Physics 101H

General Physics 1 – Honors

Lecture 41 – 11/15/23

Fluid Dynamics

Summary

Topics

Monday: Pascal's principle [chapter 14]

- Pascal's principle
- Hydraulic lift
- Measuring pressure

Today: Fluid dynamics [chapter 14]

- Continuity equation
- Bernoulli equation

Announcements

Today:	**Problem set 7 due**
	No problem set assigned
Tomorrow:	**Quiz 8**
Next week:	**All classes remote**

Quick quiz

Instructions: This quiz is for your own learning. There are three questions and each question has two columns. Write your own solution, without reference to your notes, the textbook, or your neighbour, **in the first column**. Once you have tried to answer all the questions, discuss the questions with a neighbour and fill in any incomplete answers **in the second column**. Keep your sheet for future reference.

Fluid dynamics

So far we have considered only the properties of **static** fluids. But now we let things flow!

Some key concepts in fluid dynamics:

- Laminar flow
- Turbulent flow
- Viscosity

We will focus on **ideal flow**

- Non viscous
- Laminar flow
- Incompressible fluid
- Irrotational flow

Arbitrary (turbulent, viscous, rotational) flow is governed by the **Navier-Stokes equations**, which still have not been solved in general. A general solution is worth $1 million from the Clay Mathematics Institute as one of its Millenium Problems

https://www.claymath.org/millennium-problems.

Continuity equation

"Continuity equations" are a quite general concept

- express conservation laws

Broadly speaking:

- tell us that the amount of stuff flowing into a region = amount of stuff flowing out

In this case, the continuity equation for fluid dynamics tells us that the **volume of fluid moving through a region is constant** (even if the shape of the container changes)

Bernoulli equation

Bernoulli's equation generalises the continuity equation

- pipe changes diameter
- pipe changes height

Follows from conservation of energy

Leads to the **Bernoulli effect**

Example 41.1: A farm maintains a large tank with an open top containing water. The water can drain through a hose of diameter 6.6 cm. The hose ends with a nozzle of diameter 2.2 cm. A rubber stopper is inserted into the nozzle and the water level is 7.5 m above the nozzle.

(a) Calculate the frictional force exerted on the stopper by the nozzle.

(b) If the stopper is removed, what mass of water flows out in two hours?

Want more practice?

Try the following problems **Chapter 14** of the textbook:

- Conceptual questions: 3, 5, 7, 11, 13, 17, 19, 23, 31, 33
- Pressure: 45, 51, 55, 57, **121**, **123**
- Pascal's and Archimedes' principles: 61, 65, 67, 71
- Fluid dynamics: 77, 79, 87, 89, **129**

Answers are provided for questions with **blue** numbers (odd numbered)

Click on the number to be taken to the answer.

But make sure you at least **try** the problem first!

Summary

Topics

Today: Fluid dynamics [chapter 14]

- Continuity equation
- Bernoulli equation

Tomorrow: Oscillations [chapter 15]

- Mass on a spring
- Simple harmonic motion

Announcements

Today:	**Problem set 7 due**
	No problem set assigned
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Next week:	**All classes remote**

PHYSICS 101 - HONORS

Lecture 41 11/15/23

Fluid dynamics (slide 4)

Laminar flow - smooth, steady flow
each "particle" of fluid follows a smooth path (a streamline)

↑ streamlines do not cross or appear/disappear

Turbulent flow - irregular
above a critical speed, almost all flow becomes turbulent

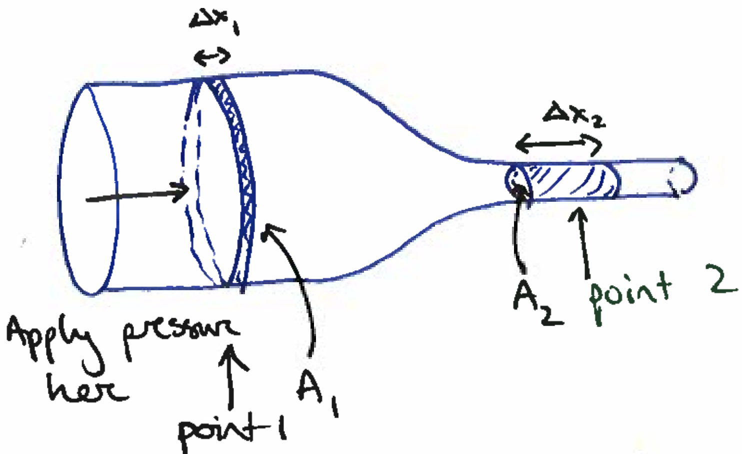
↑ full dynamics of flow governed by Navier-Stokes equations - currently unsolved in general!

