Physics 101H

General Physics 1 – Honors

Lecture 8 – 9/13/23

Uniform circular motion

Problem sets

Problem Set 2 has been posted

Due by the **start of class** on **Wednesday 20 September**

Remember

- I will **drop the lowest grade** on your weekly Problem Sets

MAKING SURE YOUR WORK IS LEGIBLE IS *YOUR* RESPONSIBILITY

Virtual Help Desk

Trevor Tingle - graduate student, course grader and 101 expert - will be hosting weekly, virtual, drop-in, help desk hours!

Wed 1:00 -3:00 PM

Thurs 1:00 - 3:00 PM

Fri 9:30 - 11:00 AM

Virtual Help Desk zoom link:

https://cwm.zoom.us/j/95241418043?pwd=WlYySDVjaWczd2laSWhGNWVwUUNPQT09

Summary

Topics

Yesterday: kinematics in 2D

Today: kinematics in 2D [chapter 4]

- Examples in 2D
- Uniform circular motion

Announcements

Today: **Problem set 1 due**
Problem set 2 assigned

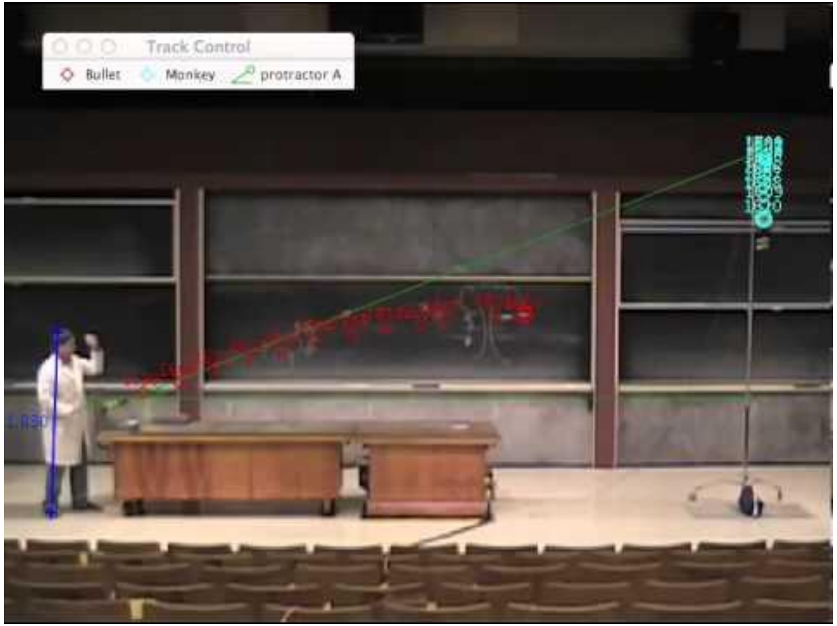
Tomorrow: **Quiz 2**

In on dimensional kinematics, vectors are distinguished from scalars by having a direction that is denoted by positive or negative values.

The boiling point of nitrogen is -195.795 C.
Is temperature a vector or a scalar quantity?

Example 8.1: A park ranger wishes to tranquilise a Virginia Northern Flying Squirrel that is sitting on the edge of a branch of a tree. The sound of the tranquiliser gun will scare the Flying Squirrel and it will jump from the branch the instant the ranger fires their tranquiliser gun. Where should the ranger aim?

Example: A park ranger wishes to tranquilise a Virginia Northern Flying Squirrel that is sitting on the edge of a branch of a tree. The sound of the tranquiliser gun will scare the Flying Squirrel and it will jump from the branch the instant the ranger fires their tranquiliser gun. Where should the ranger aim?

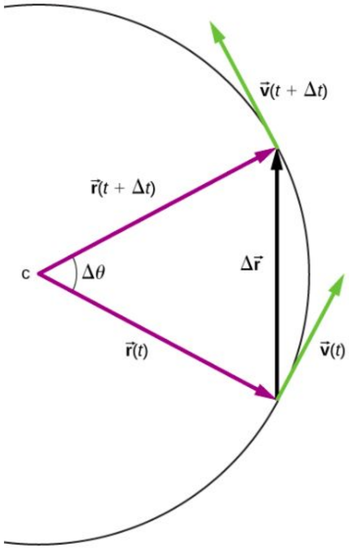


Is an object that is moving in a circle with constant speed accelerating?

