Physics 101H

General Physics 1 – Honors

Lecture 14 – 9/22/23

Newton's laws

Summary

Topics

Yesterday:

- Noninertial reference frames
- "Fictitious" forces

Today: Newton's laws [chapter 5]

- Solving problems
  (AKA a flipped classroom)

Two minute essay

Instructions: Write one paragraph on the following topic. You have two minutes. You may not use your notes and you should not consult with others around you. Your answer will not be graded; your answer is for your own learning and you don't need to share your answer.*

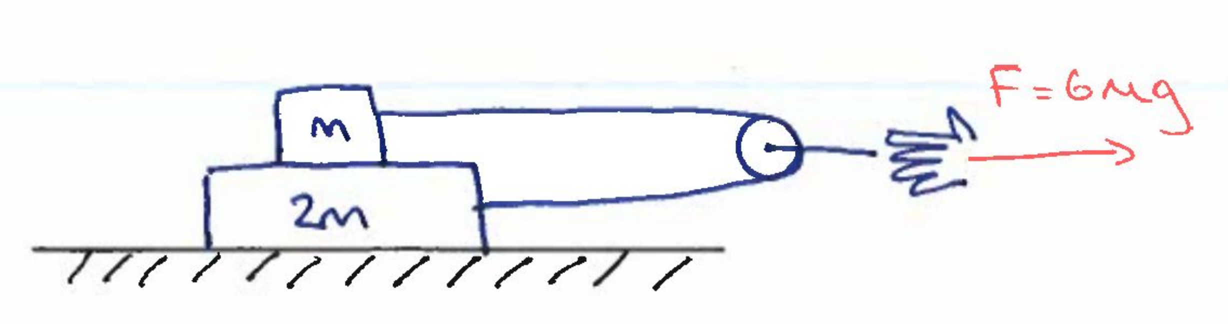
Question: Revisit your two minute essay on what happens if you hold a pendulum that is free to swing (such as a shoe on a shoestring) inside a plane accelerating down a runway during takeoff. [Remember that from Lecture 12 on Wednesday 20 September?!] Explain your reasoning in light of our discussion of fictitious forces yesterday.

Group Work

- Plan
  - 20 minutes: Work in groups on example problems
  - 10 minutes: Neatly write up solution
  - ?? minutes: Look at other groups' solutions
- Goals
  - Work with others
  - Communicate process in writing
  - Ask and get answers to questions as they come up
  - Consider the grader's perspective

Example 14.1: A block with mass m sits on top of a block with mass *2m*, which sits on a table. The coefficients of friction (both static and kinetic) between all surfaces are equal to one. A string is connected to each mass and wraps around a frictionless pulley. You pull on the pulley with a force of *6 mg*.

(a) Explain why the bottom block must slip with respect to the table.
(b) Explain why the top block must slip with respect to the bottom block.
(c) What is the acceleration of your hand?



[Hint: for parts (a) and (b), try assuming that the blocks in question don't slip and then derive a contradiction. This contradiction means that your assumption must be incorrect.]

Example 14.2: You are driving along a horizontal straight road that has a coefficient of static friction μ with your tyres. If you step on the brakes:

(a) What is your maximum possible deceleration?

(b) What is your maximum possible deceleration if, instead, you are travelling with speed v around a bend that forms a quarter circle, of radius R?

(c) Study your result for part (b) in the limits:

  (i) R -> Infinity (explain why this result makes sense!)