Viscosity - measures internal friction of the fluid
viscous fluids lose energy as internal heat

Ideal flow: nonviscous = no energy loss due to viscosity
laminar = smooth
incompressible = constant density
irrotational = no angular momentum about any point

Continuity equation

Consider an <u>ideal</u> <u>fluid</u> moving through a pipe
- nonviscous
- laminar
- incompressible
- irrotational



Consider a "disc" of volume V_1 passing point 1

In time Δt, a volume V_1 passes (and similarly at point 2)

$$V_1 = A_1 \Delta x_1 = A_1 (v_1 t) \qquad V_2 = A_2 \Delta x_2 = A_2 (v_2 t)$$

Volumes are equal, otherwise the liquid would all pile up in the middle, which is not allowed because it is incompressible

$$\Rightarrow V_1 = V_2 \quad \text{or} \quad A_1 v_1 \Delta t = A_2 v_2 \Delta t$$

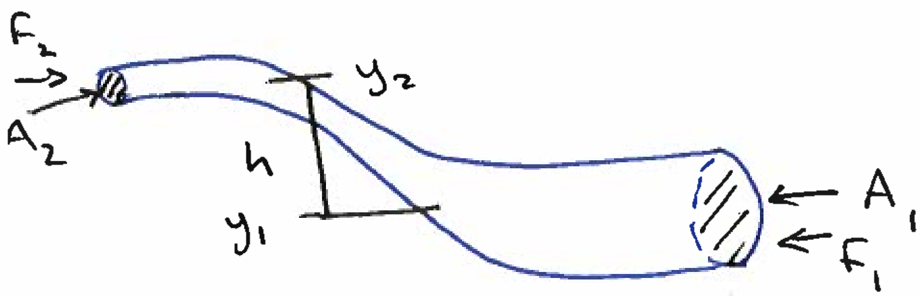
$$\Rightarrow A_1 v_1 = A_2 v_2$$

$$\Rightarrow v_2 = \frac{A_1}{A_2} v_1$$ so $v_2 > v_1$ because $A_1 > A_2$!

Fluid moves <u>more quickly</u> through the smaller pipe

Bernoulli's equation (slide 6)

Now consider a pipe that changes diameter **and** height



Apply a force at one end

$$F_1 = P_1 A_1$$

and consider work done moving a volume $V_1 = A_1 \Delta x_1$ a distance Δx_1

$$W_1 = F_1 \Delta x_1$$
$$= P_1 A_1 \Delta x_1$$
$$= P_1 V_1$$

The same volume of water moves through the other end $V_2 = V_1$, but now the work done is

$$W_2 = -F_2 \Delta x_2$$
$$= -P_2 A_2 \Delta x_2$$
$$= -P_2 V_2 = -P_2 V_1$$

The total work is

$$W = W_1 + W_2$$
$$= P_1 V - P_2 V$$

This must be equal to the total change in energy

$$W = \Delta E_K + \Delta E_p$$

$$\Rightarrow (P_1 - P_2)V = \underbrace{\tfrac{1}{2} m v_2^2 - \tfrac{1}{2} m v_1^2}_{\Delta E_K} + \underbrace{mgy_2 - mgy_1}_{\Delta E_p}$$

$$\Rightarrow P_1 - P_2 = \tfrac{1}{2}\tfrac{m}{V} v_2^2 - \tfrac{1}{2}\tfrac{m}{V} v_1^2 + \tfrac{m}{V} g y_2 - \tfrac{m}{V} g y_1 \quad \big\} \rho = \tfrac{m}{V}$$
$$= \tfrac{1}{2}\rho v_2^2 - \tfrac{1}{2}\rho v_1^2 + \rho g y_2 - \rho g y_1$$

$$\Rightarrow P_1 + \tfrac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \tfrac{1}{2}\rho v_2^2 + \rho g y_2$$

Or

$$\boxed{P + \tfrac{1}{2}\rho v^2 + \rho g y = \text{constant}}$$ ← Bernoulli's equation

For fluid at constant height

$$\Rightarrow P_1 + \tfrac{1}{2}\rho v_1^2 + \cancel{\rho g y} = P_2 + \tfrac{1}{2}\rho v_2^2 + \cancel{\rho g y}$$

