Physics 101H

General Physics 1 – Honors

Lecture 6 – 9/8/23

1D and 2D motion

Welcome!

I am Tangereen

Pronouns: they/them/their

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Problem sets

Problem Set 1 is due by the **start of class** on **Wednesday 14 September**

Remember

- There are two components – online and handwritten
- Chris will **drop the lowest grade** on your weekly Problem Sets

MAKING SURE YOUR WORK IS LEGIBLE IS *YOUR* RESPONSIBILITY

Summary

Topics

This week

- Vector products
- Describing motion in 1D
- Position, velocity, acceleration

Next week

- Circular motion
- Forces

Today: kinematics in 1 and 2D [chapters 3/4]

- Examples in 1D
- Describing motion in 2D
- Position, velocity, acceleration
- Constant acceleration
- Projectile motion

Monday: examples

Announcements

Wednesday: Problem set 1 due
Problem set 2 assigned

Summary

Kinematics is the description of the motion of objects

Dynamics is the explanation of the cause of the motion of objects

Position – vector describing where object is relative to some reference frame

Displacement – change in position [a vector]

Distance – total length of journey [a scalar]

Velocity – rate of change of position [a vector]

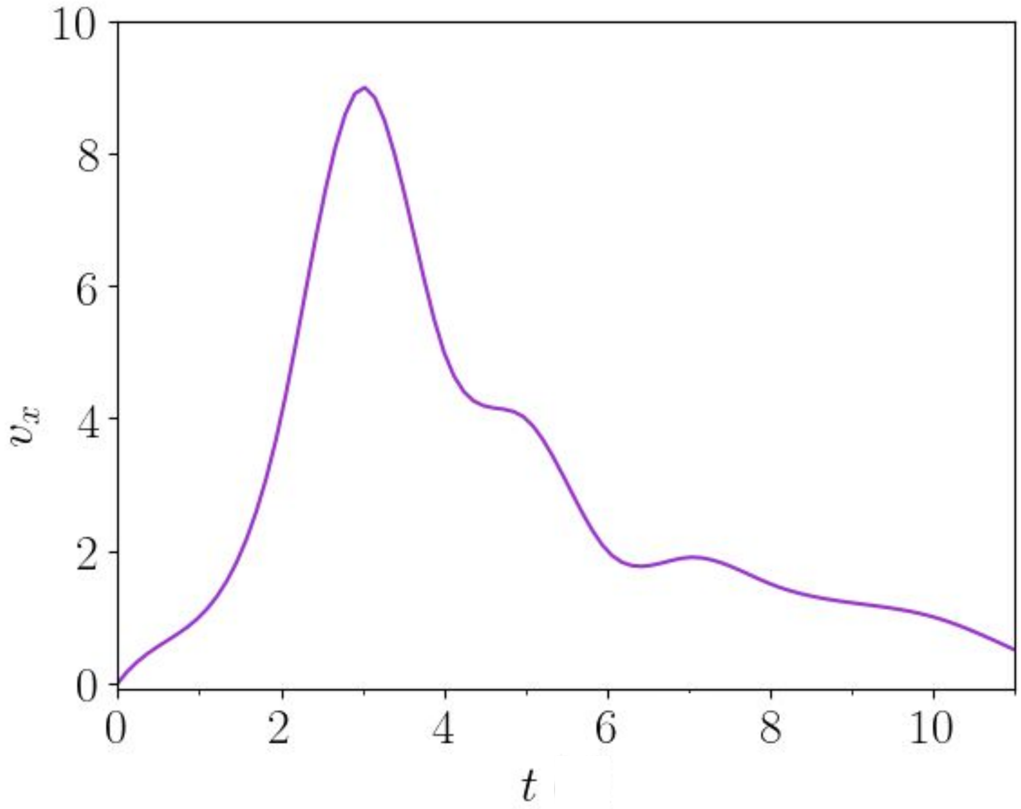
Speed – the magnitude of the velocity [a scalar]

Acceleration – rate of change of velocity [a vector]

1D motion

Example: Velocity vs. Time Graph

- What's wrong with this graph?
- What is the average acceleration between t = 0 and t = 6 s?
- When does the acceleration have its greatest positive value?
- When is the object at rest?
- When is the acceleration zero?
- About how far did the object move?