Uniform circular motion

Uniform circular motion is motion in a circle at a constant speed

Special case of motion in 2D with constant acceleration



Period – time taken to complete one revolution

Angular speed – rate at which the angle changes

Summary

Topics

Today: kinematics in 2D [chapter 4]

- Examples in 2D
- Uniform circular motion

Tomorrow: kinematics in 2D [chapter 4]

- Examples in 24

Announcements

Today:	**Problem set 1 due**
	Problem set 2 assigned
Tomorrow:	**Quiz 2**

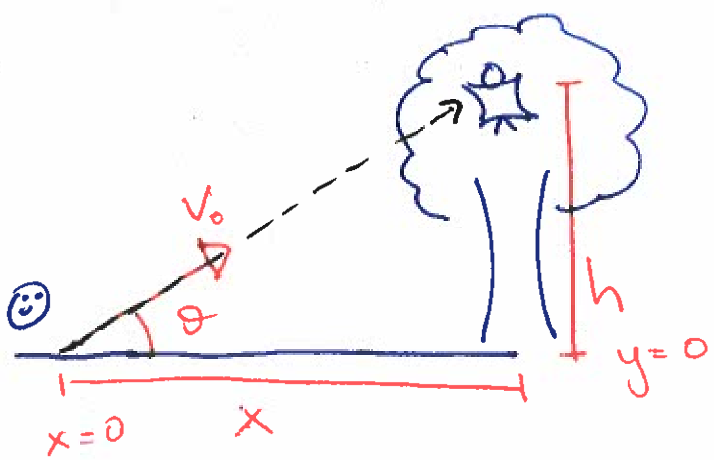
PHYSICS 101 - HONORS

Lecture 8 9/13/23

Squirrel example

Free fall occurs <u>independent</u> of horizontal motion

⇒ the squirrel falls as fast as the projectile



Kinematic equations for the dart

$$x_d = (v_0 \cos\theta)t \quad (1)$$

$$y_d = -\frac{1}{2}gt^2 + (v_0 \sin\theta)t + 0 \quad (2)$$

And for the squirrel

$$x_s = x \quad (3)$$

$$y_s = -\frac{1}{2}gt^2 + h \quad (4)$$

A hit requires $x_d = x_s$ and $y_d = y_s$ at time t

using (1): $t = \frac{x_d}{v_0 \cos\theta} = \frac{x}{v_0 \cos\theta}$ ($x_d = x_s = x$ (3))

(2) ⇒ $y_d = -\frac{g}{2}\left(\frac{x}{v_0 \cos\theta}\right)^2 + v_0 \sin\theta \cdot \left(\frac{x}{v_0 \cos\theta}\right)$

and (4) → $y_s = -\frac{g}{2}\left(\frac{x}{v_0 \cos\theta}\right)^2 + h$

Check $y_d \stackrel{?}{=} y_s$:

$$y_d = -\frac{g}{2}\left(\frac{x}{v_0 \cos\theta}\right)^2 + x\tan\theta = -\frac{g x^2}{2 v_0^2 \cos^2\theta} + x \cdot \left(\frac{h}{x}\right)$$

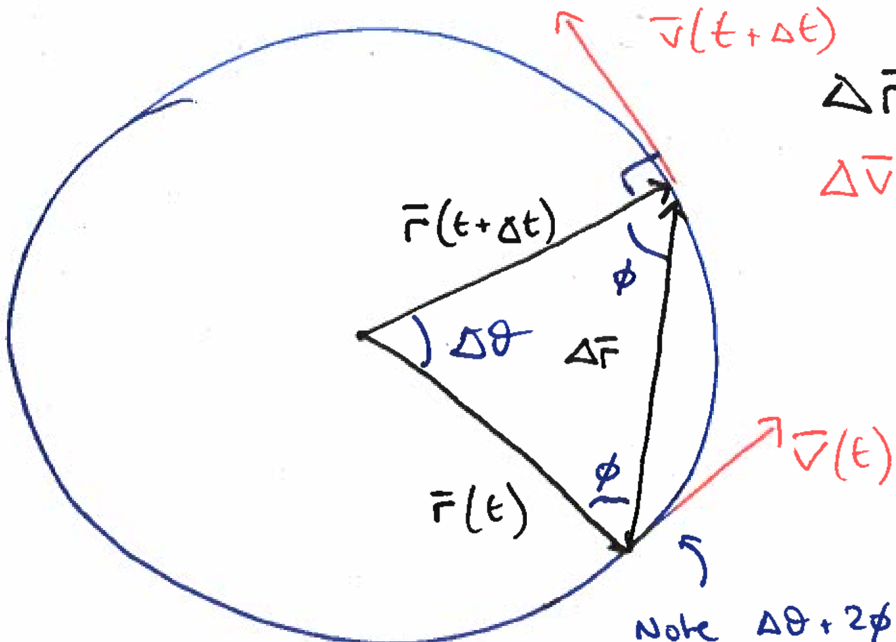
$$= -\frac{g}{2}\frac{x^2}{v_0^2 \cos^2\theta} + h = y_s \ ✓$$

