Physics 101H

General Physics 1 – Honors

Lecture 32 – 10/27/24

Static equilibrium

Summary

Topics

Yesterday: Angular motion [chapter 10/11]

- Group work

Today: Static equilibrium [chapter 12]

- Static equilibrium
- Stress and strain
- Elastic modulus

Announcements

Wednesday November 1:	**Problem Set 6 due** **No problem set assigned**
Thursday November 2:	**Quiz 7**
Friday November 3:	**Review lecture**
Monday November 5:	**Midterm 2**
Wednesday November 8:	**No class**

Quick quiz

Instructions: This quiz is for your own learning. There are three questions and each question has two columns. Write your own solution, without reference to your notes, the textbook, or your neighbour, **in the first column**. Once you have tried to answer all the questions, discuss the questions with a neighbour and fill in any incomplete answers **in the second column**. Keep your sheet for future reference.

Equilibrium

Knowing when things will not move can be important

- for example, when at the top of a ladder leaning against a wall

An object in **equilibrium** does not experience any accelerations (linear or angular):

- zero net force
- zero net torque

Static equilibrium means that the object is at rest in our chosen frame of reference

There is no real physical difference between equilibrium and static equilibrium, because the laws of physics are the same in all inertial reference frames, but sometimes it is convenient to pick a frame in which the object is at rest.

Centre of gravity

For rigid objects close to Earth, treat gravity as if it acts at the **centre of gravity**

For all practical purposes, this is the same point as the centre of mass

Recall the definition of centre of mass:

- “average” position of the mass of the object
- force acting on the centre of mass causes linear motion, but not rotation

If the centre of gravity does not lie above a point of contact with the ground, gravity generates a torque that leads to **instability**

Example: A human arm weighs 41.5 N. Determine the magnitudes of the tension forces in the deltoid (shoulder) muscle and that exerted by the shoulder on the humerus (upper arm bone) to hold the arm straight out. Assume that the deltoid acts on the arm with an angle of 12° at a distance of 8 cm from the joint, and the centre of gravity of the arm is at a distance of 29 cm from the joint.

Deformable objects

The extended objects we have been considering so far have been **rigid**

But we know that not everything is rigid! Some objects can be **deformed**.

- Even apparently rigid objects can be deformed if the applied forces are sufficient

Characterise forces on an object by

- Stress
- Strain

Elastic modulus

Property of object that measures response to stress

Different situations have different names, but all characterise the response to stress

- Young's modulus
- Shear modulus
- Bulk modulus

Elastic objects return to their original shape when the external forces are removed

Above the **elastic limit**, materials become **plastic**, and do not return to their original shape

Example 32.2: The Mariana Trench is about 11 km deep, which is very deep. The pressure at this depth is 1.13×10^8 N/m^2, which is a lot. Calculate the change in volume of 1 m^3 of seawater carried from the surface to this deepest point. Find the change in density of water at the bottom of the trench. The bulk modulus of water is approximately 0.22×10^{10} N/m^2.

Want more practice?

Try the following problems **Chapter 12** of the textbook:

- Conceptual questions: 1, 5, 7, 9, 13, 15, 17
- Static equilibrium: 25, 27, 31, 37, 41, **75**, **77**, **79**
- Stress and strain: 45, 49, 51, 59, 63

Answers are provided for questions with **blue** numbers (odd numbered)

Click on the number to be taken to the answer.

But make sure you at least **try** the problem first!

Summary

Topics

Today: Static equilibrium [chapter 12]

- Static equilibrium
- Stress and strain
- Elastic modulus

Monday: Gravity [chapter 13]

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PHYSICS 101 - HONORS

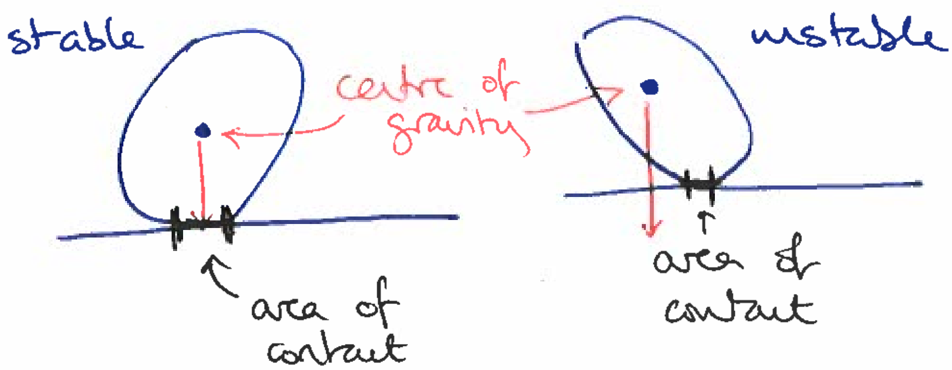
Lecture 32 10/27/23

Equilibrium and centre of gravity (slides 4/5)

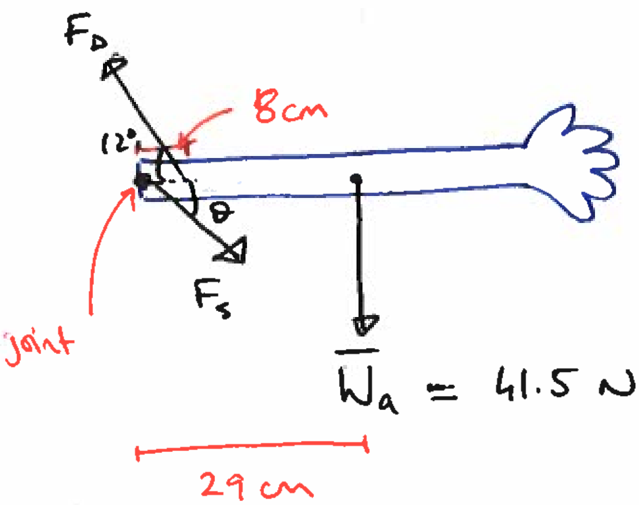
Equilibrium : $\bar{F}_{net} = \sum_i \bar{F}_i = 0$

