Physics 101H

General Physics 1 – Honors

Lecture 30 – 10/25/23

Rotational motion

Summary

Topics

Monday: Angular motion [chapter 11]
- Examples!
- Gyroscopes

Today: Angular motion [chapter 10/11]
- Group work

Announcements

Today:	**Problem set 5 due**
	Problem set 6 assigned
Tomorrow:	**Quiz 6**

Group Work

- Plan
  - 20 minutes: Work in groups on example problems
  - 10 minutes: Neatly write up solution
  - ?? minutes: Look at other groups' solutions
- Goals
  - Work with others
  - Communicate process in writing
  - Ask and get answers to questions as they come up
  - Consider the grader's perspective

Example 30.1: A bike has two wheels separated by a distance $2L$ and the centre of mass of the bike and its rider is halfway between the wheels, at a height of L above the ground. The coefficient of kinetic friction between the wheels and the ground is μ. If the brakes are applied hard enough so that skidding occurs in all of the following situations (assume the wheels always remain in contact with the ground), what is the deceleration of the bike if the brakes are applied to:

(a) Both wheels
(b) The back wheel (assume the front wheel experiences no friction whatsoever)
(c) The front wheel (assume the back wheel experiences no friction whatsoever)
(d) Which brake should you use to leave the longest skid mark?

Example 30.2: A stick has mass m, length l, and uniform mass density λ.

(a) Find the moment of inertia of the stick around an axis through one end.

The stick is held horizontal, with its left end attached to a pivot. It is then released.

(b) What is the tangential (linear) acceleration of the right end, immediately after release?

The stick is now stood up horizontally, with its bottom end attached to a pivot. It is given an infinitesimal push and allowed to swing downwards around the pivot.

(c) At the instant the stick is horizontal (that is, after a quarter turn), what is the tangential speed of the end of the stick?

Example 30.3: A uniform solid cylinder of mass m and radius R lies on top of a long board, of equal mass m. The board lies on an even longer table. There is sufficient friction between all objects to avoid slipping and the coefficient of kinetic friction between all surfaces is μ. The initial speed of both objects is v_0, which means that the cylinder is initially at rest with respect to the board. What is the acceleration of the board and the acceleration of the cylinder?

Example 30.4: A uniform solid cylinder of mass m and radius R lies on top of a long board, of equal mass m. The board lies on a plane inclined at an angle θ. The coefficient of kinetic friction between the board and the plane is μ=½ tan(θ). Assume that the coefficient of kinetic friction between the cylinder and the board is sufficiently large that the cylinder does not slip with respect to the board. What is the linear acceleration of the cylinder?

Example 30.5: If you drop a yo-yo from rest, while holding on to the end of the string, it will accelerate down, reach a maximum speed, and then slow down. Eventually it will reach its lowest point when the string is completely unwound.

(a) Explain qualitatively why the speed reaches its maximum value at an intermediate point, instead of at the bottom, as it would for a dropped ball.

(b) Show this quantitatively. To make this feasible, assume that the string is massless and wound in a single spiral that remains essentially circular at all times (so that you can assume the yo-yo has a moment of inertia of $mR^2/2$). Also assume that the string has a tiny thickness ε when viewed from the side and that the spiral starts with the radius R of the yo-yo. Find the radius of the spiral at which the yo-yo achieves its maximum speed.

Summary

Topics

Today: Angular motion [chapter 10/11]

- Group work

Tomorrow: Angular motion [chapter 10/11]

- Problem solutions

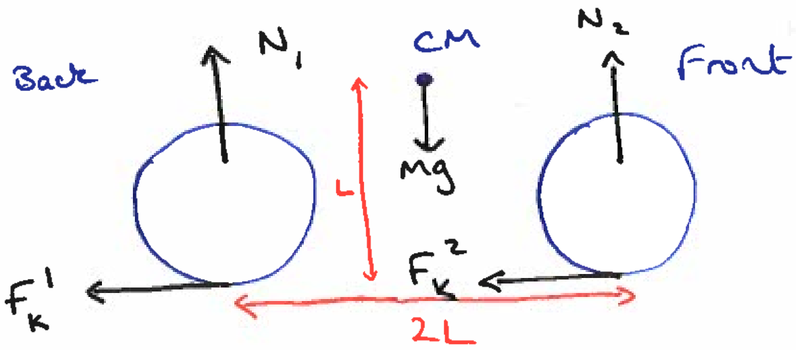
Announcements

Today:	**Problem set 5 due** **Problem set 6 assigned**
Tomorrow:	**Quiz 6**

PHYSICS 101 - HONORS

Lecture 30 10/25/23

Example 30.1 (slide 4)