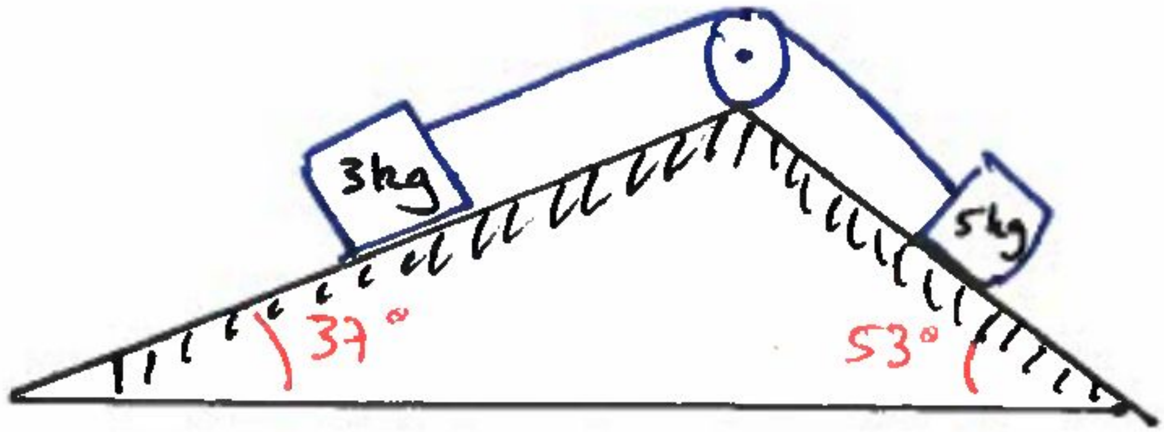
  (ii) v is very small (be sure to define small!)

Example 14.3: Two objects are connected by a light string that passes over a frictionless pulley. One object hangs from the string vertically below the pulley and the other lies on a frictionless incline plane. Find

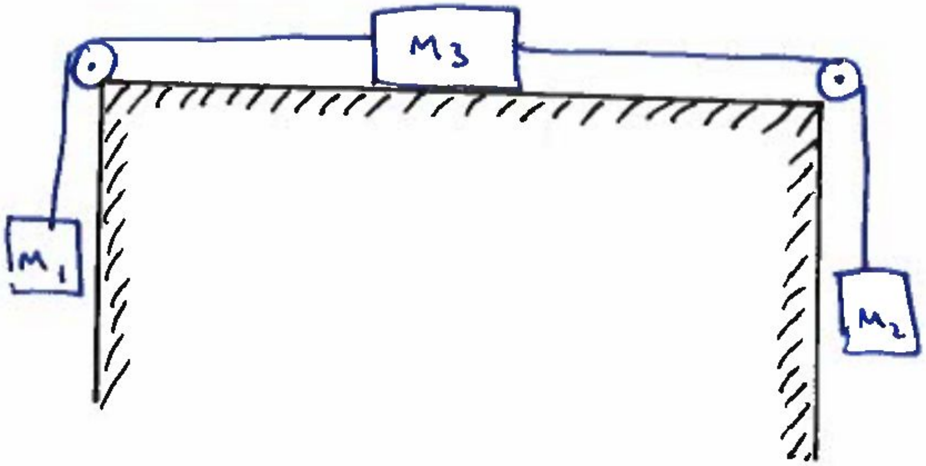
(a) The magnitude of the acceleration of the objects.
(b) The tension in the string.
(c) How do you your results change if the object on the incline plane experiences friction, with coefficients of friction (both static and kinetic) equal to one?

Example 14.4: Two blocks, of mass 3 kg and 5 kg, respectively, are connected by a massless string that passes over a frictionless pulley. The blocks are in contact with frictionless planes of angles 37° and 53°, respectively. Determine:

(a) The magnitude of the acceleration of the objects.
(b) The tension in the string.
(c) The normal force on each block?



Example 14.5: Three blocks, m_1, m_2, and m_3, are connected by massless strings that pass over two frictionless pulleys. The central block, m_3, rests on a frictionless surface.



Determine:

(a) The acceleration of the system.
(b) The tension in each string.
(c) What happens in the limit that $m_3 = 0$? Does this result make sense?

