# Physics 101H

## General Physics 1 – Honors

Lecture 43 – 11/17/23

SHM and Circular Motion

# Summary

**Topics**

**Yesterday: Oscillations [chapter 15]**
- Mass on a spring
- Simple harmonic motion

**Today: Oscillations [chapter 15]**
- SHM and uniform circular motion

**Announcements**

**Next week:** **Lectures posted to Blackboard**

# Simple harmonic motion

Key quantities:

- Amplitude
- Frequency
- Period
- Phase
- Velocity
- Maximum velocity
- Acceleration
- Maximum acceleration

# Harmonic motion

Uniform circular motion (projected onto a single axis) is secretly simple harmonic motion

This projection actually helps us understand Euler’s formula!

In fact, “everything is a harmonic oscillator”

**Example 43.1:** Show that a simple pendulum, in the small angle approximation, undergoes simple harmonic motion.

# Summary

**Topics**

**Today: Oscillations [chapter 15]**

- SHM and uniform circular motion

**"Monday": Oscillations [chapter 16]**

- Damped oscillations
- Forced oscillations

**Announcements**

**Next week:** **Lectures posted to Blackboard**

# NEXT WEEK:
# CLASSES ARE PRE-RECORDED

# PHYSICS 101 - HONORS

## Lecture 43 11/17/23

### Simple Harmonic Motion (slide 3)

Recall that SHM (← simple harmonic motion) is <u>defined</u> by the (2nd order homogeneous ordinary differential) equation

$$\ddot{x} + \omega^2 x = 0$$

And the most general solution can be written as

$$x(t) = A\cos(\omega t + \phi)$$

← generalises $x(t) = e^{\pm i\omega t}$

Note: Euler's formula tells us

$$e^{i\omega t} = \cos(\omega t) + i\sin(\omega t)$$

$$e^{-i\omega t} = \cos(\omega t) - i\sin(\omega t)$$

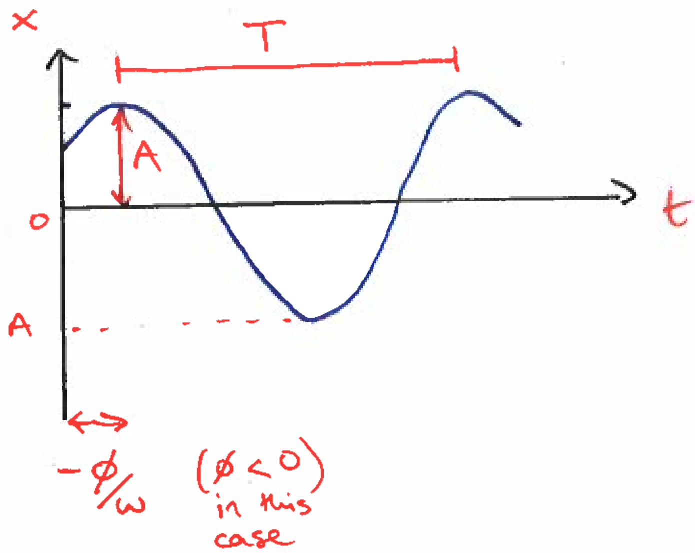
These can be solved to express $\cos(\omega t)$ and $\sin(\omega t)$ as

$$\cos(\omega t) = \frac{e^{i\omega t} + e^{-i\omega t}}{2} \qquad \sin(\omega t) = \frac{e^{i\omega t} - e^{-i\omega t}}{2i}$$

There is a theorem that says that if two functions are solutions of an ODE, then so are sums and multiples of those functions. Thus, if $e^{+i\omega t}$ and $e^{-i\omega t}$ both satisfy $\ddot{x} + \omega^2 x = 0$, then so does $\frac{e^{i\omega t} + e^{-i\omega t}}{2} = \cos(\omega t)$.

↑ which is how we obtain our general solution

Our solution can be drawn as




$$x(t) = A\cos(\omega t + \phi)$$

- $A$ — amplitude
- $\omega$ — angular frequency
- $\phi$ — phase

Amplitude – minimum/maximum value of $x(t)$

Frequency – $f = \frac{\omega}{2\pi}$ – number of oscillations per second

$$f = \frac{1}{T}$$

Period – $T = \frac{1}{f} = \frac{2\pi}{\omega}$ – time take for one oscillation

Phase – "offset" where crest or trough occurs

e.g. if $\phi = 0$ the $x(t) = A\cos(\omega t) = A$ at $t = 0$ (crest)

Velocity – $v = \frac{dx}{dt} = -A\omega \sin(\omega t + \phi)$ ← varies periodically!

