Physics 101H

General Physics 1 – Honors

Lecture 29 – 10/23/23

Rotational motion and gyroscopes

Summary

Topics

Friday: Angular motion [chapter 10/11]

- Rolling motion
- Angular momentum
- Rotational dynamics

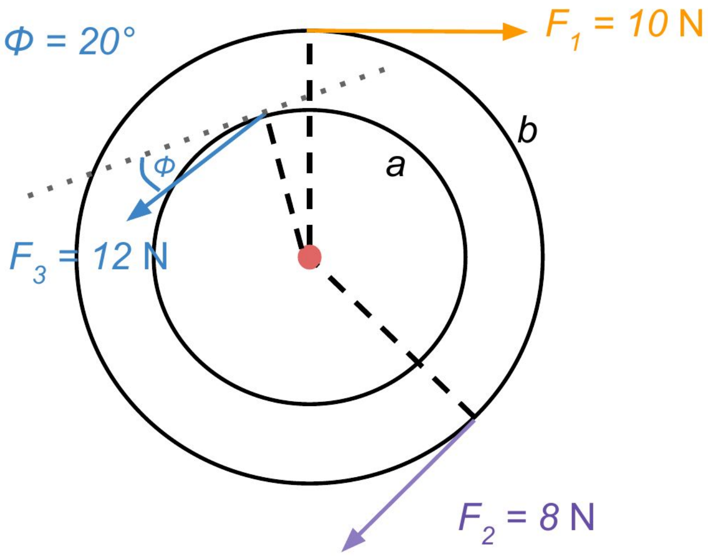
Today: Angular motion [chapter 11]

- Examples galore!
- Gyroscopes

Announcements

Wednesday:	**Problem set 5 due**
	Problem set 6 assigned
Thursday:	**Quiz 6**

Example 29.1: Find the net torque on the wheel, about the axle, if a = 7 cm and b = 21 cm. Assume that anticlockwise rotations are positive.



Example 29.2: A wheel of diameter 2.4 m rotates about its central axis with a constant angular acceleration of 3.6 rad/s^2. If it starts from rest, find the angular speed of the wheel, the tangential speed, the total acceleration, and where a point on the rim ends up after 2 s have elapsed, assuming that starts at 36.0 degrees with respect to the horizontal.

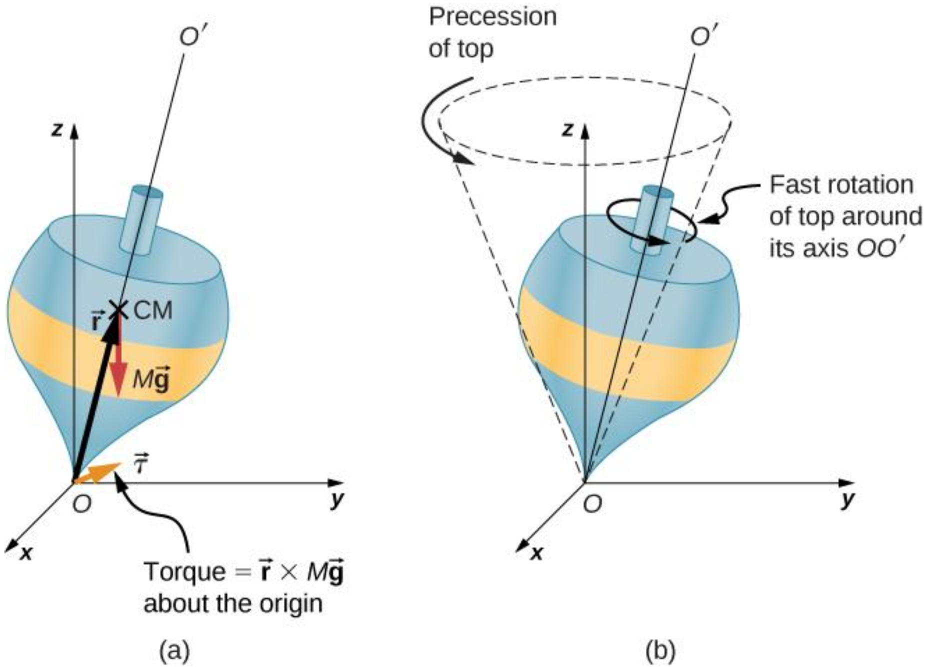
Example: Rigid rods of negligible mass connect three particles of mass 4 kg, 2 kg, and 3 kg at distances of 3 m, –2 m and –4 m from the x axis, respectively. The system rotates about the x axis with an angular speed of 1.1 rad/s. Find (a) the moment of inertia about the x axis; (b) the rotational kinetic energy of the system; (c) the tangential speed of each particle; and (d) the total translational kinetic energy of the system.