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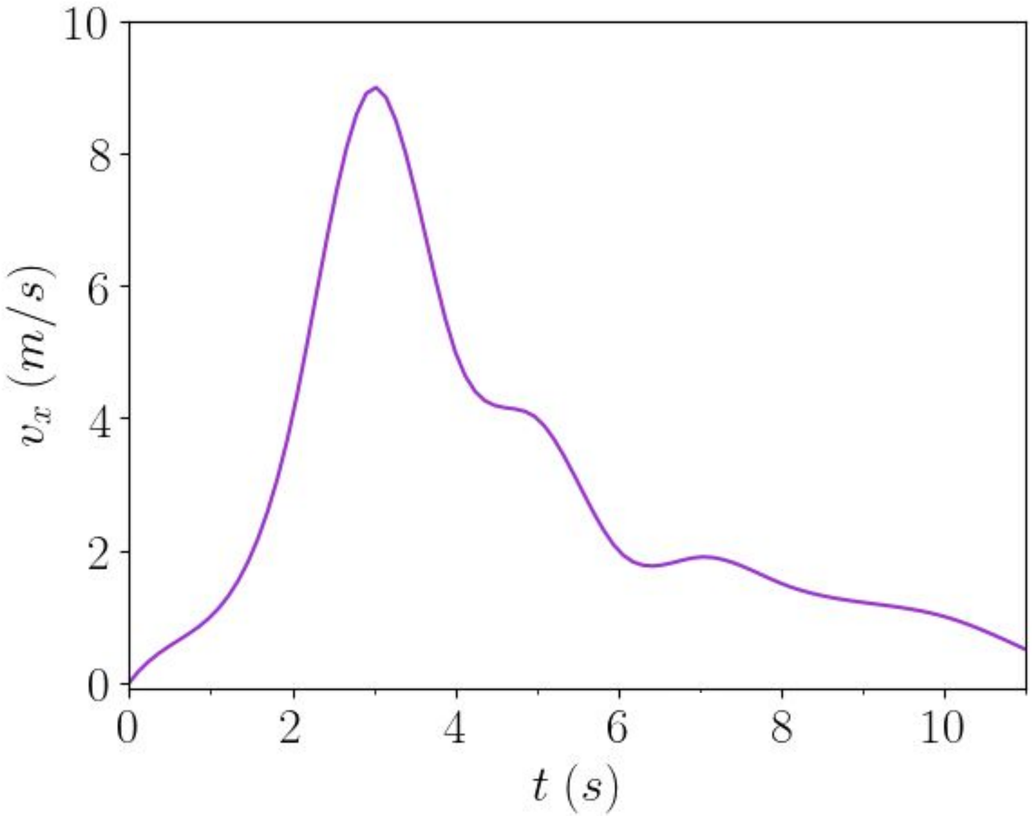

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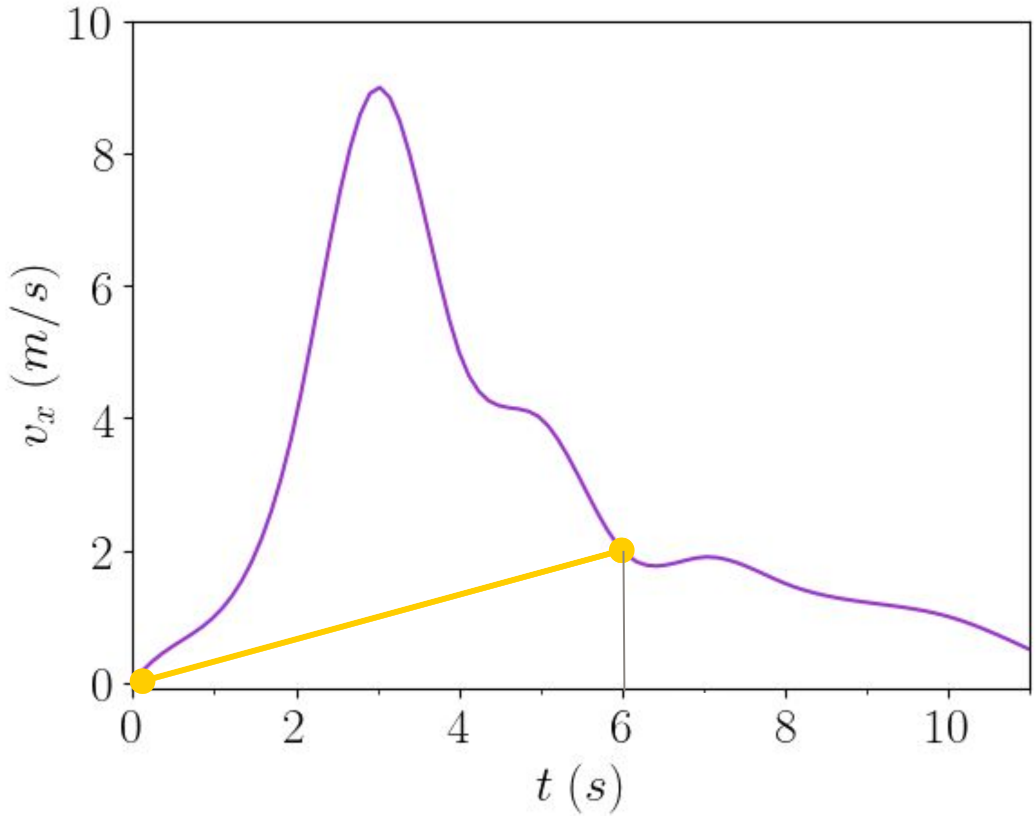

