Physics 101H

General Physics 1 – Honors

Lecture 22 – 10/6/23

Collisions

Summary

Topics

Yesterday: Momentum and collisions [chapter 9]

- Newton's third law and momentum
- Momentum and impulse
- Isolated systems
- Collisions

Today: Collisions and centre of mass [chapter 9]

- Elastic collisions
- Inelastic collisions

Announcements

Wednesday: **Problem Set 4 assigned**
Next week: **No problem set assigned**
No class on Thursday or Friday

Snooker


Top 7
Counterattacks
2021/2022
RONNIE O'SULLIVAN
JUDD TRUMP

Example 22.1: Find the final velocities in terms of the initial velocities in an elastic collision

Example 22.2: Find the final velocity in terms of the initial velocities in a perfectly inelastic collision in one dimension.

Example 22.3: Find the final velocity of a tennis ball when a tennis ball and basketball are dropped simultaneously from a height h? Assume that the tennis ball is vertically above the basketball and has a mass one tenth of the basketball, that both balls are initially stationary, and that all collisions are perfectly elastic.

Example 22.4: A pellet of mass m is fired into a block of wood of mass M, suspended from a wire. The pellet embeds itself in the block, and the combined system swings to a height h. What was the initial speed of the pellet (in terms of m, M, and h)?

Summary

Topics

Today: Collisions [chapter 9]

- Elastic collisions
- Inelastic collisions

Monday: Centre of mass [chapter 9]

- Collisions in 2D
- Centre of mass
- Calculating the centre of mass
- Centre of mass motion

Announcements

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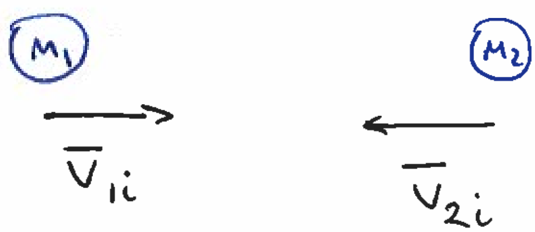
NEXT WEEK:
NO CLASS ON THURSDAY OR FRIDAY
NO LAB

PHYSICS 101 - HONORS

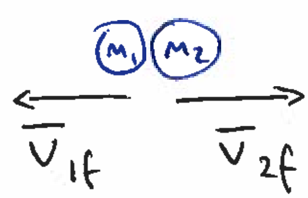
Lecture 22 10/6/23

Velocities in elastic collisions (slide 4)

Initial set-up :



Final :



Elastic collision => momentum is conserved
kinetic energy is conserved

↑ in practice these are <u>not quite true</u>,
but they are excellent approximations.

Momentum conservation $\Rightarrow \bar{p}_i = \bar{p}_f$ or $\bar{p}_i - \bar{p}_f = 0$

$$\bar{p}_i = \bar{p}_{1i} + \bar{p}_{2i} \qquad\qquad \bar{p}_f = \bar{p}_{1f} + \bar{p}_{2f}$$

$$= M_1\bar{v}_{1i} + M_2\bar{v}_{2i} \qquad\qquad = M_1\bar{v}_{1f} + M_2\bar{v}_{2f}$$

$$\Rightarrow M_1\bar{v}_{1i} + M_2\bar{v}_{2i} = M_1\bar{v}_{1f} + M_2\bar{v}_{2f}$$

or $M_1\bar{v}_{1i} - M_1\bar{v}_{1f} = M_2\bar{v}_{2f} - M_2\bar{v}_{2i}$

$$\Rightarrow M_1(\bar{v}_{1i} - \bar{v}_{1f}) = M_2(\bar{v}_{2f} - \bar{v}_{2i})$$

in 1D $M_1(v_{1i} - v_{1f}) = M_2(v_{2f} - v_{2i}) \qquad (*)$

kinetic energy conservation $\Rightarrow E_{Ki} = E_{Kf}$ or $E_{Ki} - E_{Kf} = 0$

$$E_{Ki} = E_{K1i} + E_{K2i} \qquad\qquad E_{Kf} = E_{K1f} + E_{K2f}$$

$$= \frac{1}{2}M_1v_{1i}^2 + \frac{1}{2}M_2v_{2i}^2 \qquad\qquad = \frac{1}{2}M_1v_{1f}^2 + \frac{1}{2}M_2v_{2f}^2$$

$$\Rightarrow \frac{1}{2}M_1v_{1i}^2 + \frac{1}{2}M_2v_{2i}^2 = \frac{1}{2}M_1v_{1f}^2 + \frac{1}{2}M_2v_{2f}^2$$