Summary

Topics

Today: Newton's laws [chapter 5]
- Solving problems (AKA a flipped classroom)

Monday: in-medium motion [chapter 6]
- Motion through a medium
- Models of resistance:
  - Linear and quadratic

Announcements

Next Wednesday: **No problem set assigned**
Practice exam posted

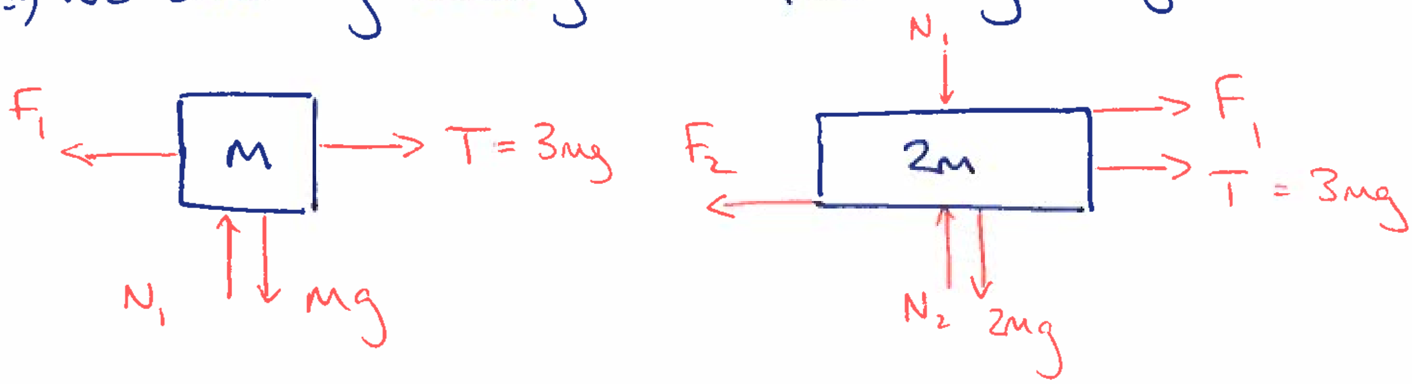
Wednesday October 4: Midterm 1

PHYSICS 101 - HONORS

Lecture 14 9/22/23

Question 14.1

a) We start by drawing two free-body diagrams



Vertically: $N_1 - mg = 0 \Rightarrow N_1 = mg$

$N_2 - N_1 - 2mg = 0 \Rightarrow N_2 = 3mg$

Now assume blocks don't slip

$$F_2^{max} = \mu_s N_2 = N_2 = 3mg$$

$$F_1^{max} = \mu_s N_1 = N_1 = mg$$

But $F_1^{max} < T \Rightarrow$ block 1 must slip!

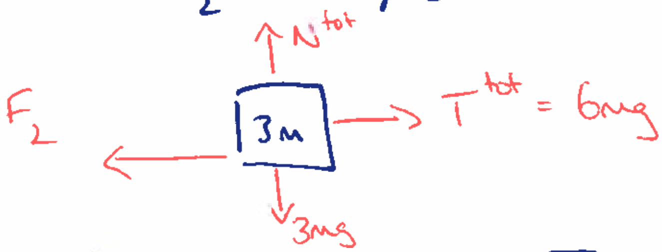
Now horizontal net forces must act on block 2, because

$$F_1 + T - F_2^{max} = mg + 3mg - 3mg = mg > 0 \Rightarrow a > 0.$$

Therefore blocks <u>must</u> slip.

(b) If the top block does not slip, we can treat the blocks as a single block with weight $3mg$

$$\Rightarrow F_2^{max} = \mu_s N^{tot} = N^{tot} = 3mg$$



comes from $N^{tot} - 3mg = 0$ which is Newton II applied vertically

Applying Newton II to the whole system horizontally:

$$T^{tot} - F_2^{max} = 6mg - 3mg = 3mg > 0$$

$= 3ma \Rightarrow a = g$

So the whole system accelerates rightward

Focussing on the top block and applying Newton II

$$T_1 - F_1 = ma \quad \text{or} \quad 3mg - F_1 = mg \Rightarrow F_1 = 2mg$$

But this isn't physically possible, because $F_1^{max} = mg$!