(a) both wheels skidding

$$\Rightarrow F_K^{tot} = F_K^1 + F_K^2 = \mu N_1 + \mu N_2 = \mu(N_1 + N_2)$$

But vertically $N_1 + N_2 - Mg = 0 \Rightarrow F_K^{tot} = \mu Mg$

And horizontally $F_K^{tot} = Ma \Rightarrow a = \frac{F_K^{tot}}{M} = \boxed{\mu g}$

(b) back wheel skidding $\Rightarrow F_K^2 = 0$

$\Rightarrow$ horizontally $F_K^{tot} = \mu N_1 = Ma$

To determine N_1, we use the fact that the total torque around the centre of mass is zero:

$N_1 L + \mu N_1 L = N_2 L \Rightarrow N_1(1+\mu) = N_2$

But $N_1 + N_2 = Mg \Rightarrow N_1(1+\mu) = Mg - N_1$

$$\Rightarrow N_1(2+\mu) = Mg$$

$$\text{or } N_1 = \frac{Mg}{2+\mu}$$

Therefore the acceleration is $a = \frac{\mu N_1}{m} = \boxed{\frac{\mu g}{2+\mu}}$

(c) front wheel skidding $\Rightarrow F_K' = 0$

$\Rightarrow F_K^{tot} = \mu N_2 = ma$

Torque around the centre of mass gives

$$N_1 L + \mu N_2 L = N_2 L \Rightarrow N_2(1-\mu) = N_1$$
$$= mg - N_2$$
$$\Rightarrow N_2(2-\mu) = mg$$
$$\Rightarrow N_2 = \frac{mg}{2-\mu}$$

Therefore the acceleration is $a = \frac{\mu N_2}{m} = \boxed{\frac{\mu g}{2-a}}$

(d) The back brake! For $0<\mu<1$, this gives the the smallest acceleration and therefore the longest distance to come to a halt = the longest skid mark.

Example 30.2 (slide 5)

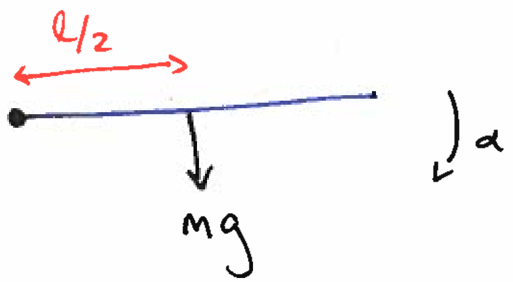
(a) $I = \int_0^{\ell} x^2 \, dm$

$$= \int_0^{\ell} x^2 \lambda \, dx$$

$$= \lambda \int_0^{\ell} x^2 \, dx$$

$$= \lambda \left. \frac{x^3}{3} \right|_0^{\ell} = \boxed{\frac{\lambda \ell^3}{3}}$$

(b)



The torque due to gravity generates an angular acceleration: $\tau = I \alpha$

In this case the torque is

$$\tau = mg \cdot \frac{\ell}{2}$$

$$\Rightarrow mg \frac{\ell}{2} = I \alpha = \frac{\lambda \ell^3}{3} \alpha$$

$$\Rightarrow \alpha = \frac{3 mg \ell}{2 \lambda \ell^3} \underset{m = \lambda \ell}{=} \frac{3 mg \ell}{2 m \ell^2} = \frac{3g}{2\ell}$$

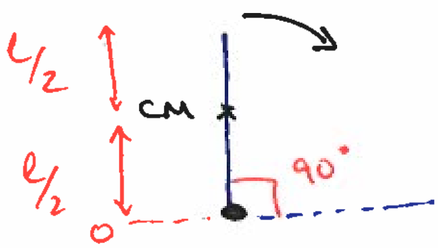
The linear acceleration is

$$a = \ell \alpha = \boxed{\frac{3g}{2}}$$

← Note $a > g$!