:)

Uniform circular motion (slide 8)

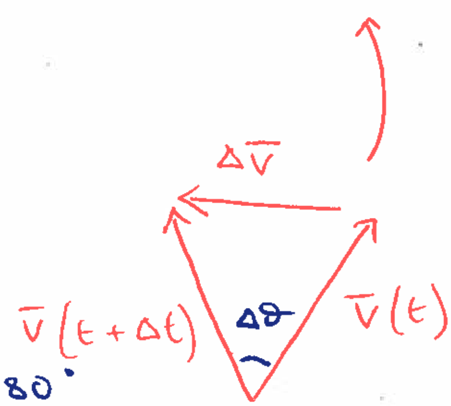
Acceleration is a <u>vector</u> ⇒ a nonzero acceleration can mean that the particle's <u>speed</u> can change or its direction!

Uniform circular motion is an example of the latter.



$$\Delta\bar{r} = \bar{r}(t+\Delta t) - \bar{r}(t)$$

$$\Delta\bar{v} = \bar{v}(t+\Delta t) - \bar{v}(t)$$



Note $\Delta\theta + 2\phi = 180°$

$\phi = 90 - \frac{\Delta\theta}{2}$

Clearly $\frac{d\bar{v}}{dt} \neq 0 \Rightarrow \bar{a} \neq 0$!

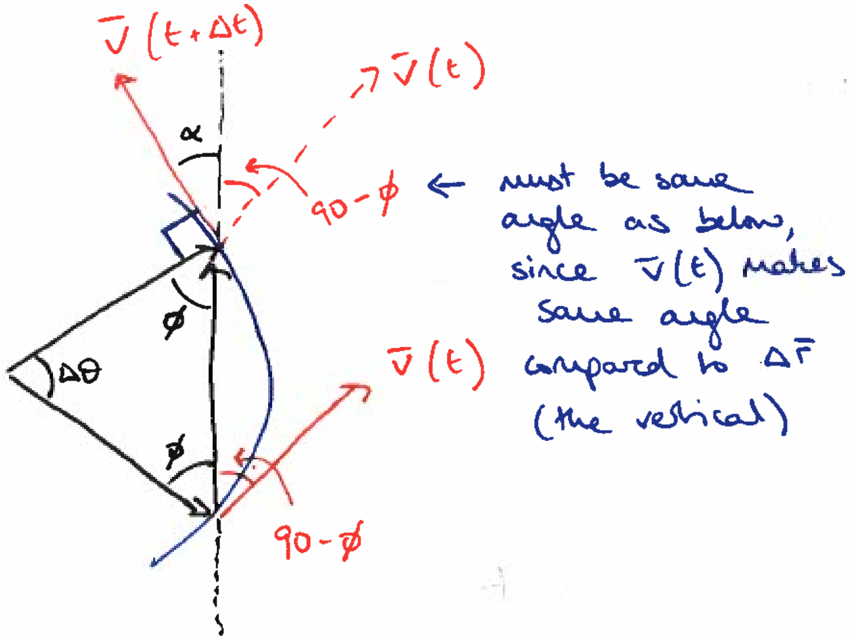
Let's see what we can figure out about $\bar{a}$