We have derived a contradiction, so our assumption that the top block does not slip is incorrect.

(c) We know that all blocks slip, so all friction forces are the kinetic friction forces

$$\Rightarrow F_1 = \mu_k N_1 = mg$$

$$F_2 = \mu_k N_2 = 2mg$$

Block 1 ; horizontally $T - F_1 = ma_1$

$$\Rightarrow 3mg - mg = ma_1$$

$$\Rightarrow a_1 = 2g$$

Block 2 ; horizontally $T + F_1 - F_2 = 2m a_2$

$$\Rightarrow 3mg + mg - 3mg = 2ma_2$$

$$\Rightarrow a_2 = g/2$$

The average position of the two blocks stays the same distance behind the pulley and therefore the same distance behind your hand

$$\Rightarrow a_{hand} = \frac{a_1 + a_2}{2}$$

$$= \frac{2g + g/2}{2}$$

$$= \boxed{\frac{5g}{4}}$$

Question 14.2

(a) The friction force causes your deceleration

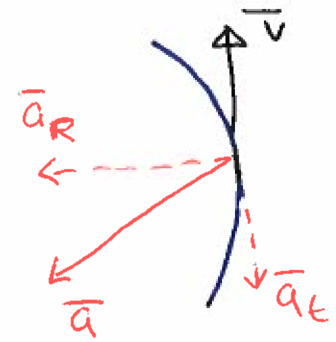
$$F_f = ma$$

We know that

$$F_f \leq \mu N = \mu m g \Rightarrow a \leq \mu g$$

The maximum deceleration is $\boxed{\mu g}$

(b) Going around a curve changes the acceleration vector



Radial component is $a_R = v^2/R$

Magnitude is $|\bar{a}| = \sqrt{a_t^2 + a_R^2}$

$$\bar{F}_f = m\bar{a} \Rightarrow |\bar{F}_f| = m|\bar{a}| = m\sqrt{a_t^2 + (v^2/R)^2}$$

This magnitude still satisfies $|\bar{F}_f| \leq \mu N$

$$\Rightarrow m\sqrt{a_t^2 + (v^2/R)^2} \leq \mu m g$$

$$\Rightarrow a_t^2 + (v^2/R)^2 \leq \mu^2 g^2 \Rightarrow \boxed{a_t \leq \sqrt{\mu^2 g^2 - (v^2/R)^2}}$$

(c) We will use a Taylor series!

$$a_t \le \underbrace{\sqrt{\mu^2 g^2 - (v^2/R)^2}}$$

$$= \mu g \sqrt{1 - \left(\frac{v^2}{\mu g R}\right)^2}$$

$$= \mu g \left(1 - \left(\frac{v^2}{\mu g R}\right)^2\right)^{1/2}$$

$$\simeq \mu g \left(1 - \frac{1}{2}\left(\frac{v^2}{\mu g R}\right) + \dots\right)$$

For $R \to \infty \Rightarrow \frac{v^2}{\mu g R} \to 0$ so $\boxed{a \simeq \mu g}$!

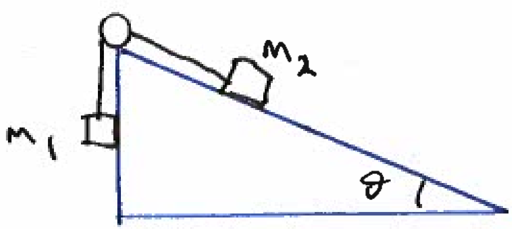
This makes sense, because $R \to \infty$ is a straight line (imagine → ) and so we reproduce the result from part (a)

v very small means $v \ll \sqrt{\mu g R}$

And we get the same result $\boxed{a \simeq \mu g}$

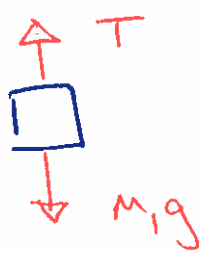
Question 14.3

(a) • start with a picture

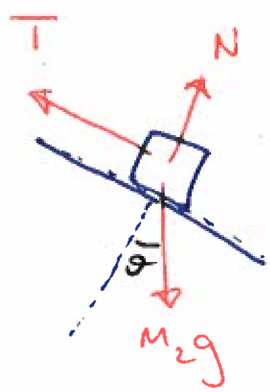


and draw free-body diagrams

m_1 :



m_2 :