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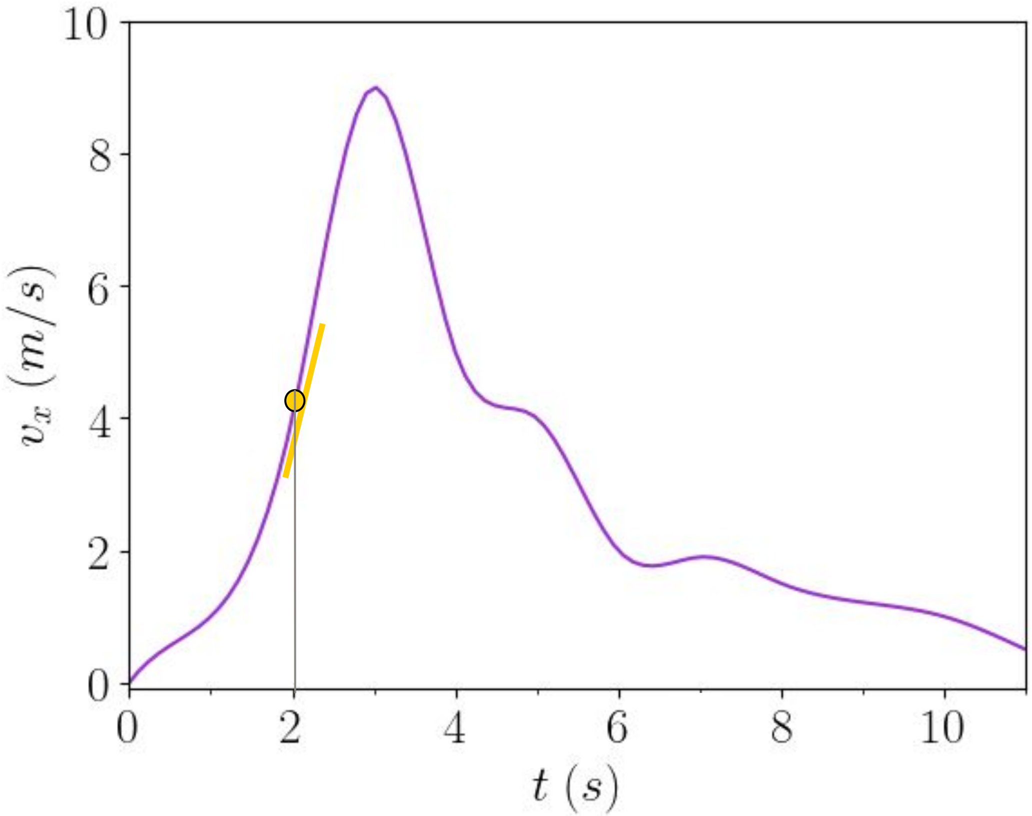

