Physics 101H

General Physics 1 – Honors

Lecture 28 – 10/20/23

Rotational motion

Summary

Topics

Yesterday: Moment of inertia [chapter 10]

- Moment of inertia calculations
- Parallel axis theorem
- Rotational kinetic energy

Today: Angular motion [chapter 10/11]

- Rolling motion and rotational kinetic energy
- Angular momentum
- Rotational dynamics

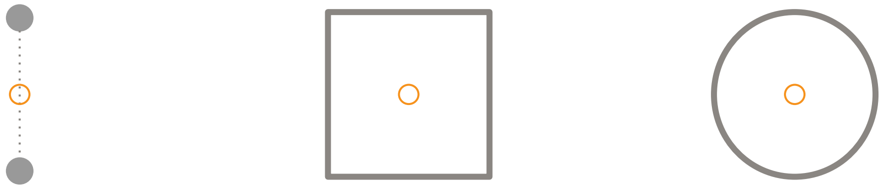
Announcements

Today: **No office hours**

Think-pair-share

Instructions: Consider the following question. After you have had a chance to think, I will ask you to raise your hands to indicate your answer. We will then discuss with our neighbours and have a second opportunity to vote.

Question: The three objects all have the same total mass and they all have the same vertical span (l). The first object is a massless rod connecting two point masses. The square and the circle are made from wires and are not solid planar objects. Which object has the largest moment of inertia around an axis that passes through the centre of mass, perpendicular to the page?



Rotational kinetic energy

Why all the fuss about the moment of inertia?

We have seen that it is the rotational analog of mass – and this analogy goes further!

Moment of inertia is central to rotational kinetic energy

Rolling motion

Imagine a cylinder rolling in a straight line along a flat surface without slipping

… what happens if we roll a cylinder down a slope?

Multiple choice

Instructions: Consider the following question. After you have had a chance to think, I will ask you to raise your hands to indicate your answer.

Question: Three objects, a solid cylinder, a ring, and a sphere are released on a slope. Which object will reach the bottom of the slope first?

(a) Solid cylinder (disc)
(b) Ring
(c) Sphere
(d) None – all will reach the bottom at the same time

Multiple choice

Question: Three objects, a solid cylinder, a ring, and a sphere are released on a slope. Which object will reach the bottom of the slope first?

(a) Solid cylinder (disc) (b) Ring (c) Sphere (d) None

Angular momentum

Angular momentum is a key quantity associated with angular motion

In fact, we can express **torque** as the rate of change of angular momentum

In the absence of external torques, angular momentum is **conserved**

Figure skating

Literally all of figure skating is explained by angular momentum conservation. Also many years of practice.

Example 28.1: A wheel of diameter 2.4 m rotates about its central axis with a constant angular acceleration of 3.6 rad/s^2. If it starts from rest, find the angular speed of the wheel, the tangential speed, the total acceleration, and where a point on the rim ends up after 2 s have elapsed, assuming that starts at 36.0 degrees with respect to the horizontal.

Summary

Topics

Today: Angular motion [chapter 10/11]

- Rolling motion and rotational kinetic energy
- Angular momentum
- Rotational dynamics

Monday: Angular motion [chapter 11]

- Examples galore!
- Gyroscopes

Announcements

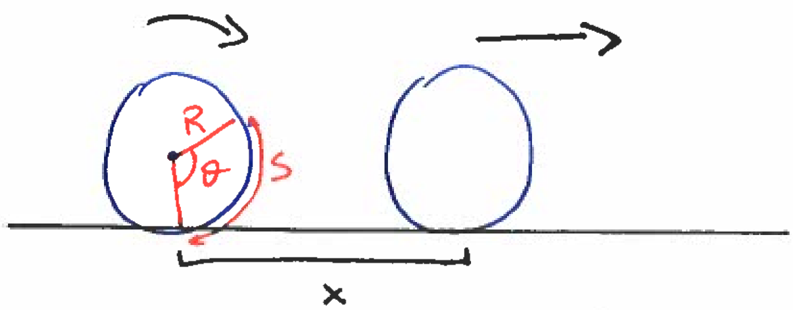
Today: **No office hours**

PHYSICS 101 - HONORS

Dumbbell	$I = mr^2$
Square	$I \leq mr^2$
Circle	$I = mr^2$

Lecture 28 10/20/23

Rolling motion (slide 5)