Acceleration – $a = \frac{d^2x}{dt^2} = -A\omega^2 \cos(\omega t + \phi)$

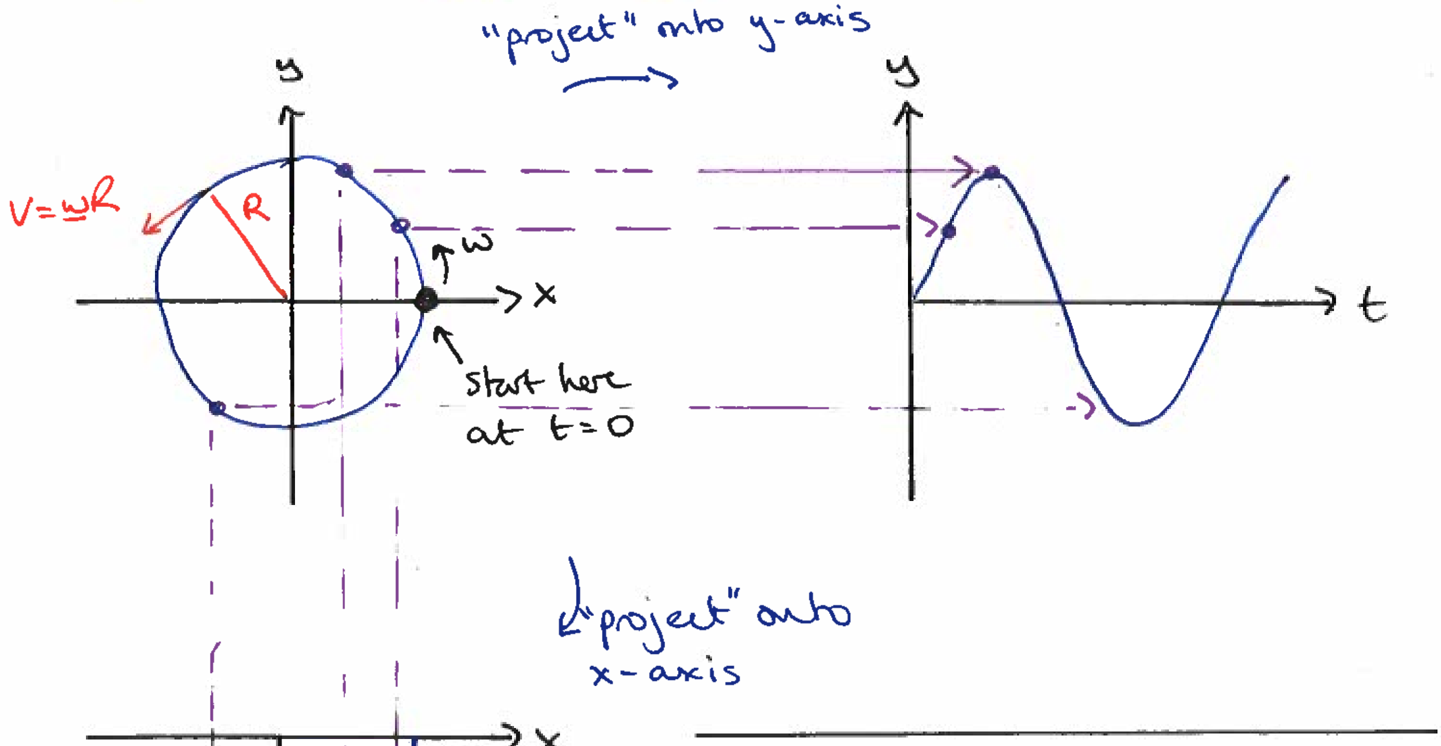
Max velocity $|v_{max}| = A\omega = \sqrt{\frac{k}{m}}A$

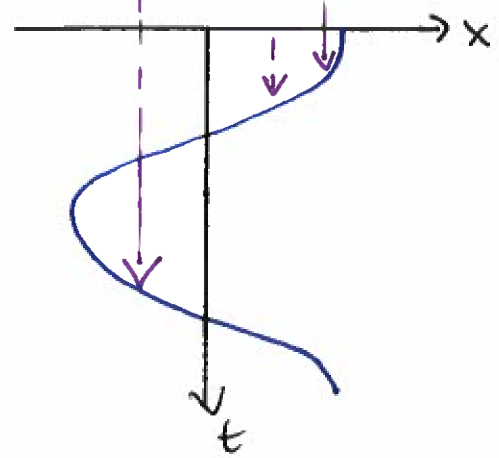
Max acceleration $|a_{max}| = A\omega^2$

# Uniform circular motion (slide 4)

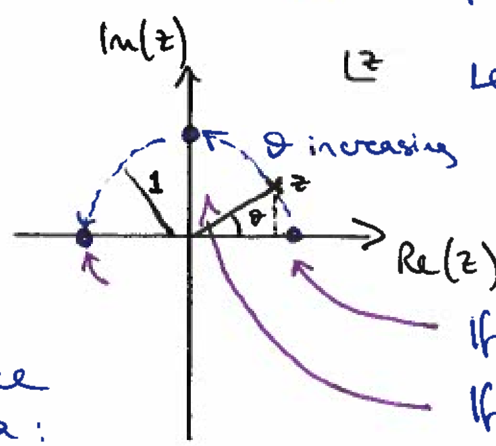
We can relate SHM to circular motion!