$\times 2$ and rearrange

$$\Rightarrow M_1(v_{1i}^2 - v_{1f}^2) = M_2(v_{2f}^2 - v_{1f}^2) \qquad (**)$$

We now have two equations for two unknowns, $\bar{v}_{1f}, \bar{v}_{2f}$

It helps to use a trick $(a^2 - b^2) = (a-b)(a+b)$ and notice that the combinations $(v_{1i} - v_{1f})$ and $(v_{2f} - v_{2i})$ appear in both $(*)$ and $(**)$ once you apply this trick to $(**)$

The results are

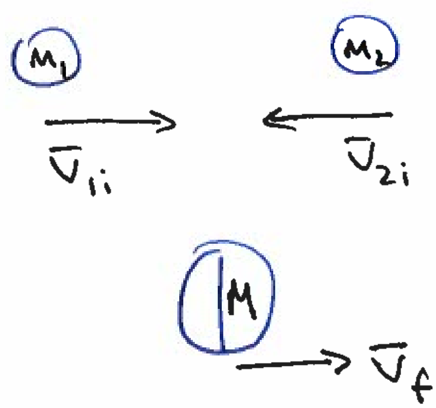
$$\boxed{\begin{aligned} v_{1f} &= \frac{(m_1 - m_2)\, v_{1i} + 2m_2 v_{2i}}{m_1 + m_2} \\ v_{2f} &= \frac{2m_1 v_{1i} + (m_2 - m_1)\, v_{2i}}{m_1 + m_2} \end{aligned}}$$

You are studying a simpler derivation (when one of the masses is initially stationary) in PS4.

Velocities in inelastic collisions (slide 5)

initial

final



inelastic => momentum is conserved
but kinetic energy is <u>not</u> conserved

<u>perfectly</u> inelastic => objects stick together
so $M = m_1 + m_2$
and $\bar{v}_{1f} = \bar{v}_{2f} = \bar{v}_f$

Momentum conservation $\Rightarrow$ $\bar{p}_i = \bar{p}_f$ or $\bar{p}_i - \bar{p}_f = 0$

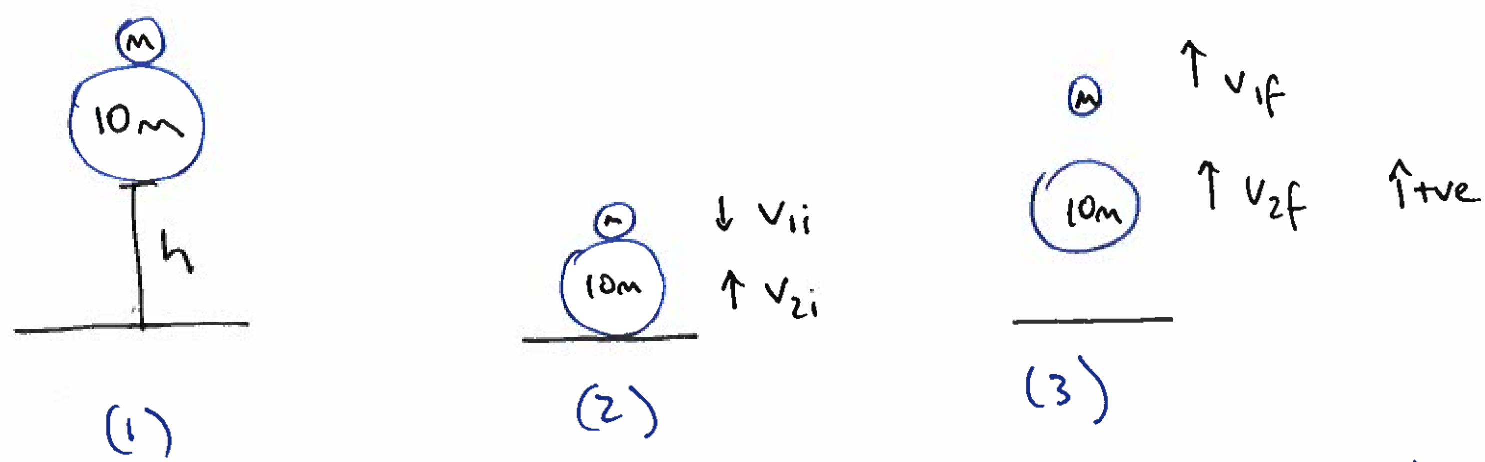
$$\bar{p}_i = m_1 \bar{v}_{1i} + m_2 \bar{v}_{2i}$$

