Physics 101H

General Physics 1 – Honors

Lecture 25 – 10/16/23

Rockets

Summary

Topics

Wednesday: Centre of mass [chapter 9]

- 2D collision examples
- Centre of mass examples

Today: Rockets [chapter 9]

- Rockets
- Propulsion and conservation of momentum
- One more centre of mass example

Announcements

This week: **Problem Set 5 posted Wednesday**
Quiz on Thursday
No office hours on Friday

Rockets

Rockets are propelled by expelling a fuel at high velocity

Question: How does conservation of momentum explain how rockets work?

Example:

82. Unreasonable Results Squids have been reported to jump from the ocean and travel 30.0 m (measured horizontally) before re-entering the water.

(a) Calculate the initial speed of the squid if it leaves the water at an angle of 20.0°, assuming negligible lift from the air and negligible air resistance.

(b) The squid propels itself by squirting water. What fraction of its mass would it have to eject in order to achieve the speed found in the previous part? The water is ejected at 12.0 m/s; gravitational force and friction are neglected.

(c) What is unreasonable about the results?

(d) Which premise is unreasonable, or which premises are inconsistent?

Example: Find the y position of the centre of mass of an isosceles triangle, of density ρ, that is h high, $2w$ wide, and t thick (a three-dimensional triangle?).

Want more practice?

Try the following problems **Chapter 9** of the textbook:

- Conceptual questions: 1, 3, 9, 13, 17
- Momentum: 19, 23, **88**
- Impulse: 27, 31, 33, **92**
- Momentum conservation: 37, 39, 41
- Collisions: 45, 49, 51, 55, 59, 61, **96**, **106**
- Centre of mass: 63, 65, **69**, **71**
- Rockets: 77, 81, **112**

Answers are provided for questions with blue numbers (odd numbered)

Click on the number to be taken to the answer.

But make sure you at least **try** the problem first!

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2024 keynote speaker

We are pleased to announce the 2024 Millie

Contact Prof. Novikova (inovikova@physics.wm.edu) if interested

Summary

Quiz 5 will cover:

Work

Power

Energy transfer

Three multiple choice **group** questions

Topics

Today: Rockets [chapter 9]

- Rockets
- Propulsion and conservation of momentum
- One more centre of mass example

Wednesday: Angular motion [chapter 10]

- Angular motion
- Moment of inertia
- Torque

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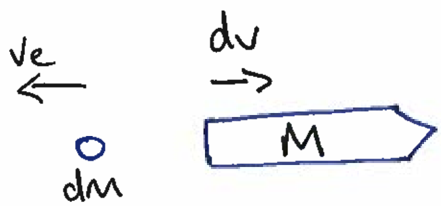
PHYSICS 101 - HONORS

Lecture 25 10/16/23

Rockets (slide 3)

Rockets eject fuel, which has momentum, and the rocket must have momentum in the opposite direction, to ensure momentum is conserved



conservation of momentum $\Rightarrow$ $v_e \, dm = M dv$

But as mass is expelled, M must change!

$$dM = -dm \Rightarrow -v_e \, dM = M dv$$

$$\Rightarrow -v_e \frac{dM}{M} = dv$$

$$\Rightarrow -v_e \int_{M_i}^{M_f} \frac{dM}{M} = \int_{v_i}^{v_f} dv$$

$$-v_e \ln M \Big|_{M_i}^{M_f} = v \Big|_{v_i}^{v_f}$$

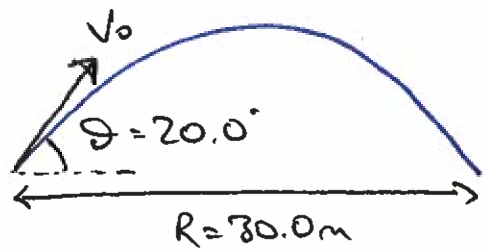
$$\Rightarrow v_f - v_i = -v_e (\ln M_f - \ln M_i)$$

or $\Delta v = v_e (\ln M_i - \ln M_f)$

$$\boxed{\Delta v = v_e \ln \frac{M_i}{M_f}}$$ ← the "rocket equation"

Squid example (slide 4)

(a)



Easiest to use the range equation

$$R = \frac{2v_0^2}{g} \sin\theta \cos\theta$$

$$\Rightarrow v_0 = \sqrt{\frac{gR}{2\sin\theta\cos\theta}} = 21.3974 \text{ m/s}$$

← keep sig figs for next part

$$\boxed{v_0 = 21.4 \text{ m/s}}$$

(b) Recall

$$\Delta v = v_e \ln \frac{M_i}{M_f} \Rightarrow \ln \frac{M_i}{M_f} = \frac{\Delta v}{v_e}$$