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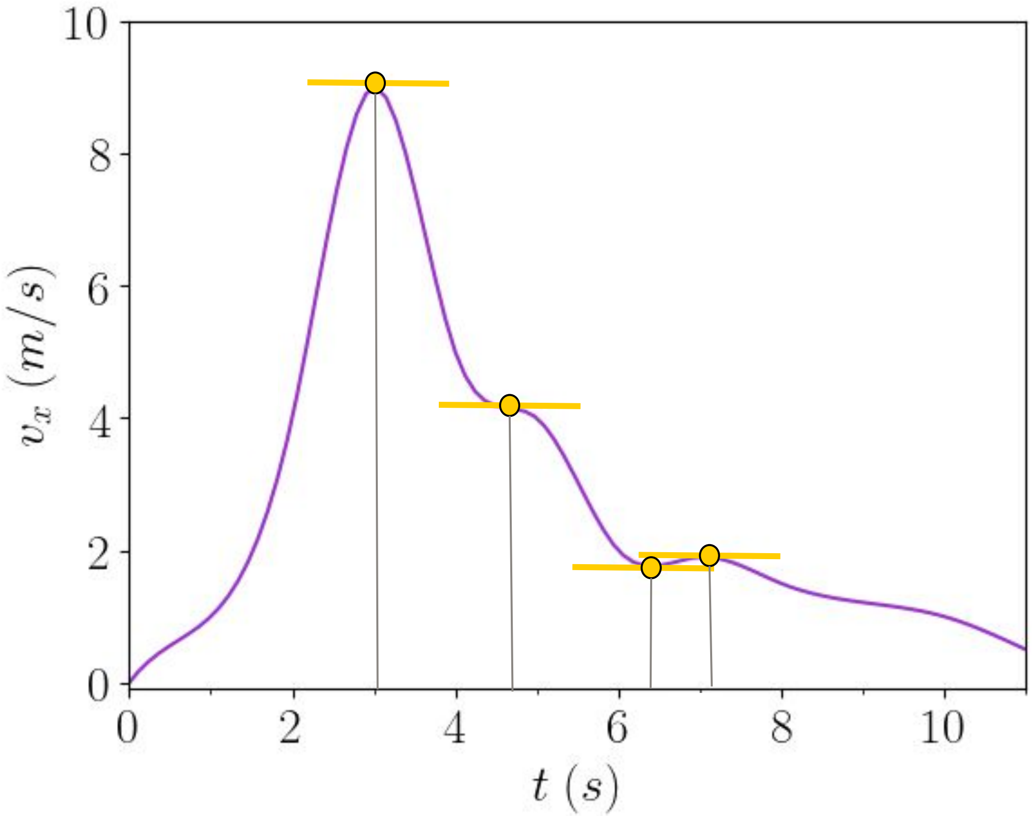

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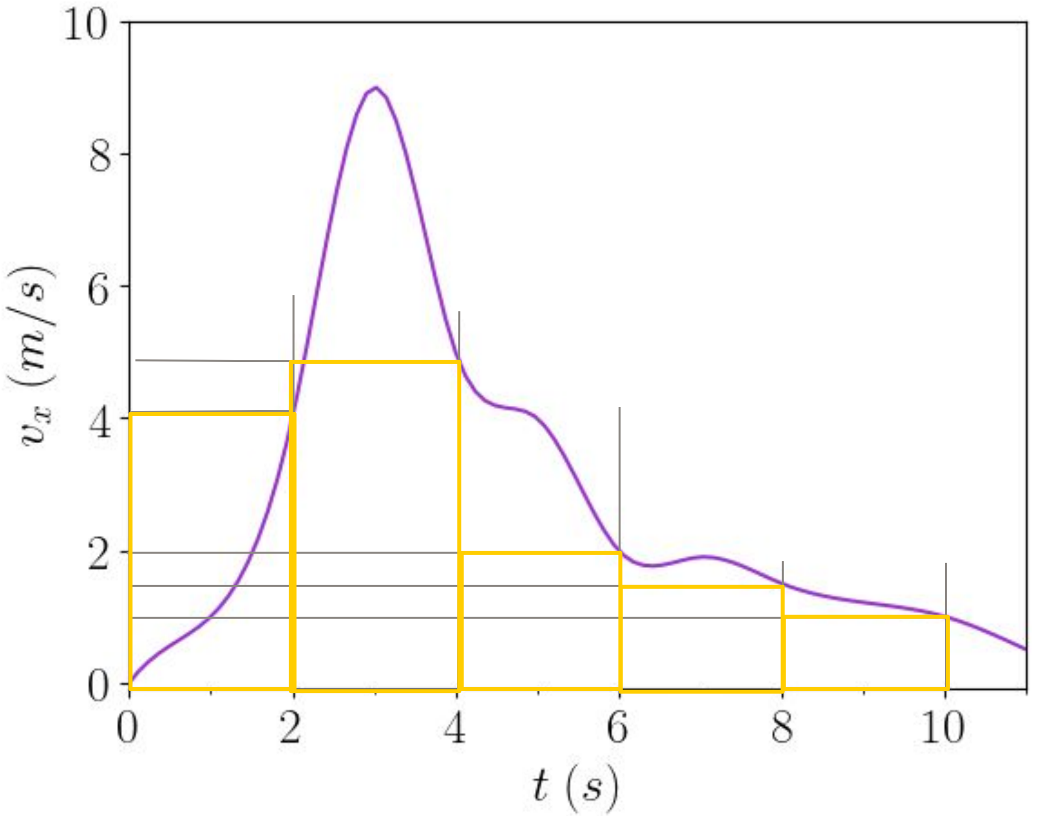


Multiple choice

Instructions: Consider the following question. After you have had a chance to think, I will ask you to raise your hands to indicate your answer.

Questions: 1. Can velocity be constant while acceleration is nonzero?