1) we will show $|\bar{a}| = \frac{v^2}{r}$

2) we will also show $\bar{a} \propto -\bar{r}$

$\Big\}$ $\bar{a} = -\frac{v^2}{r}\hat{r}$

Let's look at the geometry in more detail



The angle α must be

$$\alpha = 90 - \phi$$

because sum of angles from $\Delta\bar{r}$ to vertical is

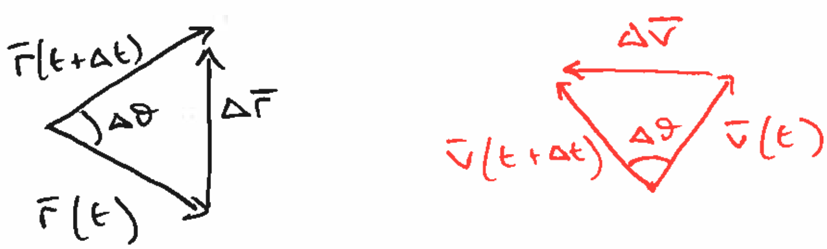
$$\phi + 90 + \alpha = 180$$

$$\Rightarrow \alpha = 90 - \phi$$

Therefore the angle between $\bar{v}(t)$ and $\bar{v}(t+\Delta t)$ is

$$\alpha + 90 - \phi = 180 - 2\phi = \Delta\theta \; !$$

We see that we have two <u>similar</u> triangles



They are isosceles triangles because $|\bar{r}(t)| = |\bar{r}(t+\Delta t)|$
and $|\bar{v}(t)| = |\bar{v}(t+\Delta t)|$

$$\Rightarrow \frac{|\Delta\bar{v}|}{|\bar{v}|} = \frac{|\Delta\bar{r}|}{|\bar{r}|}$$

$$\Rightarrow |\Delta\bar{v}| = \frac{|\Delta\bar{r}|}{|\bar{r}|}\,|\bar{v}|$$

or $$\Delta v = \frac{v}{r}\,\Delta r$$

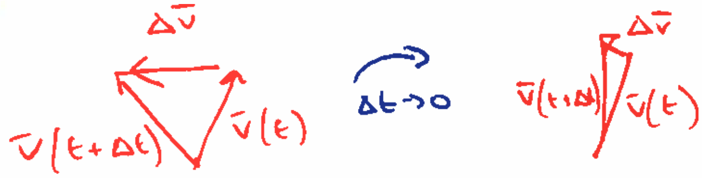
The rate of change of these quantities is

$$\frac{\Delta v}{\Delta t} = \frac{v}{r}\frac{\Delta r}{\Delta t} \quad \text{or} \quad a = \frac{v}{r}\cdot v = \frac{v^2}{r}$$

↑ for $\lim_{\Delta t \to 0}$ ← we have shown 1) !

To show 2), we note that as $\Delta t \to 0$, so does $\Delta\theta$

In this limit, $\Delta \bar{v}$ is perpendicular to $\bar{v}$



Since $\bar{v}$ is tangent to the circle, this means $\Delta\bar{v}$ must point towards the centre of the circle ! ← we have shown 2!

Period and speed

$$2\pi r = vT \quad \Rightarrow \quad \boxed{T = \frac{2\pi r}{v}}$$

$2\pi r$: circumference

vT: speed · time for one revolution (the period)

Define the angular speed $\omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}$

For uniform circular motion ω is constant

$$\Rightarrow \int \omega \, dt = \int \frac{d\theta}{dt} dt = \int d\theta$$

Integrating over one period

$$\omega \int_0^T dt = \int_0^{2\pi} d\theta \Rightarrow \omega T = 2\pi \quad \text{or} \quad \boxed{\omega = \frac{2\pi}{T}}$$

In fact, ω is also an <u>angular frequency</u>!

Since $T = \frac{2\pi r}{v}$, we also have

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{(2\pi r/v)} = \frac{v}{r} \quad \text{or} \quad \boxed{v = \omega r}$$

This means the acceleration can also be written as

$$a = \frac{v^2}{r} = \frac{(\omega r)^2}{r} = \omega^2 r \qquad \boxed{a = \frac{v^2}{r} \quad a = \omega^2 r}$$

<u>Equation Summary</u>

Period $\quad T = \frac{2\pi r}{v} \quad T = \frac{2\pi}{\omega}$

Centripetal acceleration $\quad a = \frac{v^2}{r} \quad a = \omega^2 r$

Angular speed/frequency $\quad \omega = \frac{d\theta}{dt} \quad \omega = v/r$