Physics 101H

General Physics 1 – Honors

Lecture 26 – 10/18/23

Angular motion

Summary

Topics

Monday: Rockets [chapter 9]

- Rockets
- Propulsion and conservation of momentum
- One more centre of mass example

Today: Angular motion [chapter 10]

- Angular motion
- Moment of inertia
- Torque

Announcements

Today:	**Problem Set 5 posted**
Thursday:	**Quiz 5**
Friday:	**No office hours**

Rigid bodies

All motion so far has been for point particles

- Even centre of mass motion was treated as point particle motion

Now we will consider **rigid**, **extended objects**

- Rigid = nondeformable (relative positions of internal parts do not change)
- Extended = cannot be treated as a point particle

Extended objects **can rotate**

Torque and moment of inertia

When a force is exerted on a rigid object that is allowed to rotate, the object tends to rotate about that axis

Torque

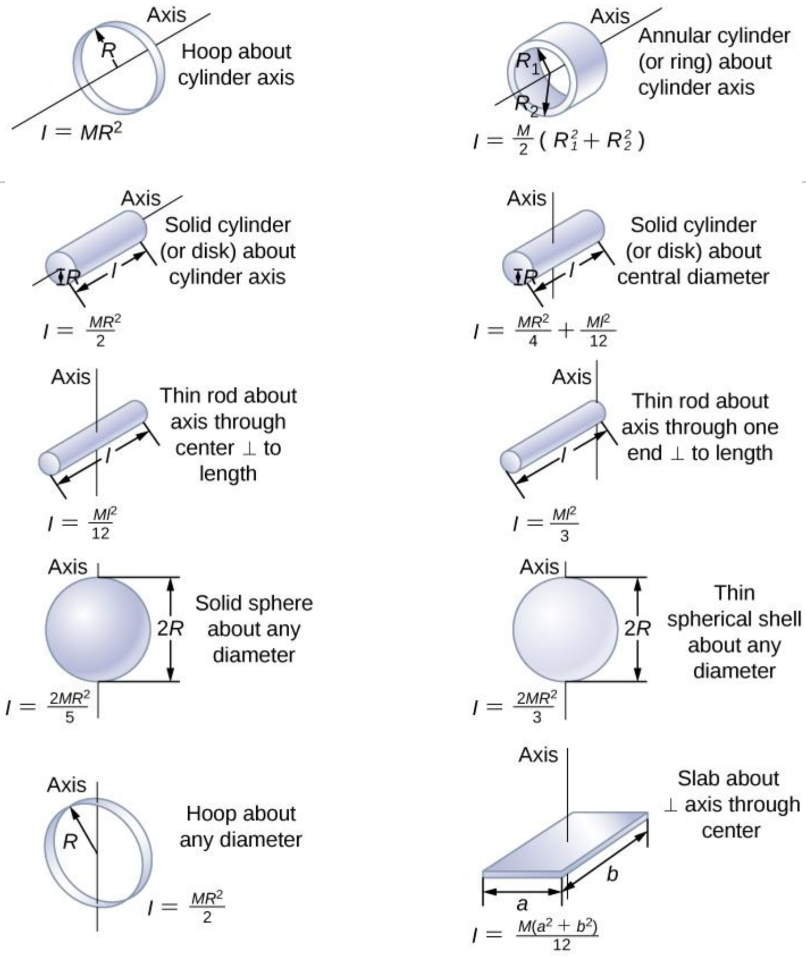
- measures the "turning power" of an applied force

Moment of inertia

- generalises our idea of mass as the resistance to linear motion
- measures the resistance to rotational motion

Common moments of inertia

Like Taylor series, we can look up common moments of inertia for simple geometrical bodies



Example: Find the moment of inertia for a rectangular plate that rotates about an axis through its center of mass.

