Physics 101H

General Physics 1 – Honors

Lecture 49 – 12/01/23

Special Relativity

Summary

See https://openstax.org/books/college-physics-2e/ chapter 28 and https://openstax.org/details/books/university-physics-volume-3 chapter 5

Topics

Yesterday: Doppler effect [chapter 17]

- Standing sound waves
- Doppler effect

Today: Special relativity

- Galilean relativity
- Special relativity
- Time dilation

Announcements

Wednesday December 6:	**Problem Set 8 due**
Monday December 11:	**Final exam 9 am to 12 midday**

Nonrelativistic dynamics

Everything we have studied so far applies to objects that are:

- Medium-sized
- Slow-moving

Once we start to study objects that fall outside of this range, **Newtonian mechanics** (Newton's laws and equivalent formulations) no longer applies

- Very heavy/large objects – require **general relativity**
- Very small objects – require **quantum mechanics**
- Very fast objects – require **special relativity**

Special relativity

Our familiar Newtonian laws of motion are low-speed approximations

More complete description is captured by **special relativity**

Based on a key insight

> Speed of light in a vacuum is the same in **all** inertial reference frames.

Relativity

Galilean relativity: laws of **motion** are the same in all reference frames

Special relativity: laws of **physics** are the same in all reference frames

Not just the laws of motion, but everything! Electricity and magnetism, thermodynamics, fluid mechanics…

This is **very different** than Galilean relativity and leads to profound differences and counter-intuitive effects (until you get used to them…)

Relativity of time

Moving observers experience **time dilation**

Need to introduce the concept of **proper time** – the time measured by an observer at rest with respect to a given clock

This effect **is real**, but you have to go **really fast** to notice it

Practice in pairs

Instructions: Discuss the following question with a neighbour. Your answers will not be graded; your discussion is for your own learning*.

Question: Estimate the level of time dilation that occurs when you are driving down the freeway at a constant speed of 100 km/h. What if you drove at ten percent of the speed of light (in a vacuum)? And how fast is that?

Summary

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Topics

Today: Special relativity

- Galilean relativity
- Special relativity
- Time dilation

Monday: Special relativity II

- Length contraction
- Lorentz transformations
- Relativistic momentum and energy

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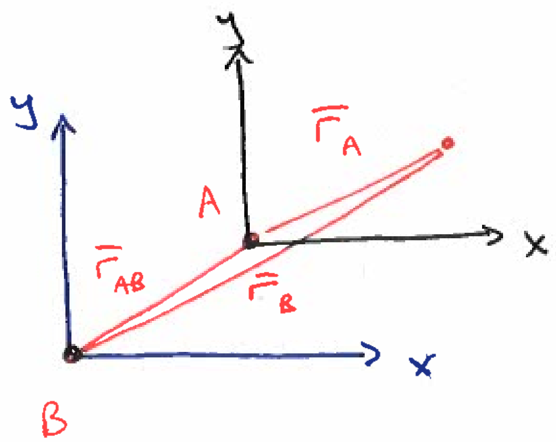
PHYSICS 101 – HONORS

Lecture 49 12/1/23

Relativity (slide 5)



Recall that if we have two inertial frames of reference in relative motion, then

$$\bar{r}_{AB} = \bar{V}_{AB}\, t$$

$$\Rightarrow \bar{r}_B = \bar{r}_{AB} + \bar{r}_A$$

$$= \bar{V}_{AB} t + \bar{r}_A$$

And for an object moving relative to the origin in A

$$\frac{d\bar{r}_B}{dt} = \bar{V}_{AB} + \frac{d\bar{r}_A}{dt} \Rightarrow \bar{u}_B = \bar{V}_{AB} + \bar{u}_A$$

← $\bar{u}_A = \frac{d\bar{r}_A}{dt}$

↑ $\bar{u}_B = \frac{d\bar{r}_B}{dt}$

↖ velocities add!

Special relativity is inconsistent with this!

Because if $\bar{u}_B = c$ then $\bar{u}_A = c$, but we have

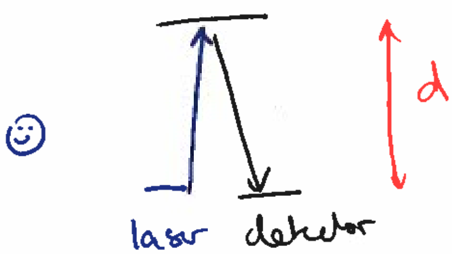
$$\bar{u}_B = \bar{V}_{AB} + \bar{u}_A \Rightarrow c = \bar{V}_{AB} + c \quad \text{for } \bar{V}_{AB} \neq 0$$

↖ not consistent!

Relativity of time (slide 6)

Special relativity leads us to the relativity of time: different observers measure different times (!!!)