Gyroscope

A **gyroscope** is a spinning object for which the axis of rotation is free to move.

Gyroscopes preserve the orientation of their axis when they are spinning and can be used to detect rotation
 – this is super useful in, for example, space and is used for navigation in autonomous vehicles and robots

Gyroscopes undergo **precession**



Two minute table

Instructions: Draw a diagram for the following topic. You have two minutes. You may use your notes and you can consult with others around you. Your answer will not be graded; your answer is for your own learning.

Question: Recall your two-column table from Lecture 27 (one column for linear/translational motion and one for rotational motion). Update this table to include all the relevant quantities, kinematic equations and dynamics equations that we have discussed for rotational motion so far. Once you have finished, compare your table with your neighbours.

Want more practice?

Try the following problems **Chapter 10** of the textbook:

- Conceptual questions: 1, 3, 5, 9, 13, 17, 19, 21, 23
- Rotational variables and kinematics: 29, 33, 39, 43**,** 49, 53, **121**
- Moment of inertia: 59, 63, 65, 69, **123**
- Torque: 71, 75, 77, 81
- Newton's second law for rotations: 85, 89, 95

Answers are provided for questions with **blue** numbers (odd numbered)

Click on the number to be taken to the answer.

But make sure you at least **try** the problem first!

Want more practice?

Try the following problems **Chapter 11** of the textbook:

- Conceptual questions: 1, 3, 5, 9, 11, 15, 17
- Rolling motion: 23, 27, 29, 33, **83**
- Angular momentum: 35, 37, 43, 45, 51, **85**, **97**
- Angular momentum conservation: 55, 57, 61, 65, 71, **81**
- Precession: 77

Answers are provided for questions with **blue** numbers (odd numbered)

Click on the number to be taken to the answer.

But make sure you at least **try** the problem first!

Summary

Quiz 6 will cover:
Rockets
Moment of inertia
Torque and angular momentum
Three problem set questions

Topics

Today: Angular motion [chapter 11]
- Examples!
- Gyroscopes

Wednesday: Angular motion [chapter 10/11]
- Group work

Announcements

Wednesday: **Problem set 5 due**
Problem set 6 assigned

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PHYSICS 101 - HONORS

Lecture 29 10/23/23

Torque example (slide 3)

Recall $\bar{\tau} = \bar{r} \times \bar{F}$

First let's identify whether torques are positive or negative, and then we can use

$$|\bar{\tau}| = |\bar{r}||\bar{F}| \sin\theta$$

F_1 ↷ ⇒ −ve

F_2 ↷ ⇒ −ve

F_3 ↶ ⇒ +ve

$$\bar{\tau}_{net} = \bar{\tau}_1 + \bar{\tau}_2 + \bar{\tau}_3$$

$$\tau_{net} = -b F_1 \sin 90^\circ - b F_2 \sin 90^\circ + a F_3 \sin(90-20)$$

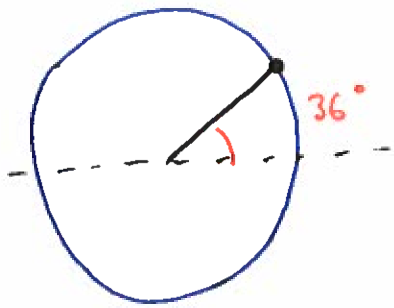
$$= -b(F_1 + F_2) + a F_3 \sin 70^\circ$$

$$= -0.21(10+8) + 0.07 \cdot 12\,N \cdot \sin 70^\circ$$

$$= -3.0 \ Nm$$

↑ clockwise!

