Physics 101H

General Physics 1 – Honors

Lecture 38 – 11/9/23

Noether's theorem

Midterm 2

mean = 85%, median = 90%, standard deviation = 22%

Mea culpa: the values in Q1 were incorrect.

I am sorry that I messed this up. To account for this, I gave everyone 15/15 on Q1.

Taking both midterms together

mean = 86%, median = 90%, standard deviation = 13%

Note for problem sets/Expert TA

mean = 74%, median = 80%, standard deviation = 14%

Summary

Topics

Thursday: Escape velocity [chapter 13]

- Conservation of energy
- Escape velocity
- Black holes

Today: Noether's theorem

- The action
- Calculus of variations
- Noether's theorem

Announcements

Yesterday:	**Problem set 7 assigned**
Today:	**No quiz**

The action

We have seen two formulations of dynamics that allow us to study motion

- Newton's laws
- Energy consideration

There is a third, equivalent approach!

- Hamilton's principle (or **principle of least action**)

Example 38.1: Use the principle of least action to find the path taken by a free particle travelling in 1D.

Noether's theorem

Noether's (first) theorem is one of the most profound results in physics

Every continuous symmetry of the action has a corresponding conserved quantity.

Most importantly, it is my favourite theorem

- relates the behaviour of physical systems to the geometry of spacetime

Emmy Noether responsible for exploring the mathematical foundations of relativity and developing entire new branches of mathematics

Summary

Topics

Today: Noether's theorem

- The action
- Calculus of variations
- Noether's theorem

Tomorrow: Fluid mechanics [chapter 14]

- Pressure
- Archimedes' Principle

Announcements

Yesterday:

Problem set 7 assigned

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The Action

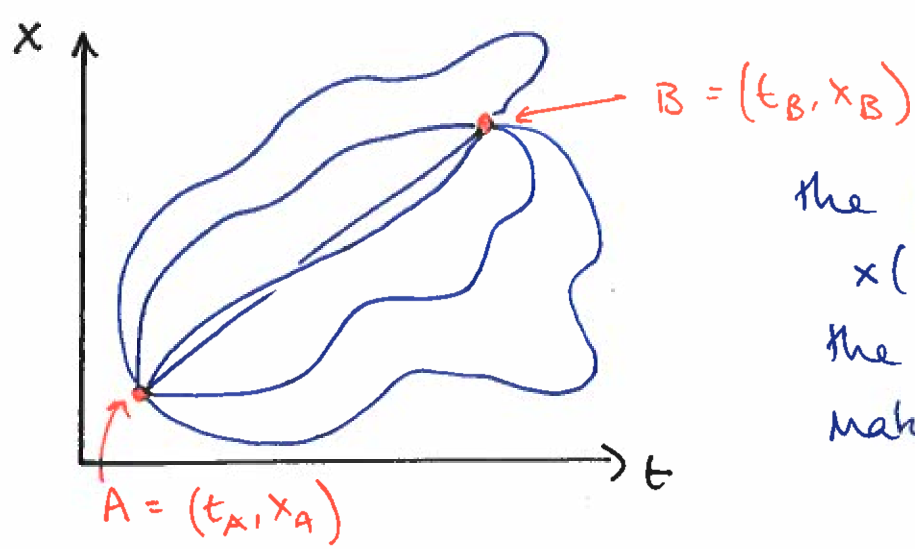
The action is defined as

$$S = \int_{t_1}^{t_2} (E_K - E_p)\, dt$$

The difference of kinetic and potential energies - called the Lagrangian

$$L \equiv E_K - E_p$$

Hamilton's principle tells us that the physical path a system takes from A to B is the path that minimises the action



the path is a function $x(t)$ - need to find the function (path) that makes S small.

What does this mean in practice?

