Physics 101H

General Physics 1 – Honors

Lecture 34 – 11/1/23

Kepler’s Laws

Summary

Topics

Monday: Gravity [chapter 13]

- Newton's laws of gravitation
- Gravitational fields and potential energy

Today: Kepler's laws [chapter 13]

- Planetary motion and orbits
- Kepler's laws

Announcements

Monday: **Practice exam posted**
Today: **Problem Set 6 due**
No problem set assigned
Thursday: **Quiz 7**
No office hours
Monday November 6: **Midterm 2**

Quick quiz

Instructions: This quiz is for your own learning. There are three questions and each question has two columns. Write your own solution, without reference to your notes, the textbook, or your neighbour, **in the first column**. Once you have tried to answer all the questions, discuss the questions with a neighbour and fill in any incomplete answers **in the second column**. Keep your sheet for future reference.

Planetary and celestial motion

Astronomy is one of the founding branches of physics

- Tends to be treated as distinct these days

Observational astronomy known since 1800 BC (at least)

- Cultures across the world have observed the stars and planets and used models to predict astronomical events

Modern Western understanding typically dated to around 1700s

- Encoded in Kepler's laws

Kepler’s laws

Formulated based on observational data (primarily by Tycho Brahe)

Derivable from Newton’s laws of motion and of gravity

1. Planets move in **elliptical orbits** with the Sun at one focus
2. Line between the Sun and a planet sweeps out **equal areas in equal times**
3. Square of the orbital period of a planet is proportional to the cube of the **semimajor axis** of the orbit

Practice in pairs

Instructions: Discuss the following question with a neighbour. Your answers will not be graded; your discussion is for your own learning.

Question: Two satellites, *A* and *B*, of the same mass orbit the Earth in concentric orbits. The distance of satellite *B* from the Earth's centre is twice the distance of *A* from Earth's centre. What is the ratio of the tangential speed of *B* to that of *A*?

(a) $\frac{1}{2}$ (b) $1/\sqrt{2}$ (c) 1 (d) $\sqrt{2}$ (e) 2

NEXT WEEK:
THE SECOND MIDTERM IS ON MONDAY NOVEMBER 6

Midterm 2

Good News: No problem set assigned today!

Bad News: Second midterm will take place on **Monday November 6**!

You will have 45 minutes to complete the exam

- 3 multiple choice questions
- 2 handwritten solution problems

Bring paper and something(s) to write with! (Spare paper will be available)

Topics cover Chapters 7 to 11 and include:

- Conservation of energy
- Conservation of momentum
- Rotational motion
- Angular momentum and torque

You may prepare your own formula sheet – **two sides** of **letter paper (215.9 x 279.4 mm)**

You may bring a calculator, but phones, tablets and laptops are not allowed

Remember you are here to learn and understand the physics!

Studying for midterm 2

Studying for the midterm:

- Work through Problem Sets
- Work through examples from class and in the textbook

When working through problems (especially someone else's solution):

- Cover up the solution and try to work out the next step in the solution
- If you can't figure that out, uncover just the first step and then try to figure out the next steps
- Try to *self-explain*, that is - write down your thought process and what principles, concepts or equations are being applied at each step.

The Society of Physics Students offers free student tutoring **Thursday 6-8pm** in Small 122

https://www.wm.edu/as/physics/undergrad/bor-undergrad-resources/sps/index.php

Remember that you are here to learn and understand the physics!

[But also remember there are two methods for calculating your final grade]

Summary

Topics

Today: Kepler’s laws [chapter 13]

- Planetary motion and orbits
- Kepler’s laws

Thursday: Escape velocity [chapter 13]

- Conservation of energy
- Escape velocity
- General relativity (briefly)

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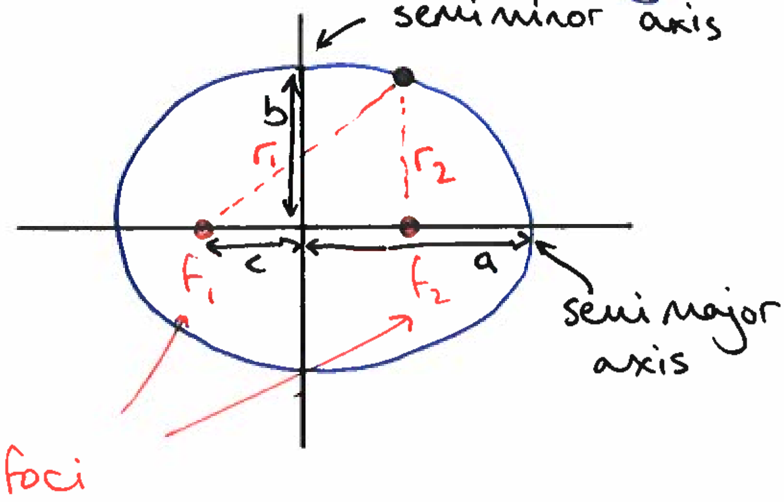
PHYSICS 101 - HONORS

Lecture 34 11/1/23

Kepler's laws (slide 4)