Summary

Topics

Today: Angular motion [chapter 10]

- Angular motion
- Moment of inertia
- Torque

Tomorrow: Moment of inertia [chapter 10]

- Moment of inertia calculations
- Parallel axis theorem
- Rotational kinetic energy

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PHYSICS 101 - HONORS

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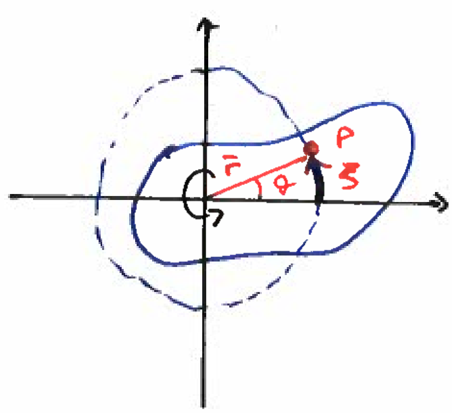
Rigid bodies (slide 3)

Translational		Rotational	
displacement	$\bar{r}$	angular displacement	$\bar{\theta}$
velocity	$\bar{v} = \frac{d\bar{r}}{dt}$	angular velocity	$\bar{\omega} = \frac{d\bar{\theta}}{dt}$
acceleration	$\bar{a} = \frac{d\bar{v}}{dt}$	angular acceleration	$\bar{\alpha} = \frac{d\bar{\omega}}{dt}$

↑ vectors point along the axis of rotation, with direction given by right hand rule

Consider motion of a rigid object around some axis



point p rotates $s = r\theta$ ← radians

with linear speed $v = \frac{ds}{dt} = \frac{d(r\theta)}{dt} = r\frac{d\theta}{dt} = r\omega$

tangential acceleration $a_t = r\frac{d\omega}{dt} = r\alpha$

Note the linear acceleration is $\bar{a} = \bar{a}_t + \bar{a}_r$

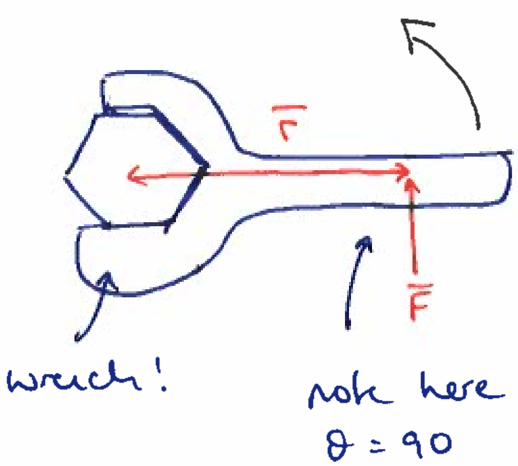
with $|\bar{a}| = \sqrt{|\bar{a}_t|^2 + |\bar{a}_r|^2} = \sqrt{r^2\alpha^2 + (v^2/r)^2} = r\sqrt{\omega^4 + \alpha^2}$

Kinematic equations for constant angular acceleration

$\alpha = \frac{d\omega}{dt} \Rightarrow \boxed{\omega = \alpha t + \omega_0}$ cf $v = a_0 t + v_0$

$\omega = \frac{d\theta}{dt} \Rightarrow \boxed{\theta = \frac{1}{2}\alpha t^2 + \omega_0 t + \theta_0}$ cf $x = \frac{at^2}{2} + v_0 t + x_0$

Torque and moment of inertia (slide 4)



Torque is defined as

$$\boxed{\bar{\tau} = \bar{r} \times \bar{F}}$$

or $|\bar{\tau}| = |\bar{r}|\,|\bar{F}| \sin\theta$

Note torque is larger when the force is further away from the axis of rotation ↖ think about opening a door

It also depends on the angle between $\bar{F}$ and $\bar{r}$ – if $\bar{F} \propto \bar{r}$ the $\bar{\tau} = 0$

↑

This tangential force causes a tangential acceleration

$$F_t = m a_t \Rightarrow \tau = F_t r = m a_t r = r(\alpha r) r = \underbrace{mr^2}\alpha$$

called the moment of inertia

Notice similarities

translational	rotational
$F = ma$	$\tau = I\alpha$

Moment of inertia measures resistance to rotational motion!