$$\bar{p}_f = M \bar{v}_f$$

$$\Rightarrow m_1 \bar{v}_{1i} + m_2 \bar{v}_{2i} = M \bar{v}_f$$

$$\Rightarrow \boxed{\bar{v}_f = \frac{m_1 \bar{v}_{1i} + m_2 \bar{v}_{2i}}{M} = \frac{m_1 \bar{v}_{1i} + m_2 \bar{v}_{2i}}{m_1 + m_2}}$$

Basketball and tennis ball example (slide 6)



Two steps: first apply conservation of energy to determine speeds just before collision

$$E_{(1)} = E_{(2)}$$

$$\cancel{E_K^{(1)}}^{\,0} + E_p^{(1)} = E_K^{(2)} + \cancel{E_p^{(2)}}^{\,0}$$

$$10mgh = \frac{10}{2} m v_B^2$$

$$\Rightarrow v_B^2 = 2gh$$

$$v_B = \sqrt{2gh}$$ right as it hits the ground

Assuming the Earth is infinitely massive, the basketball bounces upward with the same kinetic energy (speed!)

$\Rightarrow \quad V_{2i} = \sqrt{2gh}$ in diagram (2)

Assuming $h \gg$ diameter of the tennis ball and the basketball

$\Rightarrow \quad v_{1i} \simeq \sqrt{2gh}$ downwards (in diagram (2))

Then use our result from earlier

$$V_{1f} = \frac{2M_2 V_{2i} + (m_1 - M_2) v_{1i}}{m_1 + M_2}$$

$$= \frac{2 \cdot 10m \sqrt{2gh} + (m - 10m)(-\sqrt{2gh})}{m + 10m}$$

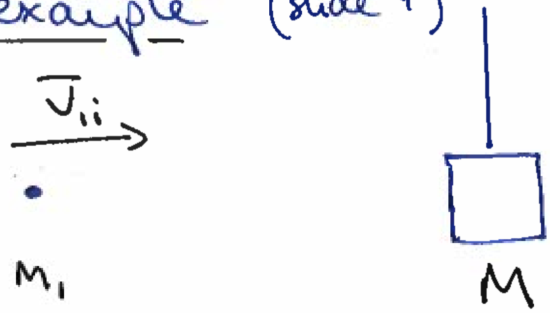
$$= \frac{(20m + 9m)}{11m} \sqrt{2gh}$$

$$= \frac{29}{11} \sqrt{2gh}$$ ← note $|v_{1i}| = \sqrt{2gh}$

$$\simeq 2.6 \cdot |v_{1i}|$$ $\Rightarrow$ 2.6 times faster!

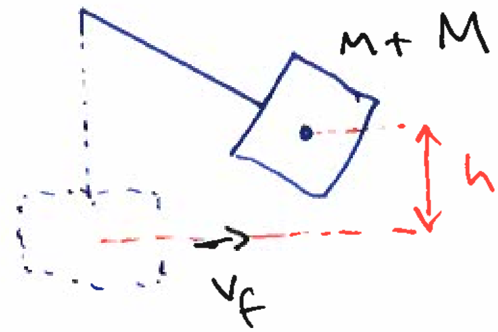
Pellet-block example (slide 7)

initial



really should draw 3 steps!

final



inelastic collision $\Rightarrow$ momentum conserved
kinetic energy not conserved

$$\bar p_i = m\bar v_{ii} + 0 = m\bar v_{ii}$$

$$\bar p_f = (M+m)\bar v_f$$

We now apply conservation of energy to find height

$$E_i = E_f \Rightarrow E_{ik} + \cancel{E_{ip}}^{\to 0} = \cancel{E_{fk}}^{\to 0} + E_{fp}$$

$$\frac{1}{2}(M+m)v_f^2 = (M+m)gh$$

$$\Rightarrow v_f = \sqrt{2gh}$$

So use this in

$$m v_{ii} = (M+m)\, v_f$$

$$v_{ii} = \left(\frac{M+m}{m}\right) v_f = \left(\frac{M+m}{m}\right)\sqrt{2gh}$$