Wheel example (slide 4)



$r = 1.2\ m \quad (d = 2.4\ m)$

$\alpha = 3.6\ rad/s^2$

$t = 2s$

$\omega_0 = 0\ rad/s$

$\theta_0 = 36.0°$

N.B.

$2\pi\ rad = 360°$

$\Rightarrow 1° = \frac{\pi}{180}\ rad$

$\Rightarrow 36° = 36 \cdot \frac{\pi}{180}\ rad$

- $\omega = \alpha t + \omega_0$

  $= 3.6 \times 2 = 7.2\ rad/s$

- $v = \omega r$

  $= 1.2 \times 7.2 = 8.64\ m/s$

- $a_{tot} = \sqrt{a_t^2 + a_r^2}$

  $= \sqrt{(\alpha r)^2 + (\omega^2 r)^2}$

  $= r\sqrt{\alpha^2 + \omega^4}$

  $= 1.2\sqrt{7.2^4 + 3.6^2}$

  $= 62.4\ m/s^2$

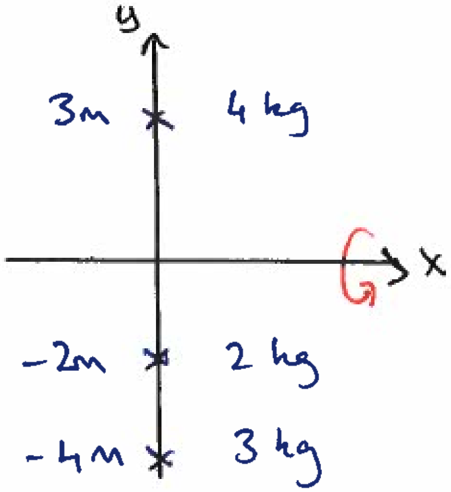
- $\theta = \frac{\alpha t^2}{2} + \omega_0 t + \theta_0$

  $= \frac{1}{2} \cdot 3.6 \cdot 2^2 + 0 + 0.628$

  $= 7.828\ rad$ ← but 2π rad is all the way around again!

$\theta = 7.828 - 2\pi = 1.54\ rad$ or $88.5°$

Three mass example (slide 5)



(a) $I = \sum_{i=1}^{3} m_i r_i^2$

$= 4 \cdot 3^2 + 2 \cdot (-2)^2 + 3 \cdot (-4$

$= \underline{92 \text{ kg m}^2}$

(b) $E_K = \frac{1}{2} I \omega^2$

$= \frac{1}{2} \cdot 92 \cdot 1.1^2$

$= \underline{55.7 \text{ J}}$

(c) $v = \omega r$

$\Rightarrow v_1 = \omega y_1 = 1.1 \cdot 3 = \underline{3.3 \text{ m/s}}$

$v_2 = \omega y_2 = 1.1 \cdot 2 = \underline{2.2 \text{ m/s}}$

$v_3 = \omega y_3 = 1.1 \cdot 4 = \underline{4.4 \text{ m/s}}$

(d) $E_K = E_K^1 + E_K^2 + E_K^3$

$= \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 + \frac{1}{2} m_3 v_3^2$

$= \frac{1}{2} \cdot 4 \cdot 3.3^2 + \frac{1}{2} \cdot 2 \cdot 2.2^2 + \frac{1}{2} \cdot 3 \cdot 4.4^2$

$= \underline{55.7 \text{ J}}$ (!)

Q: Why should this not be a surprise?

Gyroscopes (slide 6)

In the absence of rotational motion, gravity causes a top to fall over, by applying a torque about the pivot point.