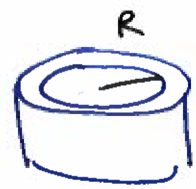
Moment of inertia:

← always measured with respect to some axis

- single particle $I = mr^2$
- system of particles $I = \sum_{i=1}^{N} m_i r_i^2$
- extended (continuous object) $I = \int r^2 \, dm$

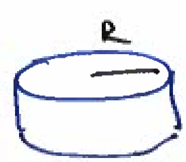
Useful results

- thin cylindrical shell



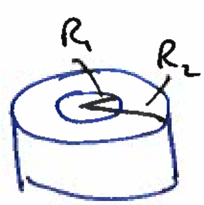
$I = MR^2$

- solid cylinder



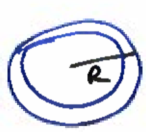
$I = \frac{1}{2}MR^2$

- hollow cylinder



$I = \frac{1}{2}M(R_1^2 + R_2^2)$

- spherical shell

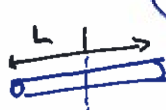


$I = \frac{2}{3}MR^2$

- sphere

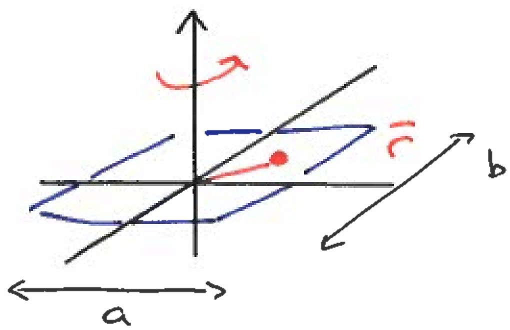
$I = \frac{2}{5}MR^2$

- rod



$I = \frac{1}{12}ML^2$

Plate example (slide 6)



$$I = \int r^2 \, dm$$

Density is $\rho = \frac{M}{V} = \frac{dm}{dV}$

$$\Rightarrow dm = \rho \, dV$$
$$= \rho \, dx \, dy \, dz$$

↑ ignore for thin sheet (dz)

$$I = \int_{-b/2}^{b/2} \int_{-a/2}^{a/2} r^2 \rho \, dx \, dy$$

$$= \rho \int_{-b/2}^{b/2} \int_{a/2}^{a/2} (x^2 + y^2) \, dx \, dy$$

$$= \rho \left[\int_{-b/2}^{b/2} dy \int_{-a/2}^{a/2} x^2 \, dx + \int_{-b/2}^{b/2} y^2 \, dy \int_{-a/2}^{a/2} dx \right]$$

$$= \rho \left[y \Big|_{-b/2}^{b/2} \frac{x^3}{3} \Big|_{-a/2}^{a/2} + y^3/3 \Big|_{-b/2}^{b/2} x \Big|_{-a/2}^{a/2} \right]$$

$$= \rho \left[\left(\tfrac{b}{2} - \left(\tfrac{-b}{2}\right)\right)\left(\frac{a^3}{3 \cdot 8} - \frac{(-a)^3}{3 \cdot 8}\right) + \left(\frac{b^3}{3 \cdot 8} - \frac{(-b)^3}{3 \cdot 8}\right)\left(\frac{a}{2} - \left(-\tfrac{a}{2}\right)\right) \right]$$

$$= \rho \left(\frac{ba^3}{12} + \frac{ab^3}{12} \right)$$

$$= \frac{\rho}{12} ab (a^2 + b^2)$$

$$\rho = \frac{M}{ab}$$

$$\Rightarrow I = \frac{M}{ab} \cdot \frac{ab}{12} (a^2 + b^2)$$

$$\boxed{I = \frac{M}{12} (a^2 + b^2)}$$

Equation summary

Angular displacement $\bar{\theta}$

Angular velocity $\bar{\omega} = \frac{d\bar{\theta}}{dt}$

Angular acceleration $\bar{\alpha} = \frac{d\bar{\omega}}{dt}$

Angular kinematics

$$\bar{\omega} = \bar{\alpha} t + \bar{\omega}_0$$

$$\bar{\theta} = \frac{1}{2}\bar{\alpha} t^2 + \bar{\omega}_0 t + \bar{\theta}_0$$

Torque $\bar{\tau} = \bar{r} \times \bar{F}$

"Newton's second law for angular motion" $\bar{\tau} = I \bar{\alpha}$

Moment of inertia

$$I = \sum_{i=1}^{n} m_i r_i^2$$

$$I = \int r^2 \, dm$$