(a) yes (b) no

2. What about speed?

(a) yes (b) no

3. Does motion in one direction affect the motion in a perpendicular direction?

(a) yes (b) no

Perpendicular coordinate system

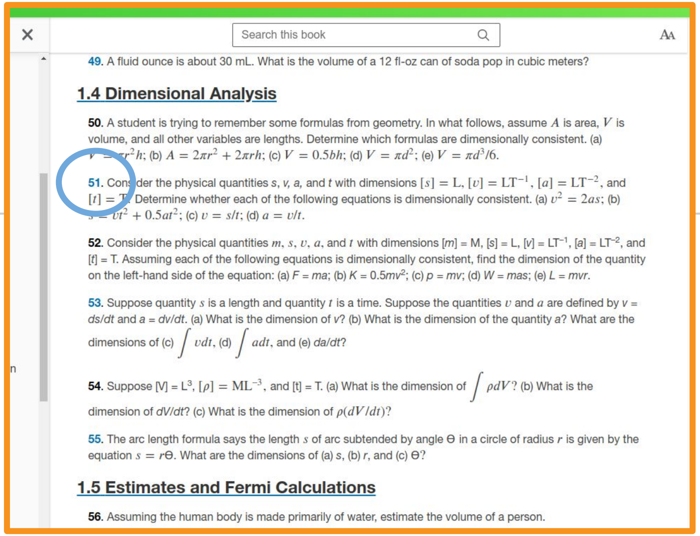
- Why do we specify perpendicular axes? Can we have nonperpendicular axes?
- Cartesian coordinates (x and y) are perpendicular, what about polar coordinates (r and θ)?

Want more practice?

Check out the following problems in the textbook

In Chapter 3:

- Conceptual questions: 5, 7, 15, 19
- Problems: 27, 31, 37, 63, 69, 79, **113**



Search this book

49. A fluid ounce is about 30 mL. What is the volume of a 12 fl-oz can of soda pop in cubic meters?

1.4 Dimensional Analysis

50. A student is trying to remember some formulas from geometry. In what follows, assume A is area, V is volume, and all other variables are lengths. Determine which formulas are dimensionally consistent. (a) $V = \pi r^2 h$; (b) $A = 2\pi r^2 + 2\pi rh$; (c) $V = 0.5bh$; (d) $V = \pi d^2$; (e) $V = \pi d^3/6$.

51. Consider the physical quantities s, v, a, and t with dimensions $[s] = \mathrm{L}$, $[v] = \mathrm{LT^{-1}}$, $[a] = \mathrm{LT^{-2}}$, and $[t] = \mathrm{T}$. Determine whether each of the following equations is dimensionally consistent. (a) $v^2 = 2as$; (b) $s = vt^2 + 0.5at^2$; (c) $v = s/t$; (d) $a = v/t$.

52. Consider the physical quantities m, s, v, a, and t with dimensions $[m] = \mathrm{M}$, $[s] = \mathrm{L}$, $[v] = \mathrm{LT^{-1}}$, $[a] = \mathrm{LT^{-2}}$, and $[t] = \mathrm{T}$. Assuming each of the following equations is dimensionally consistent, find the dimension of the quantity on the left-hand side of the equation: (a) $F = ma$; (b) $K = 0.5mv^2$; (c) $p = mv$; (d) $W = mas$; (e) $L = mvr$.

53. Suppose quantity s is a length and quantity t is a time. Suppose the quantities v and a are defined by $v = ds/dt$ and $a = dv/dt$. (a) What is the dimension of v? (b) What is the dimension of the quantity a? What are the dimensions of (c) $\int v\,dt$, (d) $\int a\,dt$, and (e) da/dt?

54. Suppose $[V] = \mathrm{L^3}$, $[\rho] = \mathrm{ML^{-3}}$, and $[t] = \mathrm{T}$. (a) What is the dimension of $\int \rho\,dV$? (b) What is the dimension of dV/dt? (c) What is the dimension of $\rho(dV/dt)$?

55. The arc length formula says the length s of arc subtended by angle Θ in a circle of radius r is given by the equation $s = r\Theta$. What are the dimensions of (a) s, (b) r, and (c) Θ?

1.5 Estimates and Fermi Calculations

56. Assuming the human body is made primarily of water, estimate the volume of a person.

Note that answers are provided for questions with blue numbers (odd numbered)

Click on the number to be taken to the answer.

But make sure you at least **try** the problem first!