• Choose vertical + horizontal directions for m_1

$\Rightarrow$ Vertical : $T - m_1 g = m_1 a \leftarrow$ assume a is upward

$\rightarrow T = m_1(a+g)$

• Choose parallel + perpendicular directions for m_2

$\Rightarrow$ Parallel : $T - m_2 g \sin\theta = m_2(-a) \leftarrow$ a is down slope

Perpendicular : $m_2 g \cos\theta - N = 0$

We have two equations for T and a

$$\left.\begin{aligned} T &= m_1(a+g) \\ T &= m_2(-a + g\sin\theta) \end{aligned}\right\} \Rightarrow m_1(a+g) = m_2(-a+g\sin\theta)$$

$$\Rightarrow m_1 a + m_2 a = -m_1 g + m_2 g \sin\theta$$

$$a(m_1 + m_2) = m_2 g \sin\theta - m_1 g$$

$$\boxed{a = \frac{g}{m_1 + m_2}\left(m_2 \sin\theta - m_1\right)}$$

(b) Now we can use this to find tension from

$$T = m_1(a + g)$$

$$= m_1\left(\frac{g}{m_1 + m_2}\left(m_2 \sin\theta - m_1\right) + g\right)$$

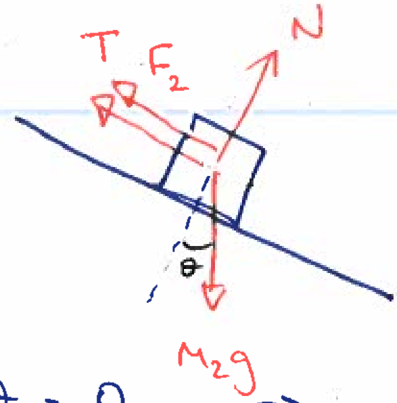
$$= m_1 g\left(1 + \frac{m_2 \sin\theta - m_1}{m_1 + m_2}\right)$$

$$= m_1 g\left(\frac{m_1 + m_2 - m_1 + m_2 \sin\theta}{m_1 + m_2}\right)$$

$$= \frac{m_1 g}{m_1 + m_2}\left(m_2 + m_2 \sin\theta\right)$$

$$\Rightarrow \boxed{T = \frac{m_1 m_2 g}{m_1 + m_2}\left(1 + \sin\theta\right)}$$

(c) Now we need a new free-body diagram for M_2



Perpendicular : $N - M_2 g \cos\theta = 0 \Rightarrow N = M_2 g \cos\theta$

Parallel : $T + F_2 - M_2 g \sin\theta = M_2(-a)$

Extra unknown now! Need to use

$$F_2 = \mu_K N = 1 \cdot M_2 g \cos\theta = M_2 g \cos\theta$$

Now two equations, two unknowns:

$$T = M_1(a+g)$$
$$T = M_2(g\sin\theta - g\cos\theta - a)$$

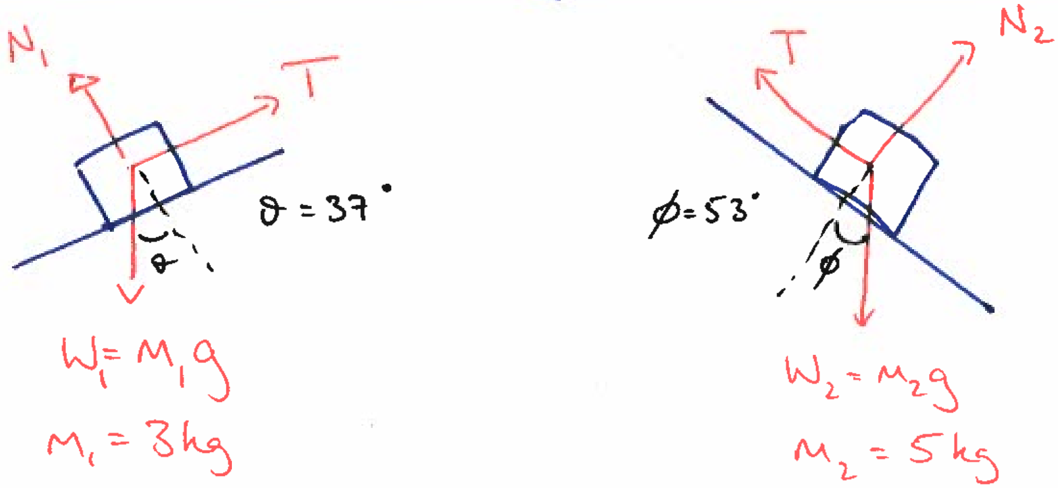
$$\Rightarrow M_1(a+g) = M_2(g\sin\theta - g\cos\theta - a)$$

$$\Rightarrow \boxed{a = \frac{g}{M_1+M_2}(M_2\sin\theta - M_2\cos\theta - M_1)}$$

$$\Rightarrow \boxed{T = \frac{M_1 M_2 g}{M_1+M_2}(1+\sin\theta-\cos\theta)}$$

Question 14.4

(a) start with free-body diagrams!



Note that $T_1 = T_2 = T$

and $a_1 = a_2 = a$

For m_1

↳ Perpendicular to surface : $N_1 - W_1 \cos\theta = 0$

$$N_1 = m_1 g \cos\theta$$

Parallel to surface : $T - m_1 g \sin\theta = m_1 a \Rightarrow T = m_1(a + g\sin\theta)$

For m_2

↳ Perpendicular to surface : $N_2 - W_2 \cos\phi = 0$

$$N_2 = m_2 g \cos\phi$$

Parallel to surface : $T - m_2 g \sin\phi = m_2(-a)$

$$\Rightarrow T = m_2(g\sin\phi - a)$$

Equate expressions for T:

$$m_1(a + g\sin\theta) = m_2(g\sin\phi - a)$$

$\theta = 37°$ $\quad m_1 = 3\,kg$

$\phi = 53°$ $\quad m_2 = 5\,kg$

$$\Rightarrow a(m_1 + m_2) = g(m_2 \sin\phi - m_1 \sin\theta)$$

$$a = \frac{g}{m_1 + m_2}(m_2 \sin\phi - m_1 \sin\theta) = \boxed{2.7\,m/s^2}$$

(b) We can use our answer to directly find T:

$$T = m_1(a + g\sin\theta)$$

$$= m_1\left[\frac{g}{m_1+m_2}(m_2\sin\phi - m_1\sin\theta) + g\sin\theta\right]$$

$$= \frac{m_1 g}{m_1+m_2}\left[m_2\sin\phi - m_1\sin\theta + (m_1+m_2)\sin\theta\right]$$

$$= \frac{m_1 g}{m_1+m_2}\left[m_2\sin\phi + m_2\sin\theta\right]$$

$$= \frac{m_1 m_2 g}{m_1+m_2}\left[\sin\phi + \sin\theta\right]$$

$$\Rightarrow \boxed{T = 3.6\ \text{N}}$$

(c) The normal forces are found from

$$N_1 = m_1 g\cos\theta$$
$$= \underline{23.5\ \text{N}}$$

$$N_2 = m_2 g\cos\phi$$
$$= \underline{29.5\ \text{N}}$$

(b) We can now use this to find the tensions

$$T_1 = m_1 (a + g)$$

$$= m_1 \left[\frac{(m_2 - m_1) g}{m_1 + m_2 + m_3} + g \right]$$

$$= \frac{m_1}{m_1 + m_2 + m_3} \left[m_2 g - m_1 g + m_1 g + m_2 g + m_3 g \right]$$