(c) For this question, we use conservation of energy

The centre of mass falls a distance $\ell/2$



Conservation of energy

$$\Rightarrow \quad mg\frac{\ell}{2} = \frac{1}{2} I \omega^2$$

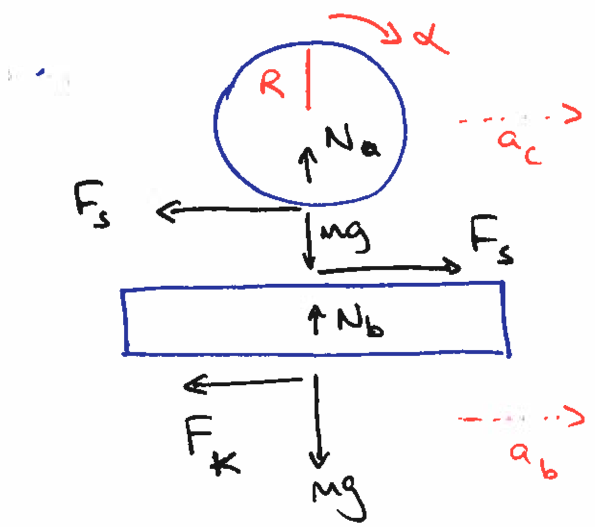
$$= \frac{1}{2} \cdot \frac{\lambda \ell^3}{3} \omega^2$$

The angular speed is therefore $\omega^2 = \frac{3mg\ell}{\lambda \ell^3} = \frac{3g}{\ell}$

The linear speed is $v = \omega r$

$$\Rightarrow \quad v = \sqrt{\frac{3g}{\ell}} \cdot \ell = \boxed{\sqrt{3g\ell}}$$

Example 30.3 (slide 6)



No slipping => static friction between the cylinder and the board!

For the board: $F_s - F_K = M a_b$ horizontally

$$\Rightarrow F_s - 2\mu_K M g = M a_b \quad (1)$$

vertically $N_b - 2Mg = 0$!

and $F_K = \mu_K N_b$

For the cylinder: $-F_s = m a_c \quad (2)$

$$F_s R = \frac{mR^2}{2} \cdot \alpha \quad (3)$$

To solve these equations we need one more relation, which comes from no slipping: $a_c = a_b + R\alpha \quad (4)$

$$\left.\begin{array}{l} (4) \Rightarrow R\alpha = a_c - a_b \\ (3) \Rightarrow R\alpha = \frac{2F_s}{m} \end{array}\right\} \Rightarrow F_s = \frac{m}{2}(a_c - a_b) \quad (5)$$

$$(5) \rightarrow (1) \Rightarrow -\frac{m}{2}(a_c - a_b) = m a_c$$

$$\text{or } \frac{m}{2} a_b = \frac{3m}{2} a_c \Rightarrow 3a_c = a_b \quad (6)$$

$$(2) \rightarrow (1) \Rightarrow -M a_c - 2\mu_K M g = M a_b$$

$$\Rightarrow -2\mu_K g = a_b + a_c \quad (7)$$

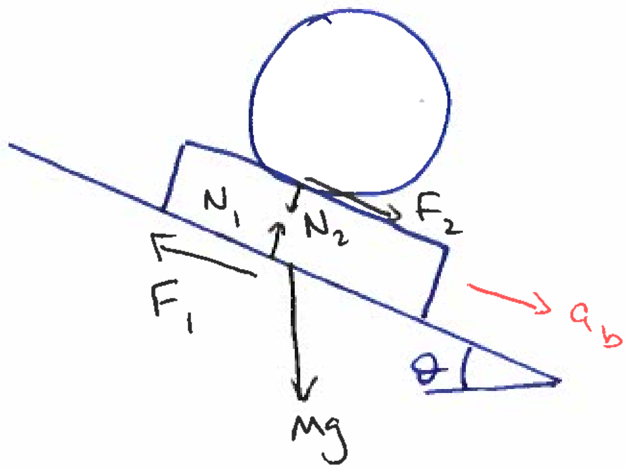
$$(6) \to (7) \Rightarrow \quad -2\mu_k g = 3a_c + a_c$$

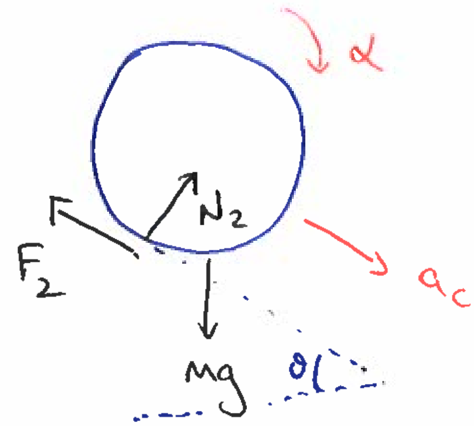
or $$\boxed{a_c = -\frac{\mu_k g}{2}}$$

and $$-2\mu_k g = a_b + a_b/3$$

or $$\boxed{a_b = -\frac{3\mu_k g}{2}}$$

Example 30.4





For the cylinder : $N_2 - Mg\cos\theta = 0 \Rightarrow N_2 = Mg\cos\theta$

For the board (perpendicular to plane) : $N_1 - N_2 - Mg\cos\theta = 0 \Rightarrow N_1 = 2Mg\cos\theta$