2D motion

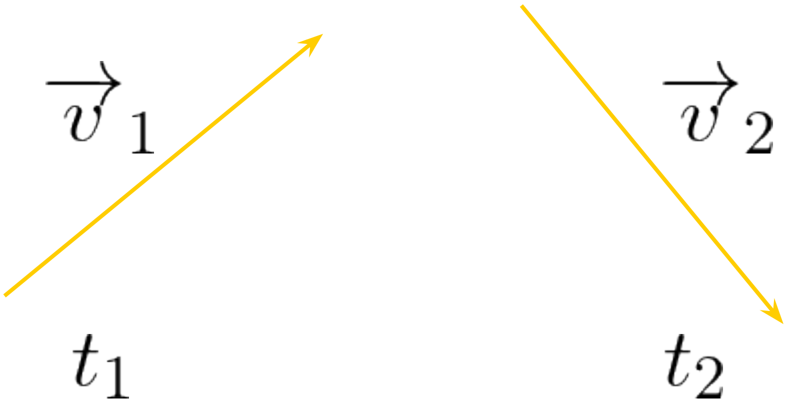
Motion gets a little more interesting

Acceleration

Acceleration is the rate of change of velocity

But velocity is a vector and can therefore change in two ways!

- Magnitude can change
- Direction can change



$$\vec{a}_{ave} = \frac{\Delta \vec{v}}{\Delta t}$$

$$\vec{a}_{inst} = \frac{d\vec{v}}{dt}$$

Constant acceleration in 2D

Motion in perpendicular directions is **totally independent**!

Write vectors in terms of components along perpendicular directions and treat them completely separately

Our 1D equations from Chapter 3 apply to each component separately

$$\vec{a} = a_x\hat{x} + a_y\hat{y}$$

$$v_x(t) = a_x t + v_{x,0} \qquad v_y(t) = a_y t + v_{y,0}$$

$$\vec{v}(t) = v_x(t)\hat{x} + v_y(t)\hat{y}$$

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$$v_x(t) = a_x t + v_{x,0} \qquad v_y(t) = a_y t + v_{y,0}$$

$$x(t) = \frac{1}{2} a_x t^2 + v_{x,0} t + x_0 \qquad y(t) = \frac{1}{2} a_y t^2 + v_{y,0} t + y_0$$

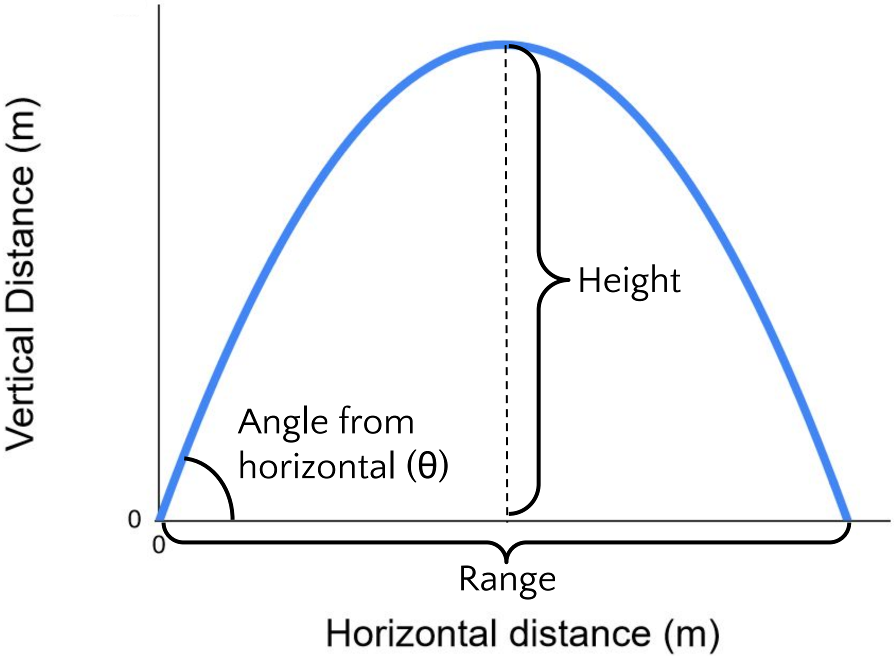
$$\vec{d}(t) = x(t)\hat{x} + y(t)\hat{y}$$

Projectile motion: Constant vertical acceleration due to gravity, no horizontal acceleration

A **projectile** will follow a **parabolic trajectory**

The distance travelled is the **range**

The maximum height reached is the **height**



Projectile motion: Constant vertical acceleration due to gravity, no horizontal acceleration