$$\Rightarrow \boxed{T_1 = \frac{m_1 g \,(2m_2 + m_3)}{m_1 + m_2 + m_3}}$$

$$T_2 = m_2 (g - a)$$

$$= m_2 \left[g - \frac{(m_2 - m_1) g}{m_1 + m_2 + m_3} \right]$$

$$\Rightarrow \boxed{T_2 = \frac{m_2 g \,(2m_1 + m_3)}{m_1 + m_2 + m_3}}$$

(c) if $m_3 = 0$ then $T_1 = \dfrac{2 m_1 m_2 g}{m_1 + m_2}$

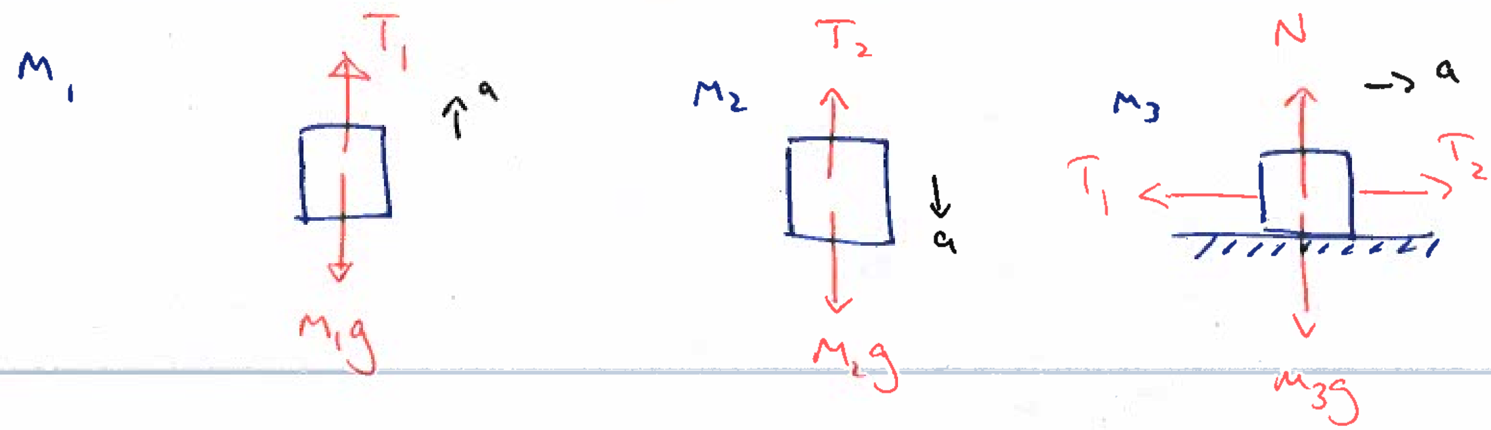
$$T_2 = \frac{2 m_1 m_2 g}{m_1 + m_2}$$

$\Rightarrow$ magnitude of T_1 = magnitude of T_2

This <u>does</u> make sense, because m_3 has no effect, it is as if it were just part of the massless string

Question 14.5

(a) We start with free-body diagrams



Note $a_1 = a_2 = a_3 = a$ but $T_1 \neq T_2$!

For M_1 : $T_1 - M_1 g = M_1 a \Rightarrow T_1 = M_1(a+g)$

For M_2 : $T_2 - M_2 g = M_2(-a) \Rightarrow T_2 = M_2(g-a)$

For M_3 : $T_2 - T_1 = M_3 a$

Plug our expressions for T_1 and T_2 into this last equation

$$M_2(g-a) - M_1(g-a) = M_3 a$$

$$\Rightarrow M_2 g - M_1 g = M_3 a + M a + M_2 a$$

$$(M_2 - M_1) g = (M_3 + M_1 + M_2) a$$

$$\Rightarrow \boxed{a = \frac{(M_2 - M_1)\, g}{M_1 + M_2 + M_3}}$$