- choose a path from $A = (t_A, x_A)$ to $B = (t_B, x_B)$

  ↑ this defines our function $x(t)$

- break up the total time interval $T = t_B - t_A$ into (infinitesimal) time steps Δt

- for each (infinitesimal) time step Δt, calculate $x(\Delta t)$ and $\frac{dx}{dt}(\Delta t)$

- from these values, calculate $E_K\left(\frac{dx}{dt}\Big|_{\Delta t}\right)$ and $E_P\left(x|_{\Delta t}\right)$

- from these values, calculate $L = E_K - E_P \big|_{\Delta t}$

- sum over all time steps $S = \sum L \, \Delta t$

- in the limit that the number of time steps $\rightarrow \infty$ and $dt \rightarrow 0$, we have $S = \int_{t_A}^{t_B} L \, dt$

- repeat this process until we find $S_{minimum}$

Process is an example of the calculus of variations

More complicated than single variable calculus!

↑ see PHYS 208

It turns out the condition that extremises the action is the Euler-Lagrange equations

$$\frac{\partial L}{\partial x} - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) = 0$$

(partial derivative → ∂; $\dot{x} = dx/dt$)

Free particle example (slide)

We know the answer!

Free particle $\Rightarrow \bar{F}_{net} = 0 \Rightarrow \bar{a} = 0$

↑ Newton II

$\Rightarrow \bar{v} = \text{constant} \Rightarrow$ moving in a straight line at constant speed

↑ just Newton I

Now let's use our calculus of variations to find the same answer

$\bar{F}_{net} = 0 \Rightarrow E_p = \text{constant}$, so choose $E_p = 0$

↑ $F = -\frac{dE_p}{dx}$

$\Rightarrow L = E_K - E_p = E_K = \frac{1}{2} m v^2 = \frac{1}{2} m \left(\frac{dx}{dt}\right)^2$

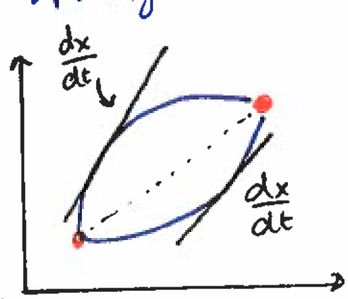
Therefore the action is $S = \int_{t_1}^{t_2} \frac{1}{2} m \left(\frac{dx}{dt}\right)^2 dt = \frac{m}{2} \int_{t_1}^{t_2} \left(\frac{dx}{dt}\right)^2 dt$

Notice that $\left(\frac{dx}{dt}\right)^2 > 0$!

So the minimum value is when $\left(\frac{dx}{dt}\right)^2 = m^2$ or $\frac{dx}{dt} = m$

But this is a constant velocity! $\Rightarrow$ straight line motion

cf → sum of average slope² always larger than straight path



Noether's theorem (slide)

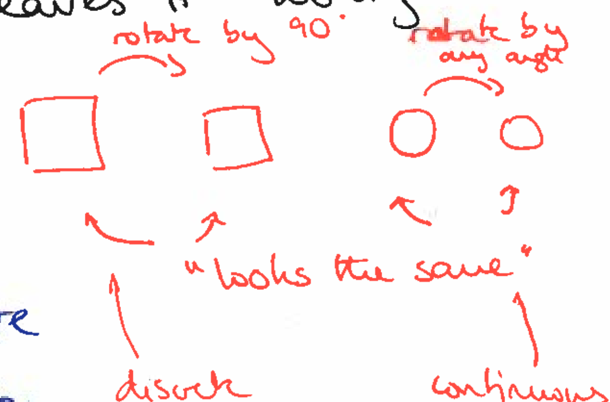
Example : time translation

Time translation means changing our "origin" of time

↑ i.e. when we start measuring a time interval

If a system is symmetric with respect to time translation this means the physics "looks the same" whether we conduct the experiment now, tomorrow or yesterday

↖ A symmetry is a transformation that leaves a system invariant ie a change to our system that leaves it "looking the same"



This means the action cannot depend on time

$t' = t + \tau$ ↖ shift

$\Rightarrow S(t') = S(t)$

Noether's theorem tells us this symmetry of the action implies there is a corresponding conserved quantity

It turns out that in this case the conserved quantity is total energy.

Other examples:

- spatial translations $x' = x + X$

If $S(x') = S(x)$ then momentum is conserved

- rotations $\theta' = \theta + \phi$

If $S(\theta') = S(\theta)$ then angular momentum is conserved