When spinning, there is still a torque

$\bar{\tau} = \bar{r} \times \bar{F}$

$|\bar{\tau}| = |\bar{r}||\bar{F}| \sin\theta$

$= rmg \sin\theta$ ← points $\perp$ to $\bar{r}$ and $\bar{F}$

eg if $\bar{F}_g \propto \hat{z}$ and $\bar{r}$ is in y-z plane, then $\bar{\tau} \propto \hat{x}$

This torque causes angular momentum to change

because $\bar{\tau} = \dfrac{d\bar{L}}{dt}$ ← $\bar{L}$ is $\parallel$ to $\bar{r}$

Since $\bar{\tau}$ is $\not\parallel$ to $\bar{L}$, it only changes the direction of $\bar{L}$, not $|\bar{L}|$

↑ axis of rotation rotates around z axis ← precession

Angular speed of precession is

$$\omega_p = \frac{mgr}{I\omega}$$

← proof is in section 11.4 of textbook.

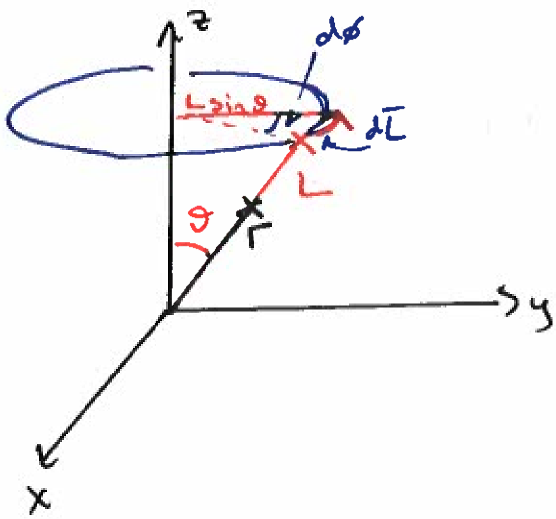
⌈ For completeness

$$\frac{dL}{dt} = rMg\sin\theta \Rightarrow dL = rMg\sin\theta\, dt$$

but $$d\phi = \frac{dL}{L\sin\theta} = \frac{rMg\sin\theta\, dt}{L\sin\theta} = \frac{rMg}{L}dt$$

$$\Rightarrow \omega = \frac{d\phi}{dt} = \frac{rMg}{L} = \frac{rMg}{I\omega}$$

Note



⌋

Translational and rotational motion

Rotational

$\bar{\theta}$

$\bar{\omega} = \frac{d\bar{\theta}}{dt}$

$\bar{\alpha} = \frac{d\bar{\omega}}{dt} = \frac{d^2\bar{\theta}}{dt^2}$

$\bar{\omega} = \bar{\alpha}t + \bar{\omega}_0$

$\bar{\theta} = \frac{\bar{\alpha}t^2}{2} + \bar{\omega}_0 t + \bar{\theta}_0$

$I = \int r^2 dm$ or $I = \sum_i m_i r_i^2$

$E_K^{rot} = \frac{I\omega^2}{2}$

$\bar{L} = \bar{r} \times \bar{p} = I\bar{\omega}$

$\bar{\tau} = \frac{d\bar{L}}{dt} = \bar{r} \times \bar{F}$

$\bar{\tau}_{net} = I\bar{\alpha}$

Translational

$\bar{r}$

$\bar{v} = \frac{d\bar{r}}{dt}$

$\bar{a} = \frac{d\bar{v}}{dt} = \frac{d^2\bar{r}}{dt^2}$

$\bar{v} = \bar{a}t + \bar{v}_0$

$\bar{x} = \frac{\bar{a}t^2}{2} + \bar{v}_0 t + \bar{x}_0$

m

$E_K^{lin} = \frac{mv^2}{2}$

$\bar{p} = m\bar{v}$

$\bar{F} = \frac{d\bar{p}}{dt}$

$\bar{F}_{net} = m\bar{a}$