- textbooks or other reference resources;
- course notes or slides.

The first three questions are multiple choice. Your answer to these multiple choice questions should be written out and submitted as part of the rest of your solutions. For example, you could write, "Problem 1: my answer is **(a)**." Do **not** circle the options on the exam itself; I will not collect the exams and you will not receive credit for your answer.

The remaining questions require written solutions. You should show all your working and include important intermediate steps, equations, and results. You can receive partial credit for these problems, even if you don't complete the problem or provide a correct final answer. Please ensure that you highlight or emphasise your final answer (for example, by circling or underlining the final answer).

You are responsible for ensuring your solutions are legible. Present your solutions legibly and as logically as you can. What this means in practice is the following:

1. Write down what quantities you know.
2. Write down the relevant equations.
3. When carrying out manipulations or substituting values into equations, try to write each equality on separate lines or otherwise distinguish them clearly.
4. Circle or underline your final answers to identify them clearly.

Some hints for tackling problems in general:

1. Try to identify what "kind" or "type" of question is being asked, for example "projectile motion", "conservation of energy", or "two dimensional collision".
2. Draw a labelled diagram.
3. Write down what quantities you know.
4. Write down the relevant equations.
5. Only substitute values at the end of your calculation and try to carry out all manipulations symbolically.
6. Double check the order of magnitude of your answer.
7. Double check the units of your answer.
8. Double check the number of significant figures of your answer. Remember that I am only looking for approximately the correct number of significant digits. If quantities are given to two or three significant digits, quote your answer to two or three (not one or five). Similarly if quantities are given to eight significant digits, do not quote your answer to two.

You do not have to tackle the questions in order. Briefly read through them all and then start on one!

Short questions

Remember, your answer to these multiple choice questions should be written out and submitted as part of the rest of your solutions.

Question 1 **10pts**

Two blocks with masses 2.0 kg and 1.0 kg lie on a frictionless table. A force of 4.5 N is applied as shown in figure 1. What is the normal force between the blocks?

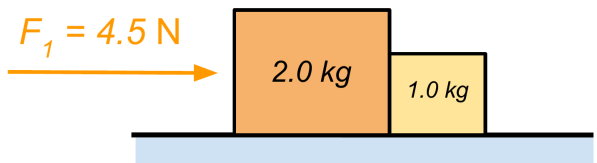

Figure 1: Diagram for Question 1.

(a) 0.5 N.

(b) 1.0 N.

(c) 1.5 N.

(d) 2.0 N.

(e) 2.5 N.

Solution 1

The answer is (c). The pair of blocks have a total mass of 3 kg, and so move with acceleration

$$a = \frac{F}{m} = \frac{4.5}{3} = 1.5\,\mathrm{m/s}.$$

The only horizontal force that the smaller block experiences is the normal force from the larger block. Therefore this normal force must be the net force that causes the smaller block to accelerate with this acceleration. This means the normal force is

$$N = F_{\mathrm{net}} = ma = 1.0 \cdot 1.5 = 1.5\,\mathrm{N}.$$

Question 2 **10pts**

Two cylinders **of the same mass and radius** roll without slipping down an incline plane. One cylinder is solid and of uniform density. The other cylinder is hollow and the mass is uniformly distributed in a cylindrical shell (that is, like a hollow pipe). Both cylinders are released simultaneously from the same height and are initially at rest. Which cylinder reaches the bottom of the slope first, and why?

(a) The solid cylinder reaches the bottom first, because the moment of inertia is larger and therefore the cylinder rolls faster.

(b) The solid cylinder reaches the bottom first, because the moment of inertia is smaller and therefore the translational speed is larger.

(c) The hollow cylinder reaches the bottom first, because the moment of inertia is smaller and therefore the cylinder rolls faster.

(d) The hollow cylinder reaches the bottom first, because the moment of inertia is larger and therefore the translational speed is larger.

(e) Both cylinders reach the bottom at the same time, because they have the same mass and the acceleration due to gravity is independent of mass.

Solution 2

The answer is (b). The solid cylinder has a smaller moment of inertia, because the mass is distributed closer to the axis of rotation. The total kinetic energy is composed of rotational, $I\omega^2/2$, and translational kinetic energy, $mv^2/2$. At the bottom of the motion, conservation of energy ensures that the total kinetic energy is the same for both cylinders. The rotational kinetic energy is smaller for the solid cylinder (because I is smaller), so the translational kinetic energy must be larger, and therefore the solid cylinder has a larger translational speed.

Question 3 **10pts**

A very light table tennis ball bounces elastically head-on off a very heavy bowling ball that is initially at rest. The fraction of the table tennis ball's initial kinetic energy that is transferred to the bowling ball is approximately

(a) 0

(b) 1/4

(c) 1/2

(d) 3/4

(e) 1

Solution 3

The answer is (a), because the momentum is conserved and therefore the bowling ball hardly moves. This can also be done quantitatively, by using the final expression for the bowling ball's speed, which is

$$v_{bb} = \frac{2mv}{m+M}.$$

This means the final kinetic energy of the bowling ball is

$$E_K^{bb} = \frac{M}{2}\left(\frac{2mv}{m+M}\right)^2 = \frac{2m^2Mv^2}{(m+M)^2} = \frac{2m^2v^2}{M(1+m/M)^2}.$$

This goes to zero for $M \to \infty$.

Longer questions

Remember, present your solutions legibly and as logically as you can. Highlight your final answer by underlining or circling it.

Question 4 — 30pts

A 11.0 kg block of wood is at rest on a horizontal floor. A blop of clay, of mass 0.3 kg, is thrown horizontally at the block, and sticks to the block. The block and blob slide across the floor 20 cm before coming to rest.

(a) If the coefficient of friction between the block and the floor is 0.56, what is the magnitude of the work done by friction?