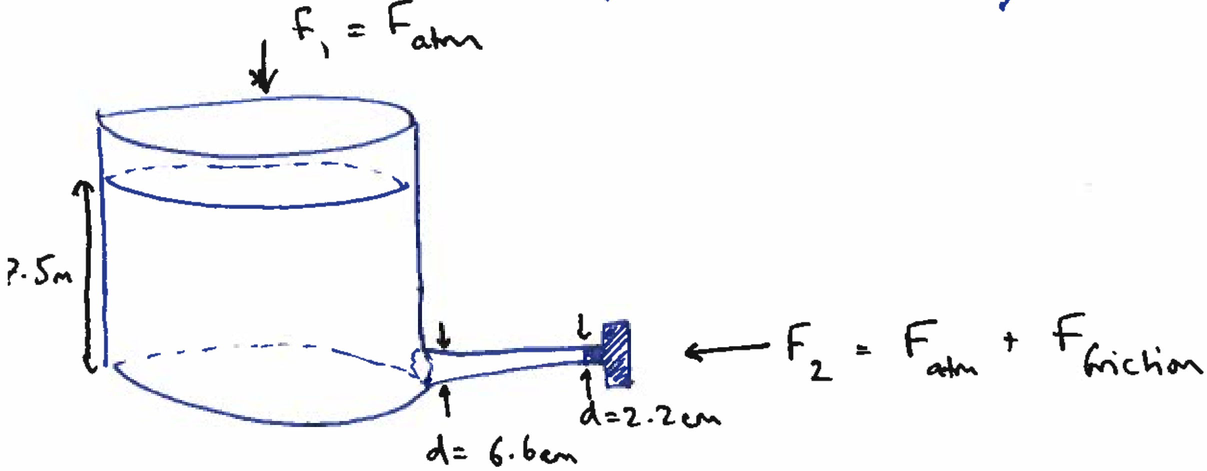
$$\Rightarrow P_1 + \tfrac{1}{2}\rho v_1^2 = P_2 + \tfrac{1}{2}\rho v_2^2$$

Bernoulli effect ↙

Fluids travelling faster have lower pressure!

If $v_1 > v_2$ then $P_1 < P_2$

Water tank example (slide 7)



Apply Bernoulli's equation

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \frac{1}{2}\rho v_2 + \rho g y_2$$

With stopper we have

$v_1 = v_2 = 0$ $\quad$ $y_1 = 7.5m$ $\quad$ $P_2 = P_{atm} + P_{friction}$

$P_1 = P_{atm}$ $\quad$ $y_2 = 0$ $\quad$ $\left(= \frac{F_{atm}}{A_{stopper}} + \frac{F_{friction}}{A_{stopper}}\right)$

$$\Rightarrow P_{atm} + \rho g y_1 = P_{atm} + P_{friction}$$

$$\Rightarrow P_{friction} = \rho g y_1 \quad \text{or} \quad \frac{F_{friction}}{A_{stopper}} = \rho g y_1$$

$$F_{friction} = \rho g y_1 A_{stopper}$$

$$= 1000 \cdot 9.81 \cdot 7.5 \cdot \left(\pi \left(\frac{d}{2}\right)^2\right)$$

$$= 73575 \cdot \pi \cdot 0.011^2$$

$$= 28.0 \text{ N}$$

Once the stopper is removed we have

$P_2 = P_{atm}$ and $v_2 \neq 0$

We assume that we can approximate v_1 as 0, because the tank is so enormous (recall $v_1 = \frac{A_2 v_2}{A_1}$)

$$\Rightarrow \quad P_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2$$

($v_1 = 0$, $y_2 = 0$)

$$\Rightarrow \quad P_{atm} + \rho g y_1 = P_{atm} + \frac{1}{2}\rho v_2^2$$

$$\Rightarrow \quad \frac{1}{2}\rho v_2^2 = \rho g y_1$$

or $v_2^2 = 2 g y_1$

$$\Rightarrow \quad v_2 = \sqrt{2 g y_1}$$

In a time Δt, the water moves a distance $\Delta x_2 = v_2 \Delta t$

This corresponds to a volume of water $= A_2 \Delta x_2$

$$\Rightarrow \quad V = A_2 v_2 \Delta t$$

This volume has mass $= \rho V$ or

$$M = \rho A_2 v_2 \Delta t$$

$$= 1000 \cdot \underbrace{(\pi \cdot 0.011^2)}_{= A_2} \cdot \underbrace{\sqrt{2 \cdot 9.81 \cdot 7.5}}_{= v_2} \cdot (2 \cdot 60 \cdot 60)$$

(2 hours, 1 hour = 60 mins, 1 min = 60s)

$= \underline{\underline{33200 \text{ kg}}}$

Key equations

Pressure (in general)	$P = F/A$
Pressure (as a function of depth in a fluid)	$P = P_0 + \rho g h$
Archimedes principle	buoyant force = weight of fluid displaced
Pascal's principle	pressure transferred equally throughout fluid
continuity equation	$V_1 = V_2$
Bernoulli's equation	$P + \frac{1}{2}\rho v^2 + \rho g y = \text{constant}$