Consider a clock built from a laser and a mirror (and a detector)



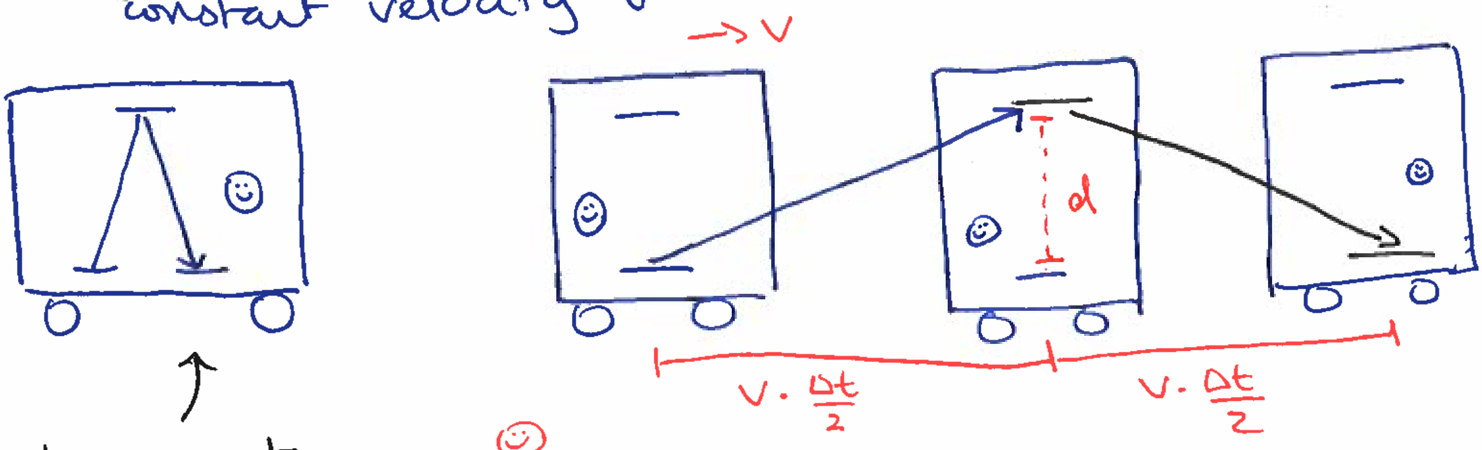
the time taken for the laser to travel up to the mirror, reflect and be measured in the detector is

$$\Delta t_p = \frac{2d}{c}$$

← distance

← speed

Now imagine we place this clock on a smooth railway track in a closed container, moving at constant velocity $\bar{v}$



observer at rest with respect to the clock

observer moving with respect to the clock

The observer at rest measures the time taken as

$$\Delta t_p = \frac{2d}{c}$$

But an observer outside sees that the light travels a longer distance! The moving observer sees that the light now has to travel a total distance

$$2 \cdot \sqrt{d^2 + \left(\frac{v\Delta t}{2}\right)^2}$$

and this must equal the time times the speed, so

$$2\sqrt{d^2 + \left(\frac{v\Delta t}{2}\right)^2} = c\Delta t$$

We now solve for Δt

$$\sqrt{d^2 + \frac{v^2\Delta t^2}{4}} = \frac{c\Delta t}{2}$$

$$\Rightarrow \quad d^2 + \frac{v^2\Delta t^2}{4} = \frac{c^2\Delta t^2}{4}$$

$$\Rightarrow \quad d^2 = \frac{c^2}{4}\Delta t^2 - \frac{v^2}{4}\Delta t^2$$

$$\Rightarrow \quad d^2 = \left(\frac{c^2}{4} - \frac{v^2}{4}\right)\Delta t^2$$

$$\Rightarrow \quad 4d^2 = (c^2 - v^2)\Delta t^2$$

$$\Rightarrow \quad \Delta t^2 = \frac{4d^2}{c^2 - v^2} \quad \Rightarrow \quad \Delta t = \frac{2d}{\sqrt{c^2 - v^2}}$$

We can write this as

$$\Delta t = \frac{2d}{\sqrt{c^2} \cdot \sqrt{1 - v^2/c^2}}$$

$$= \frac{2d}{c\sqrt{1 - v^2/c^2}}$$

Now we can introduce a new quantity

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$

$$\Rightarrow \Delta t = \frac{2d}{c} \cdot \gamma$$

or $\Delta t = \gamma \, \Delta t_p$

Δt is time of clock tick for observer on outside of train

Δt_p is time of clock tick for observer inside the train

Since $v^2 < c^2 \Rightarrow \gamma > 1$ so $\Delta t > \Delta t_p$!

time is dilated

The time of a clock tick for a moving observer is different than for the stationary observer

Δt_p is the proper time – the "internal time" as measured by an observer at rest with respect to the clock

Time dilation example (slide 7)

Speed of light is 3×10^8 m/s

$v = 100$ km/h

$= 100 \cdot 1000/3600$ m/s

(1000: # m in 1 km; 3600: # s in 1 hr)

$= 27.8$ m/s

$\Rightarrow \gamma = \dfrac{1}{\sqrt{1 - v^2/c^2}} = 1.000\,000\,000\,000\,004\,000$ ← !!!

If $v = 0.1c$ (10%) then

$\gamma = \dfrac{1}{\sqrt{1-(0.1c)^2/c^2}} = \dfrac{1}{\sqrt{1-0.01}} = \dfrac{1}{\sqrt{0.99}} = 1.00504$

↑ 0.5% effect

This is $0.1 \times 3 \times 10^8$ m/s $= 3 \times 10^7$ m/s

↑ 300,0000 m/s !