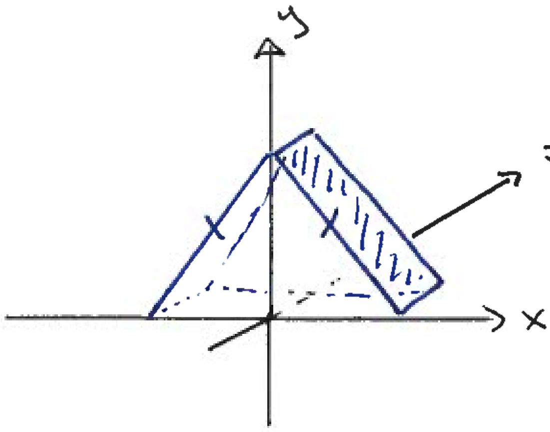
$$\Rightarrow \frac{M_i}{M_f} = \exp\left(\frac{\Delta v}{v_e}\right)$$

More natural to express this as

$$\frac{M_f}{M_i} = \frac{1}{\exp\left(\frac{\Delta v}{v_e}\right)} = e^{-\Delta v/v_e} = \exp\left(-\frac{21.3974}{12.0}\right)$$

$$= 0.168$$ ⇒ it would have to eject $\boxed{83\%}$ of its mass!

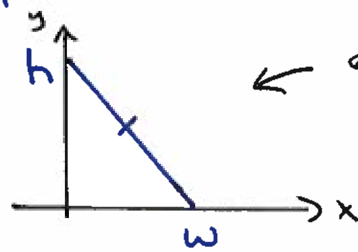
Triangle example (slide 5)



Symmetry tells us x-component must lie along line from base to point, which we define to be the y-axis

The z-component must be $t/2$.

For the y component, consider just one half of the triangle



← equation of line $y = mx + b$

in this case $y = -\frac{h}{w}x + h$

other half: $y = \frac{h}{w}x + h$

$y_{cm} = \frac{1}{M}\int y\,dm$ ← what is dm?

Density is $\rho = \frac{M}{V} \Rightarrow \rho = \frac{dm}{dV}$

$\Rightarrow dm = \rho\,dV$

$= \rho\,dx\,dy\,dz$

$\Rightarrow y_{cm} = \frac{2}{M}\iiint y\rho\,dx\,dy\,dz$

But what are the limits of integration?

$z \in [0, t]$ independent of x, y

x range depends on y

$y \in [0, h]$, once we take care of x

At a given y value, we integrate from

0 to $-\frac{h}{w}x + h$ $x \geq 0$ $\Rightarrow x = -\frac{w}{h}(y-h) = \frac{w}{h}(h-y)$

$\frac{h}{w}x + h$ to 0 $x < 0$ $\Rightarrow x = \frac{w}{h}(y-h)$

Thus

$$y_{cm} = \frac{1}{M}\int_0^t \int_0^h \left[\int_{\frac{w}{h}(y-h)}^0 + \int_0^{\frac{w}{h}(h-y)} \right] y\rho \, dx\,dy\,dz$$

$$= \frac{\rho}{M}\int_0^t dz \int_0^h y \left[\int_{\frac{w}{h}(y-h)}^0 dx + \int_0^{\frac{w}{h}(h-y)} dx \right] dy$$

$$= \frac{\rho}{M}\int_0^t dz \int_0^h y \int_{\frac{w}{h}(y-h)}^{\frac{w}{h}(h-y)} dx\, dy$$

$$= \frac{\rho}{M} \cdot t \cdot \int_0^h y \left[x \Big|_{\frac{w}{h}(y-h)}^{\frac{w}{h}(h-y)} \right] dy$$

$$= \frac{\rho t}{M} \int_0^h y \left(\frac{w}{h}(h-y) - \frac{w}{h}(y-h) \right) dy$$

$$= \frac{\rho t 2w}{M h} \int_0^h y(h-y)\, dy$$

$$= \frac{2\rho t w}{M h} \left[\frac{hy^2}{2} - \frac{y^3}{3} \right]_0^h$$

$$= \frac{2\rho t w}{M h} h \cdot h^2 \left(\frac{1}{2} - \frac{1}{3} \right) = \frac{2\rho t w h^2}{M 6}$$

Note $\rho = \frac{M}{V} = \frac{M}{2 \cdot (\frac{1}{2} w h) t} = \frac{M}{wht}$ $\Rightarrow y_{cm} = \frac{M}{wht} \cdot \frac{twh^2}{3M}$

$$= \boxed{\frac{h}{3}}$$

Equation summary

Conservation of momentum $\Delta \bar{p} = \bar{p}_f - \bar{p}_i = 0$

Conservation of kinetic energy $\Delta E_K = E_K^f - E_K^i = 0$

Rocket equation $\Delta v = v_e \ln \frac{M_i}{M_f}$

Centre of mass $\bar{r}_{cm} = \sum_{i=1}^{n} \frac{m_i \bar{r}_i}{M}$

$$\bar{r}_{cm} = \frac{1}{M} \int \bar{r}\, dm$$