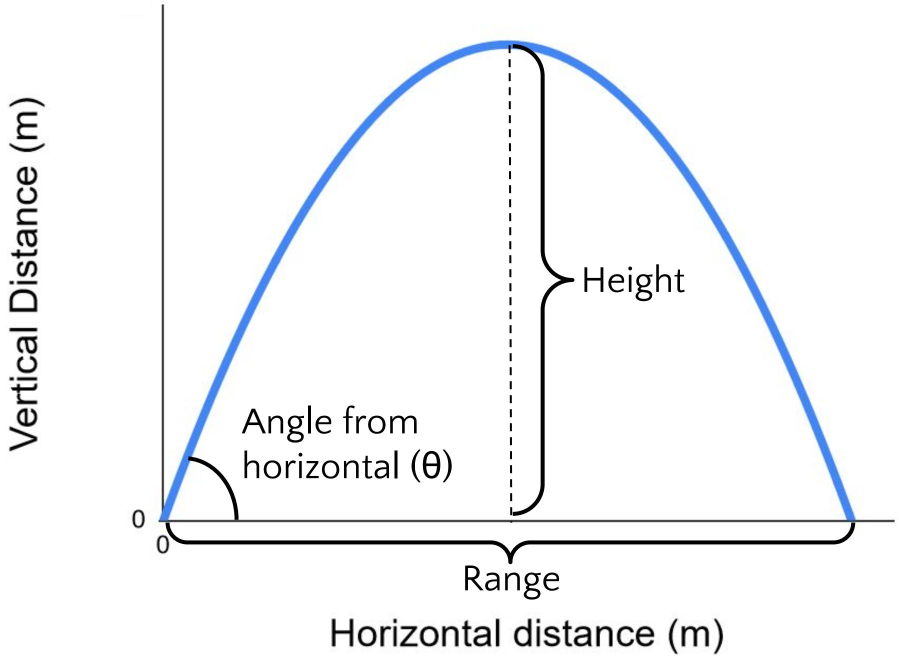
$$\vec{a} = -g\hat{y}$$

$$v_x(t) = v_{x,0}$$

$$x(t) = v_{x,0}t + x_0$$

$$v_y(t) = -gt + v_{y,0}$$

$$y(t) = -\frac{1}{2}gt^2 + v_{y,0}t + y_0$$



Here, g=9.8 m/sˆ2, so acceleration due to gravity is –g

Monday

- Break into small (randomly generated) groups
- Work through examples
- Write results neatly
- Review other groups' results

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Physics 101 - Honors

Lecture 6 9/8/23

1D Motion (Slides 6-14)

* What is wrong with the graph on slide 7?
  The units of the variables on the axes aren't labeled!

* What is the average acceleration between $t = 0s$ and $t = 6s$?
  The average acceleration is the difference in ~~acceleration~~ velocity between the two end points, divided by the difference in time

$$a_{ave} = \frac{\Delta v}{\Delta t} = \frac{2m/s - 0m/s}{6s - 0s} = \frac{1}{3}\ m/s^2 = 0.67\ m/s^2$$

* When does the acceleration have its greatest positive value?
  When the tangent to the velocity has the greatest positive slope.

* When is the object at rest?
  When the velocity is 0m/s, so only at $t = 0s$.

* When is the acceleration zero?
  When the slope of the tangent to the velocity is zero.

* About how far did the object move?
  Velocity is the derivative of position with respect to time, so position is the integral of velocity with respect to time, or

$$\Delta x = \int_{t_i}^{t_f} v(t)\, dt.$$

The integral of a function measures the area below its curve, so we can estimate the total distance by estimating the area under the velocity curve. Using the boxes on slide 12,

$$\Delta x \approx (2s)\left(4\frac{m}{s} + 5\frac{m}{s} + 2\frac{m}{s} + 1.5\frac{m}{s} + 1\frac{m}{s}\right)$$
$$= 27m.$$

We could improve this estimate by using more, narrower boxes.

⇒ Conceptual Questions (Slide 13)

* Can velocity be constant while acceleration is nonzero?
  No! Acceleration is the rate of change of velocity, so if velocity doesn't change, there is no acceleration.

* Can speed be constant while acceleration is nonzero?
  Yes! If the direction is changing.

* Does motion in ~~a perpendicular~~ one direction affect the motion in a

Perpendicular Coordinate Systems (Slide 14)

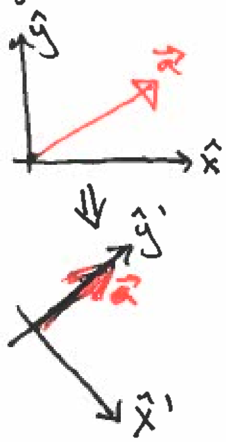
★ Why do we specify axes? Can we have nonperpendicular axes?

Mathematically, yes, but it's very annoying for our purposes. We are always free to choose our coordinate system, so we are free to choose one that makes our life easiest, such as perpendicular axes.