Plot the x position of a particle undergoing circular motion and it traces out a cosine (or sine) wave = SHM!




This helps us visualise $e^{i\pi}$!




Let $z = e^{i\theta}$
↑ complex number with magnitude 1

If $\theta = 0$ then $z = 1$

If $\theta = \pi/2$ then $z = i$

If $\theta = \pi$ then $z = -1$

Notice these values make sense in Euler's formula:

$$e^{i\cdot 0} = \cos(0) + i\sin(0) = 1 + i\cdot 0 = 1$$

$$e^{i\cdot \pi/2} = \cos(\pi/2) + i\sin(\pi/2) = 0 + i\cdot 1 = i$$

$$e^{i\pi} = \cos(\pi) + i\sin(\pi) = -1 + i\cdot 0 = -1$$

We can also use trig to understand the real and imaginary parts of z ...

eg $Re(z) = 1\cdot\cos\theta = \cos\theta$ ✓

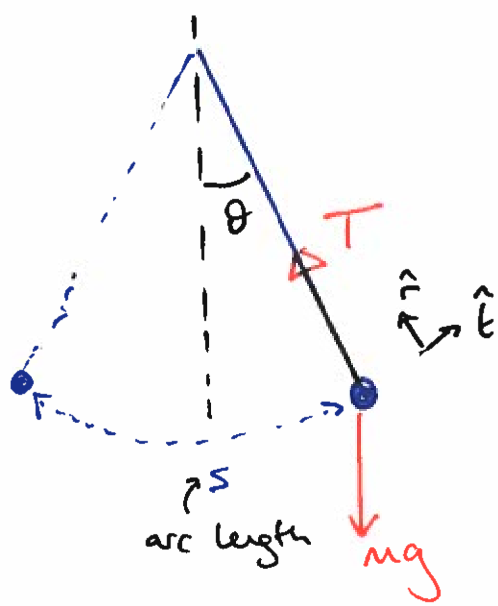
Actually, any "well-behaved" function can be written as an infinite sum of sines and cosines

$$f(x) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty} a_n \cos(nx) + \sum_{n=1}^{\infty} b_n \sin(nx)$$

called the <u>Fourier series</u> of a function

The essence of this is that you can break down complicated problems into the sum of a bunch of harmonic oscillators of different frequencies (and amplitudes).

<u>Simple pendulum example</u> (slide 4)




$\bar{T} = T\hat{r}$ and $\bar{F}_g = -mg\cos\theta\,\hat{r} - mg\sin\theta\,\hat{t}$ are the only forces acting on the pendulum

Newton's second law tells us

$\bar{F}_{net} = m\bar{a}$

$\Rightarrow \bar{T} + \bar{F}_g = m\bar{a}$ or $T\hat{r} + (-mg\cos\theta\,\hat{r} - mg\sin\theta\,\hat{t}) = m\bar{a}$

The acceleration in this case is $\bar{a} = a_r\hat{r} + a_t\hat{t}$

⇒ in radial direction:

$$T - mg\cos\vartheta = ma_r = \frac{mv^2}{r}$$

↑ $a_r = a_c = mv^2/r$

and in the tangential direction

$$-mg\sin\vartheta = ma_t \quad \Rightarrow \quad a_t = -g\sin\vartheta$$

But we know that

$$a_t = \frac{d^2 s}{dt^2} \quad \text{and since } s = L\vartheta \Rightarrow a_t = \frac{d^2}{dt^2}(L\vartheta) = L\frac{d^2\vartheta}{dt^2}$$

So we have

$$L\frac{d^2\vartheta}{dt^2} = -g\sin\vartheta \quad \Rightarrow \quad \frac{d^2\vartheta}{dt^2} = -\frac{g}{L}\sin\vartheta$$

Now introduce $\omega^2 \equiv g/L \Rightarrow \frac{d^2\vartheta}{dt^2} = -\omega^2\sin\vartheta$ ← valid for <u>any</u> angle $\vartheta$

If we use the Taylor series

$$\sin\vartheta \simeq \vartheta - \frac{1}{3!}\vartheta^3 + \frac{1}{5!}\vartheta^5 + \dots$$

then for small angles we can approximate $\sin\vartheta \approx \vartheta$

$$\Rightarrow \boxed{\frac{d^2\vartheta}{dt^2} = -\omega^2\vartheta}$$

← solution is $\theta(t) = A\cos(\omega t + \phi)$ with $\omega = \sqrt{\frac{g}{L}}$ the angular frequency

↖ But this is equation for simple harmonic motion!