Kinetic friction for board : $F_1 = \mu N_1$

$$= \frac{1}{2}\tan\theta \cdot 2Mg\cos\theta$$

$$= Mg\sin\theta$$

For the cylinder : $Mg\sin\theta - F_2 = Ma_c$

For the board (parallel to plane) : $Mg\sin\theta - F_1 + F_2 = Ma_b$

$\Rightarrow Mg\sin\theta - Mg\sin\theta + F_2 = Ma_b$

$\Rightarrow F_2 = Ma_b$

For the cylinder (rotationally) : $F_2 R = \frac{MR^2}{2} \cdot \alpha \leftarrow \tau = I\alpha$ with $I = \frac{MR^2}{2}$

$\Rightarrow F_2 = \frac{MR\alpha}{2}$

No-slip condition : $a_c = a_b + R\alpha$

Thus $\quad m a_b = \frac{mR\alpha}{2}$

$\Rightarrow \quad a_b = \frac{R}{2}\alpha$

And so $\quad a_c = a_b + R\alpha$

$= \frac{R}{2}\alpha + R\alpha$

$= \frac{3R\alpha}{2} \quad \Rightarrow \quad a_c = 3a_b$

$\Rightarrow \quad F_2 = m a_b$

$= \frac{m a_c}{3}$

$\Rightarrow \quad mg \sin\theta - F_2 = m a_c$

$\Rightarrow \quad mg \sin\theta - \frac{m a_c}{3} = m a_c$

$\Rightarrow \quad \boxed{a_c = \frac{3}{4} g \sin\theta}$

Example 30.5 (slide 8)

(a) As the yo-yo falls, potential energy is converted to kinetic energy. This kinetic energy is both translational and rotational. As the yo-yo falls, the radius of the loop of string decreases, so the yo-yo must spin faster to generate a given vertical displacement in a given time. This means that, as the yo-yo falls, a larger fraction of the kinetic energy is rotational. At some point in time, the fraction of kinetic energy that is translational is decreasing faster than the total kinetic energy is increasing, so the linear speed starts to decrease.

(b) At some time t after release, the cross-sectional area of string unwound is

$$\pi R^2 - \pi r^2$$

r radius at time t

If ℓ is the length of hanging string, then the cross-sectional area of unwound string is $\ell \epsilon$

$$\Rightarrow \ell = \frac{\pi}{\epsilon}(R^2 - r^2)$$

Potential energy lost is $mg\ell$

Kinetic energy gained is $\frac{1}{2}(mv^2 + I\omega^2)$

As the yo-yo unwinds, it is essentially "rolling" down the vertical "plane" formed by the hanging string. This means the speed of the yo-yo at the point of "contact" is $v = \omega r$

$$\Rightarrow \quad mg\ell = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

becomes

$$mg\frac{\pi}{\epsilon}\left(R^2 - r^2\right) = \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{1}{2}mR^2\right)\left(\frac{v}{r}\right)^2$$

$$g\frac{\pi}{\epsilon}\left(R^2 - r^2\right) = v^2 \cdot \frac{1}{2}\left(1 + \frac{R^2}{2r^2}\right)$$

$$\Rightarrow \quad v^2 = \frac{2g\pi}{\epsilon}\,\frac{(R^2 - r^2)}{(1 + R^2/2r^2)}$$

The maximum v^2 occurs for $\frac{d}{dr}v^2 = 0$

$$\Rightarrow \quad \frac{2g\pi}{\epsilon}\left[(-2r)\left(1 + \frac{R^2}{2r^2}\right) - \left(\frac{-2R^2}{2r^3}\right)(R^2 - r^2)\right] = 0$$

$$\Rightarrow \quad (-2r)\left[r + \frac{R^2}{2r} - \frac{R^2}{2r^3}(R^2 - r^2)\right] = 0$$

$$\Rightarrow \quad r + \frac{R^2}{2r} - \frac{R^4}{2r^3} + \frac{R^2}{2r} = 0$$

$$r^4 + R^2r^2 - \frac{R^4}{2} = 0$$

This is a quadratic in r^2 and the positive solution is

$$r^2 = \frac{R^2}{2}\left(\sqrt{3} - 1\right)$$

or

$$\boxed{r = \frac{R}{\sqrt{2}}\sqrt{\sqrt{3} - 1}}$$