First law

Ellipses are defined by $r_1 + r_2 = \text{constant}$



$$\text{or} \quad \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

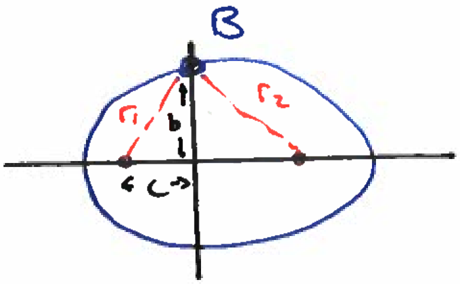
Define eccentricity

$e = \frac{c}{a}$

$e = 0$ circle

$e \to 1$ very squished

Consider point B

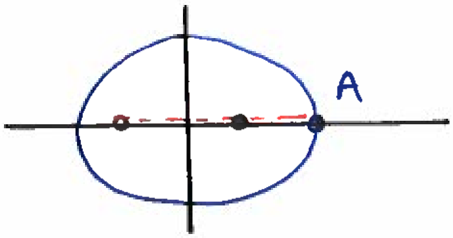


$r_1 = \sqrt{c^2 + b^2} = r_2$

$\Rightarrow r_1 + r_2 = 2r_1 = \text{constant}$

or $2\sqrt{c^2 + b^2} = \text{constant}$

Consider point A



$r_1 = a + c$

$r_2 = a - c$

$r_1 + r_2 = a + c + a - c = 2a$

$= \text{constant}$

$\Rightarrow 2\sqrt{c^2 + b^2} = 2a$ or $\underline{a^2 = b^2 + c^2}$

Second law

Force due to gravity points along $\hat{r}$

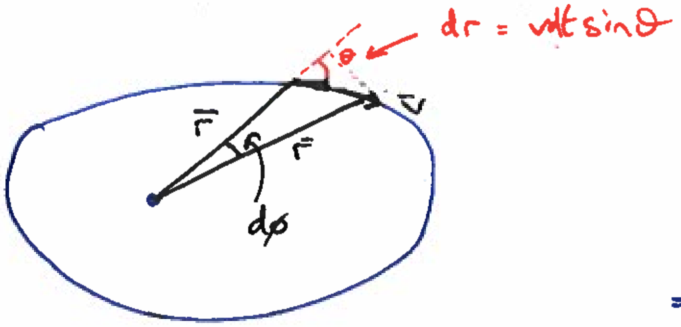
$$\Rightarrow \bar{r} \times \bar{F}_g = 0$$

Angular momentum is therefore conserved ↙ remember $\frac{d\bar{L}}{dt} = \bar{\tau}_{net}$

$$\Rightarrow \bar{L}_{planet} = \bar{r} \times \bar{p} = \bar{r} \times M_{planet} \bar{v}$$

or $|\bar{L}_{planet}| = M_{planet} |\bar{r}| |\bar{v}| \sin\theta$ $\quad (L_p = M_p r v \sin\theta)$

Now let's look at the <u>area swept out by $\bar{r}$</u>



area of the triangle is

$$dA = \frac{1}{2} \underset{\text{base}}{r} \underbrace{v dt \sin\theta}_{\text{height}}$$

$$\Rightarrow \frac{dA}{dt} = \frac{1}{2} \underbrace{r v \sin\theta}_{= \frac{L_{planet}}{M_{planet}}}$$

Thus $$\frac{dA}{dt} = \frac{L_{planet}}{2M_{planet}} = \text{a } \underline{\text{constant}}$$

$$\boxed{\frac{dA}{dt} = \text{constant}}$$

Third law

In the approximation that $e = 0$ (circular orbit)

$$F_c = F_g \leftarrow \text{gravitational force} \quad \text{or} \quad m a_c = \frac{GmM}{r^2} \Rightarrow \frac{mv^2}{r} = \frac{GmM}{r^2}$$

↑ centripetal force

$$\Rightarrow \frac{v^2}{r} = \frac{GM}{r^2} \Rightarrow v^2 = \frac{GMr}{r^2} = \frac{GM}{r}$$

But we know that $v = \frac{2\pi r}{T}$

$$\Rightarrow \left(\frac{2\pi r}{T}\right)^2 = \frac{GM}{r}$$

Rearranging for T^2:

$$\frac{GM}{r}T^2 = (2\pi r)^2 \quad \text{or} \quad \boxed{T^2 = \frac{4\pi^2 r^3}{GM}}$$

This tells us that $T^2 \propto r^3$

For an ellipse this becomes $T^2 \propto a^3$ ↖ semimajor axis

Satellite example (slide 5)

$$F_c = F_g \Rightarrow v^2 = \frac{GM}{r} \Rightarrow v_A = \sqrt{\frac{GM}{r_A}} \quad \text{and} \quad v_B = \sqrt{\frac{GM}{r_B}}$$

$$\Rightarrow v_B / v_A = \sqrt{\frac{GM}{r_B}} \Big/ \sqrt{\frac{GM}{r_A}} = \sqrt{\frac{r_A}{r_B}} = \sqrt{\frac{r_A}{2r_A}} = \frac{1}{\sqrt{2}} \qquad \underline{\underline{(b)}}$$