No slipping means
$s = x$
but $s = R\theta$
$\Rightarrow R\theta = x$

$$v_{cm} = \frac{dx}{dt} = \frac{d}{dt}(R\theta) = R\frac{d\theta}{dt} = R\omega$$

$$a_{cm} = \frac{dv_{cm}}{dt} = \frac{d}{dt}(R\omega) = R\frac{d\omega}{dt} = R\alpha$$

Kinetic energy is ← N.B. $W^{rot} = \Delta E_K^{rot}$

$$E_K = \frac{1}{2} I_g \omega^2$$

where I_g is the moment of inertia around an axis at the point of contact with the ground.

Parallel axis theorem $\Rightarrow I_g = I_{cm} + mR^2$

$$\Rightarrow E_K = \frac{1}{2}(I_{cm} + mR^2)\omega^2$$

$$= \frac{1}{2} I_{cm}\omega^2 + \frac{m}{2}(R\omega)^2$$

$$= \frac{1}{2} I_{cm}\omega^2 + \frac{m}{2} v_{cm}^2$$

← linear kinetic energy of centre of mass motion!

Thus

$$\boxed{E_K^{total} = E_K^{rot} + E_K^{linear}}$$

← the total kinetic energy is the sum of rotational and translational kinetic energies

Angular momentum (slide 8)

Recall that torque is $\overline{\tau} = \overline{r} \times \overline{F}$.

But $\overline{F} = \frac{d\overline{p}}{dt}$!

$$\Rightarrow \overline{\tau} = \overline{r} \times \frac{d\overline{p}}{dt}$$

Note, too that

$$\overline{v} \times \overline{p} \left(= \frac{d\overline{r}}{dt} \times \overline{p} \right) = \overline{v} \times m\overline{v} = m(\overline{v} \times \overline{v}) = 0$$

Therefore

$$\overline{\tau} = \overline{r} \times \frac{d\overline{p}}{dt} = \overline{r} \times \frac{d\overline{p}}{dt} + 0 = \overline{r} \times \frac{d\overline{p}}{dt} + \frac{d\overline{r}}{dt} \times \overline{p}$$

$$= \frac{d}{dt}\underbrace{(\overline{r} \times \overline{p})}$$

Define $\overline{L} = \overline{r} \times \overline{p}$ as the angular momentum

$$\Rightarrow \boxed{\begin{aligned} \overline{L} &= \overline{r} \times \overline{p} \\ \overline{\tau} &= \frac{d}{dt}\overline{L} \end{aligned}}$$

← torque is the rate of change of angular momentum (cf $\overline{F} = \frac{d\overline{p}}{dt}$)

But remember that

$$\bar{\tau} = I\bar{\alpha}$$

so

$$\frac{d\bar{L}}{dt} = I\bar{\alpha} = I\frac{d\bar{\omega}}{dt} = \frac{d}{dt}(I\bar{\omega})$$

$\Rightarrow$ $\boxed{\bar{L} = I\bar{\omega}}$ cf. $\bar{p} = m\bar{v}$

If there are no external torques, then $\bar{\tau} = 0$ and

$\frac{d\bar{L}}{dt} = 0 \Rightarrow \bar{L} = \text{constant}$

↖ or "$\bar{L}$ is conserved"

Note that if:

- $\bar{L}$ is constant
- I changes (mass distribution changes)

$\Rightarrow \omega$ changes !

Equation summary

Parallel axis theorem	$I = MR^2_{cm} + I_{cm}$
Kinetic energy	$E_K^{total} = E_K^{rot} + E_K^{lin}$
Angular momentum	$\bar{L} = \bar{r} \times \bar{p}$
Torque	$\bar{\tau} = \frac{d}{dt}\bar{L}$
	$\bar{L} = I\bar{\omega}$