(b) What is the initial speed of the clay?

Solution 4

(a) The work done by friction is

$$F_F \cdot \Delta x = \mu_K N \cdot \Delta x = \mu_K (m_1 + m_2) g \cdot \Delta x = 0.56 \cdot (11.0 + 0.3) \cdot 9.81 \cdot 0.2 = \boxed{12.4\,\mathrm{J}}.$$

(b) Conservation of momentum tells us that the final momentum of the block plus blob must be equal to the initial momentum of the blob (this is a perfectly inelastic collision, so kinetic energy is not conserved). This means that

$$m_1 v_1 = (m_1 + m_2) v_2,$$

where m_1 is the mass of the blob and m_2 the mass of the block. Thus the initial speed is

$$v_1 = \frac{m_1 + m_2}{m_1} v_2.$$

Once the block and blob are moving, the work done by friction on the block converts the kinetic energy to heat dissipated as the block slides. The work done by friction must be equal to the initial kinetic energy of the block and blob, so

$$\frac{1}{2}(m_1 + m_2) v_f^2 = \mu_K (m_1 + m_2) g \Delta x.$$

Therefore the final speed (after the collision and right as the block and blob start to slide) is

$$v_f = \sqrt{2 \mu_K g \Delta x}.$$

We can now plug this into our expression for the initial speed of the blob, to obtain

$$\begin{aligned} v_1 &= \frac{m_1 + m_2}{m_1} v_2 \\ &= \frac{m_1 + m_2}{m_1} \sqrt{2\mu_K g \Delta x} \\ &= \frac{0.3 + 11.0}{0.3} \sqrt{2 \cdot 0.56 \cdot 9.81 \cdot 0.2} \\ &= \boxed{55.8\,\mathrm{m/s}}. \end{aligned}$$

Question 5 **40pts**

Two objects are attached to ropes that are attached to wheels that are attached to a common axle through their centers, shown in figure 2. The total moment of inertia of the two wheels is I.

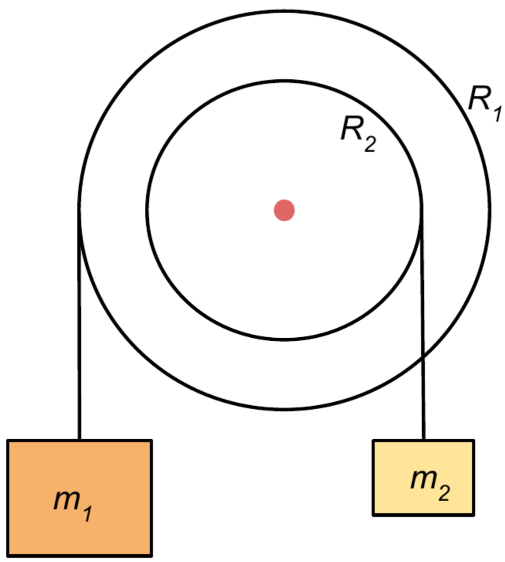

Figure 2: Diagram for Question 5.

(a) Find the value of m_1, expressed in terms of m_2, R_1, and R_2, that ensures the system is in equilibrium.

(b) If an additional mass, m_3, is added on top of m_1, what is the angular acceleration of the system?

Solution 5

(a) Equilibrium means that there is no linear or angular acceleration. Applying Newton's second law, we have

$$T_1 - m_1 g = 0,$$

and

$$T_2 - m_2 g = 0.$$

To ensure there is no angular acceleration, we must also have that the net torque is zero. The torques are

$$|\tau_1| = T_1 R_1 \sin 90^\circ,$$

and

$$|\tau_2| = T_2 R_2 \sin 90^\circ,$$

so

$$\vec{\tau}_1 + \vec{\tau}_2 = 0$$

means

$$T_1 R_1 = T_2 R_2.$$

Since we have

$$T_1 = m_1 g,$$

and

$$T_2 = m_2 g,$$

we can combine these equations as

$$m_1 g R_1 = m_2 g R_2.$$

Thus

$$\boxed{m_1 = \frac{m_2 R_2}{R_1}}.$$

(b) With the presence of m_3, the system will no longer be in equilibrium and will accelerate. The m_1 mass will accelerate downwards with linear accleration $|a| = \alpha R$. The m_2 mass will accelerate upwards. Applying Newton's second law, we have

$$T_1 - (m_1 + m_3)g = (m_1 + m_3)(-a),$$

and

$$T_2 - m_2 g = m_2 a.$$

We can rearrange these as

$$T_1 = (m_1 + m_3)(g - a),$$

and

$$T_2 = m_2(a + g).$$

The net torque generates an angular acceleration

$$\tau_1 - \tau_2 = I\alpha,$$

or

$$T_1 R_1 - T_2 R_2 = I\alpha.$$

Plugging in our expressions for the tensions, we have

$$(m_1 + m_3)(g - a)R_1 - m_2(a + g)R_2 = I\alpha.$$

We can now eliminate the linear acceleration as

$$(m_1 + m_3)(g - \alpha R_1)R_1 - m_2(\alpha R_2 + g)R_2 = I\alpha.$$

Rearranging this, we have

$$(m_1 + m_3)gR_1 - m_2 gR_2 = \alpha(m_1 + m_3)R_1^2 + \alpha m_2 R_2^2 + I\alpha,$$

or

$$g\left[(m_1 + m_3)R_1 - m_2 R_2\right] = \alpha\left[(m_1 + m_3)R_1^2 + m_2 R_2^2 + I\right].$$

Hence

$$\boxed{\alpha = \frac{g\left[(m_1 + m_3)R_1 - m_2 R_2\right]}{(m_1 + m_3)R_1^2 + m_2 R_2^2 + I}}.$$