We can also move and rotate our axes to simplify things. Suppose I had an object with acceleration

$$\vec{a} = a_x \hat{x} + a_y \hat{y}$$

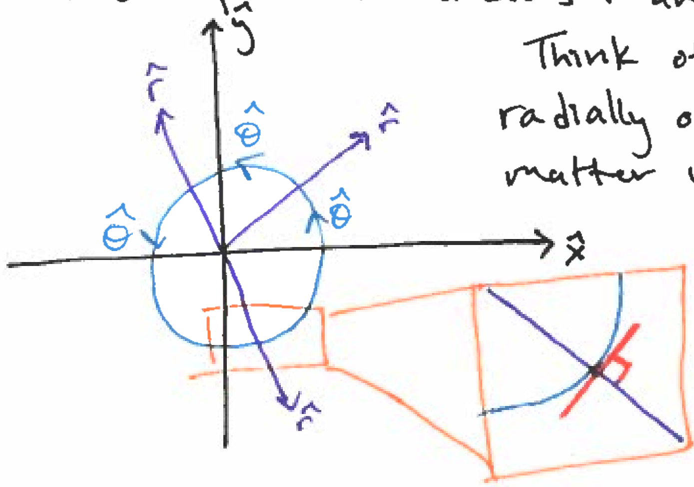
for some coordinate system $\hat{x}, \hat{y}$.



I can choose to define a new coordinate system so that one of the axes is parallel to $\vec{a}$. Then all of the acceleration is in that one direction and there is no acceleration in the perpendicular direction. Now

$$\vec{a} = 0\hat{x}' + \sqrt{a_x^2 + a_y^2}\,\hat{y}'.$$

★ Are the polar coordinates $\hat{r}$ and $\hat{\theta}$ perpendicular?



Think of the $\hat{r}$ direction as pointing radially out from the origin. It doesn't matter where in Cartesian space. Then think of the $\hat{\theta}$ "axis" like a circle centered on the origin. At any point where an $\hat{r}$ "axis" intersects the $\hat{\theta}$ circle, the two are perpendicular.

This means that, like Cartesian coordinates, we can treat the polar coordinates $\hat{r}$ and $\hat{\theta}$ independently.

Acceleration in 2D (Slide 17)

To move from 1D motion to 2D (or higher) we change from scalars to vectors, but the basic definitions are still the same. For a displacement vector $\vec{d}$, the velocity is still defined as

$$\vec{v} = \frac{d\vec{d}}{dt}$$

and the acceleration as

$$\vec{a} = \frac{d\vec{v}}{dt}.$$

Since velocity is a vector, it can change in magnitude or direction or both.

Constant Acceleration in 2D (Slides 18 and 19)

Since motion in perpendicular directions is independent, we can decompose vectors into their perpendicular components and deal with them individually. For an acceleration

$$\vec{a} = a_x \hat{x} + a_y \hat{y}.$$

where a_x and a_y are constant, we can integrate to find the velocity, just like in 1D. Then

$$v_x(t) = a_x t + v_{x,0}$$

and

$$v_y(t) = a_y t + v_{y,0}.$$

Putting them back together, the velocity vector is

$$\vec{v}(t) = v_x(t)\hat{x} + v_y(t)\hat{y}$$
$$= (a_x t + v_{x,0})\hat{x} + (a_y t + v_{y,0})\hat{y}.$$

Integrating again, we get the displacement vector

$$\vec{d}(t) = x(t)\hat{x} + y(t)\hat{y}$$
$$= \left(\tfrac{1}{2}a_x t^2 + v_{x,0}t + x_0\right)\hat{x} + \left(\tfrac{1}{2}a_y t^2 + v_{y,0}t + y_0\right)\hat{y}.$$

Projectile Motion (Slides 20 and 21)

A special case of constant acceleration in 2D is when we have only acceleration in the vertical direction, due to gravity, and no acceleration in the horizontal direction. This results in parabolic motion: the object follows a parabola.

The total distance travelled horizontally is often called the range. The maximum height reached in the vertical direction is called the height. We use θ to describe the angle that the initial velocity makes with the horizontal.

For a coordinate system with up vertically as the positive $\hat{y}$ direction,

$$\vec{a} = 0\hat{x} - g\hat{y}.$$

Then our kinematic equations are

$$v_x(t) = v_{x,0} \qquad v_y(t) = -gt + v_{y,0}$$

$$x(t) = v_{x,0}t + x_0 \qquad y(t) = -\frac{1}{2}gt^2 + v_{y,0}t + y_0 .$$

Conceptual Question (Slide 22)

A cart on a roller coaster rolls down the track in the figure. As the cart passes the point shown, what happens to its speed and acceleration in the direction of motion?