Physics 101H

General Physics 1 – Honors

Lecture 15 – 9/25/23

Motion through a medium

Summary

Topics

Friday: Newton's laws [chapter 5]

- Solving problems (AKA a flipped classroom)

Today: in-medium motion [chapter 6]

- Motion through a medium
- Models of resistance:
  - Linear and quadratic

Announcements

Wednesday:	**No problem set assigned**
	Practice exam posted
Thursday:	**Quiz 4**
Next Wednesday:	**Midterm 1**

Practice in pairs

Instructions: Discuss the following question with a neighbour. Your answers will not be graded; your discussion is for your own learning*.

Question: A block is at rest on a plane inclined at angle θ. The forces on it are the gravitational, normal, and friction forces (not drawn to scale!). Which of the following statements is *always* true, for any θ?

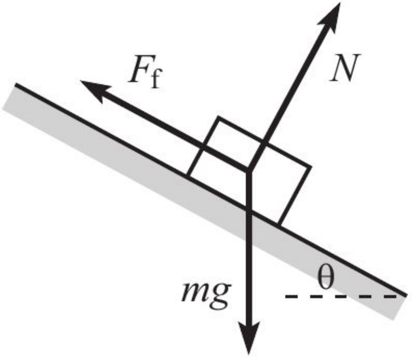
(a) $mg \leq N$ and $mg \leq F_f$

(b) $mg \geq N$ and $mg \geq F_f$

(c) $F_f = N$

(d) $F_f + N = mg$

(e) $F_f > N$ if $\mu_s > 1$



Motion through a medium

Many of our examples specify "frictionless" planes and pulleys and so on

But real objects experience friction when moving through a medium

- For example: air drag or viscosity
- Resistive force due to the medium
- Opposes the relative motion of the object and the medium
- Magnitude of the resistive force depends on the relative speed, possibly in some complicated (nonlinear) way

Motion through a medium

Resistance model

At low speeds, we can approximate the resistive force as linear in the speed

Leads to **terminal velocity** – constant velocity at which the projectile travels

At higher speeds, we can model the resistive force as quadratic in the speed

Linear resistance model

Quadratic resistance model

Summary

Quiz 4 will cover:

Forces

Newton's laws

Inertial and noninertial reference frames

Four multiple choice questions

Topics

Today: in-medium motion [chapter 6]

- Motion through a medium
- Models of resistance:
  - Linear and quadratic

Wednesday: Work [chapter 7]

- Work done
- Constant force
- Varying force

Announcements

Wednesday:	**No problem set assigned**
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PHYSICS 101 - HONORS

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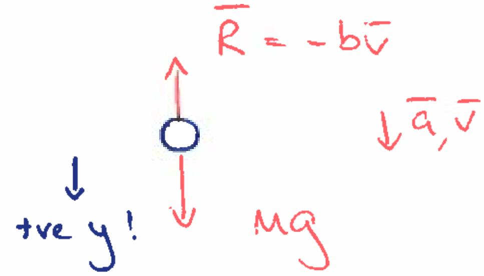
Linear resistance model (slide 5)

Assume $\bar{R} = -b\bar{v}$ $\Rightarrow$



1D motion with air resistance

$$\underbrace{\bar{F}_g + \bar{R}}_{} = m\bar{a}$$

$$\bar{F}_{net} = \sum_i \bar{F}_i = \bar{F}_g + \bar{R} = \bar{F}_g - b\bar{v}$$

$mg - bv = ma$ in y direction

$$a = g - \frac{b}{m}v = -\frac{b}{m}\left(v - \frac{mg}{b}\right)$$

This is an ordinary differential equation (ODE)

$$\frac{dv}{dt} = -\frac{b}{m}\left(v - \frac{mg}{b}\right) \Rightarrow \int \frac{dv}{v - \frac{mg}{b}} = -\frac{b}{m}\int dt$$

$\nwarrow$ $\int \frac{dx}{x} = \ln x$

$$\ln\left(v - \frac{mg}{b}\right) = -\frac{b}{m}t + c$$

$$\exp\left[\ln\left(v - \frac{mg}{b}\right)\right] = \exp\left[-\frac{b}{m}t + c\right]$$

& define $D = e^{-c}$ just another arbitrary constant

$$v - \frac{mg}{b} = De^{-bt/m}$$

$$v = De^{-bt/m} + \frac{mg}{b}$$

& define $A = \frac{bD}{mg}$ yet another arbitrary constant

$$\Rightarrow v(t) = \frac{mg}{b}\left(1 + Ae^{-bt/m}\right)$$

Let's try to determine A, assuming some boundary conditions. Let's assume that at $t=0$, $v=0$. Then

$$v = 0 \Rightarrow v(t=0) = \frac{mg}{b}\left(1 + A e^{-b\cdot 0/m}\right) = 0$$

or $\frac{mg}{b}(1 + A) = 0 \Rightarrow 1 + A = 0$

because $\frac{mg}{b} \neq 0$

So $A = -1$

$$\Rightarrow v(t) = \frac{mg}{b}\left(1 - e^{-bt/m}\right)$$

Note that as $t \to \infty$

$$v(t \to \infty) \to \frac{mg}{b}\left(1 - \underbrace{e^{-b\infty/m}}_{\to 0}\right) = \frac{mg}{b}$$

Velocity tends to a constant, called the terminal velocity

Quadratic resistance model (slide 7)

Take $\bar{R} = -\frac{1}{2} D \rho A v^2 \hat{v}$

- $-$: antiparallel to $\hat{v}$
- D: drag coefficient
- ρ: density of medium
- A: area of projectile
- v^2: quadratic in v (ie proportional to $|\bar{v}|^2$)

Now we have $\bar{F}_g + \bar{R} = m\bar{a}$

In 1D $mg - \frac{D}{2}\rho A v^2 = ma$

Terminal velocity requires $a = 0$

$$\Rightarrow mg - \frac{D}{2}\rho A v^2 = 0$$

or $$v^2 = \frac{2mg}{D\rho A}$$

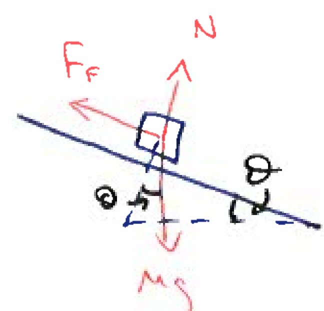
$$\Rightarrow v_T = \sqrt{\frac{2mg}{D\rho A}}$$

But what about the full ODE ?

$$m\frac{dv}{dt} = mg - \frac{D\rho A}{2} v^2$$

This is much more complicated, but we can solve it.
In 2D, it gets much more interesting! We will revisit this in a problem set soon!

<u>Plane example</u> (slide 3)



Parallel: $mg\sin\theta - F_f = 0$

Perpendicular: $N - mg\cos\theta = 0$

$\Rightarrow N = mg\cos\theta \Rightarrow N \leq mg$ because $\cos\theta \leq 1$ $\Rightarrow$ (a) is wrong

and $F_f = mg\sin\theta$

$\Rightarrow F_f \leq mg$ ⇖ **Answer (b)**

If $\theta = 0 \Rightarrow F_f = 0 \Rightarrow$ (c) is wrong!
$\theta = 90 \Rightarrow F_f = mg \Rightarrow$ (e) is wrong!

Note (d) $\Rightarrow \sin\theta + \cos\theta = 1$ which is <u>wrong</u>!