$\bar{\tau}_{net} = \sum_i \bar{\tau}_i = 0$ around any axis

Stability : centre of gravity lies in area of contact with ground (or other surface)



Arm example (slide 6)



Equilibrium $\Rightarrow \sum_i \overline{F}_i = 0$

$\Rightarrow \overline{F}_D + \overline{F}_s + \overline{W}_a = 0$

Horizontally: $F_D \cos 12 = F_s \cos \theta$ (*)

Vertically: $F_D \sin 12 - F_s \sin \theta - W_a = 0$ (**)

Consider rotation around axis through joint, ↺ = +ve

$\sum_i \overline{\tau}_i = 0 \Rightarrow \overline{\tau}_D + \overline{\tau}_a = 0$

$0.08 \cdot F_D \sin 12 - 0.29 \cdot W_a \sin 90 = 0$

$\Rightarrow 0.08\, F_D \sin 12 = 0.29\, W_a$

$\Rightarrow F_D = \frac{0.29 \cdot 41.5}{0.08 \sin 12} = 723.564$ N

$F_D = 724$ N

(**) $\Rightarrow F_s \sin\theta = -W_a + F_D \sin 12$

combine with (*) by dividing

$\tan\theta = \frac{F_D \sin 12 - W_a}{F_D \cos 12} \Rightarrow \theta = \arctan\left(\frac{723.564 \sin 12 - 41.5}{723.564 \cos 12}\right)$

$= 8.75°$

$\Rightarrow F_s = \frac{F_D \cos 12}{\cos\theta} = \frac{723.564 \cos 12}{\cos 8.75} = 716$ N

Deformable objects (slide 7)

Stress – external force acting on an object per unit cross-sectional area

$$\sigma = \frac{F_\perp}{A} \leftarrow \text{units } N/m^2 \text{ (pressure!)}$$

Strain – fractional change in length

$$\epsilon = \frac{\Delta L}{L_0} \leftarrow \text{unitless}$$

↑ the result of tensile forces (compression or stretching) – result of a stress

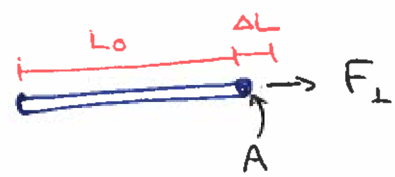
Shear strain	$\frac{\Delta x}{L}$
Bulk strain	$\frac{\Delta V}{V_0}$

Elastic modulus (slide 8)

Young's modulus characterises resistance of a solid to changes in length – in other words, how responsive is it to external forces

$$Y = \frac{\text{stress}}{\text{strain}} = \frac{F_\perp / A}{\Delta L / L_0} = \frac{\sigma}{\epsilon}$$

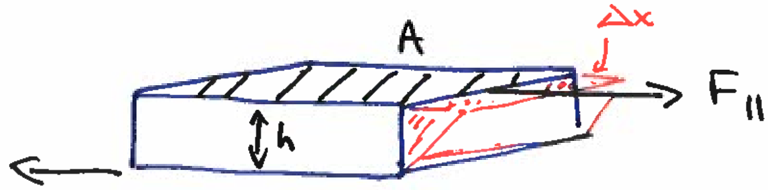
N.B. does not apply to liquids!



Shear modulus – resistance of a solid to shear forces

↖ not relevant to liquids

$$S = \frac{\text{stress}}{\text{strain}} = \frac{F_{\parallel}/A}{\Delta x/h}$$

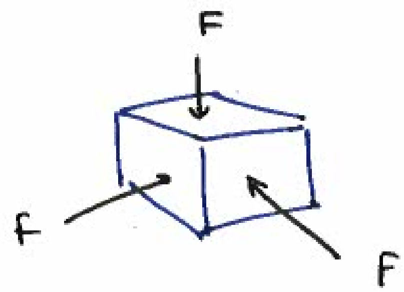


Bulk modulus – resistance to changes in volume

↖ applies to liquids too?

$$B = \frac{\text{stress}}{\text{strain}} = -\frac{F/A}{\frac{\Delta V}{V}} = -\frac{P}{\frac{\Delta V}{V}}$$

↑ minus sign ensures B is positive for ordinary materials



– compressibility is inverse of bulk modulus

$$k = \frac{1}{B} = -\frac{\Delta V/V_0}{P}$$

Mariana Trench example (slide 9)

$$B = -\frac{P}{\Delta V / V} \Rightarrow \frac{\Delta V}{V} = -\frac{P}{B} \Rightarrow \Delta V = -\frac{P}{B} V$$

The water starts at atmospheric pressure

$$P_{atm} = 1.01 \times 10^5 \ N/m^2$$

and occupies volume $V = 1 \ m^3$

$$\Delta V = -\frac{1.13 \times 10^8}{0.21 \times 10^{10}} \cdot 1 = -0.054 \ m^3$$

The change in density can be expressed as

$$\frac{\rho_{deep}}{\rho_{atm}} = \frac{M/V_{deep}}{M/V_{atm}} = \frac{M V_{atm}}{M V_{deep}} = \frac{V_{atm}}{V_{deep}}$$

But $V_{deep} = V_{atm} + \Delta V$

$$\Rightarrow \frac{\rho_{deep}}{\rho_{atm}} = \frac{V_{atm}}{V_{atm} + \Delta V}$$

$$= \frac{1}{1 - 0.054} = 1.0568$$

$\Rightarrow$ increase in pressure of 5.7 %