Physics 101H

General Physics 1 – Honors

Lecture 10 – 9/15/23

2D motion

Summary

Topics

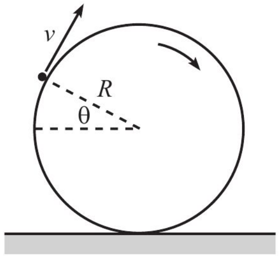
Yesterday: kinematics in 2D [chapter 4]

- Power outage!

Today: kinematics in 2D [chapter 4]

- Circular motion
- Relative velocity
- Galilean transformations
- Lorentz transformations

Example 10.1: A wheel of radius R is stuck in the mud, spinning in place with the rim moving at speed v. Bits of the mud depart from the wheel at various random locations, as in the figure. Find the maximum height that the mud reaches, assuming $v^2 > gR$.



Is motion in a vertical circle uniform circular motion or nonuniform circular motion?

Nonuniform motion

Generalise our decomposition of the acceleration to a more general case:

- motion along a curved path with variable speed

Acceleration can be broken into **radial** and **tangential acceleration**

Relative motion

Motion gets even more interesting

Relative velocity

Position is defined by first choosing a specific reference frame

Velocity is therefore defined with respect to a specific reference frame

So how do we compare observations made by experimenters in different reference frames that are moving with respect to each other?

Frame transformations

Galilean transformations – relate observations in moving reference frames

Transformations relevant to **nonrelativistic motion** (slowly moving objects)

Lorentz transformations – also relate observations in moving reference frames!

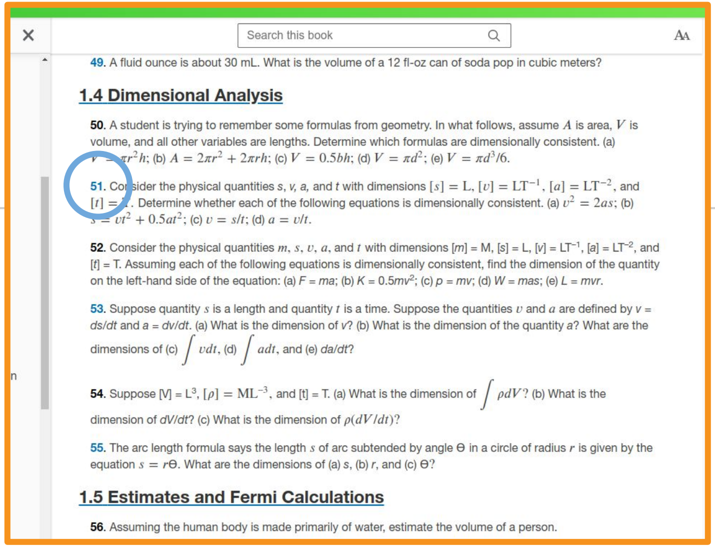
Transformations relevant to **special relativity**

- Describes fast moving objects
- Entangle both space and time (i.e. **spacetime**), not just space
- Ensure objects with mass cannot travel faster than light in a vacuum

Want more practice?

Try the following problems **Chapter 4** of the textbook:

- Conceptual questions: 5, 7, 11, 15
- 2D kinematics as vectors: 19, 23, 27, 31
- Projectile motion: 33, 35, 39, 43, 51, **99**
- Uniform circular motion: 61, 63, 67, **87**
- Relative motion: 73, 77



Search this book

49. A fluid ounce is about 30 mL. What is the volume of a 12 fl-oz can of soda pop in cubic meters?

1.4 Dimensional Analysis

50. A student is trying to remember some formulas from geometry. In what follows, assume A is area, V is volume, and all other variables are lengths. Determine which formulas are dimensionally consistent. (a) $V = \pi r^2 h$; (b) $A = 2\pi r^2 + 2\pi rh$; (c) $V = 0.5bh$; (d) $V = \pi d^2$; (e) $V = \pi d^3/6$.

51. Consider the physical quantities s, v, a, and t with dimensions $[s] = \mathrm{L}$, $[v] = \mathrm{LT}^{-1}$, $[a] = \mathrm{LT}^{-2}$, and $[t] = \mathrm{T}$. Determine whether each of the following equations is dimensionally consistent. (a) $v^2 = 2as$; (b) $s = vt^2 + 0.5at^2$; (c) $v = s/t$; (d) $a = v/t$.

52. Consider the physical quantities m, s, v, a, and t with dimensions $[m] = \mathrm{M}$, $[s] = \mathrm{L}$, $[v] = \mathrm{LT}^{-1}$, $[a] = \mathrm{LT}^{-2}$, and $[t] = \mathrm{T}$. Assuming each of the following equations is dimensionally consistent, find the dimension of the quantity on the left-hand side of the equation: (a) $F = ma$; (b) $K = 0.5mv^2$; (c) $p = mv$; (d) $W = mas$; (e) $L = mvr$.

53. Suppose quantity s is a length and quantity t is a time. Suppose the quantities v and a are defined by $v = ds/dt$ and $a = dv/dt$. (a) What is the dimension of v? (b) What is the dimension of the quantity a? What are the dimensions of (c) $\int v\,dt$, (d) $\int a\,dt$, and (e) da/dt?

54. Suppose $[V] = \mathrm{L}^3$, $[\rho] = \mathrm{ML}^{-3}$, and $[t] = \mathrm{T}$. (a) What is the dimension of $\int \rho\,dV$? (b) What is the dimension of dV/dt? (c) What is the dimension of $\rho(dV/dt)$?

55. The arc length formula says the length s of arc subtended by angle Θ in a circle of radius r is given by the equation $s = r\Theta$. What are the dimensions of (a) s, (b) r, and (c) Θ?

1.5 Estimates and Fermi Calculations

56. Assuming the human body is made primarily of water, estimate the volume of a person.

Answers are provided for questions with blue numbers (odd numbered)

Click on the number to be taken to the answer.

But make sure you at least **try** the problem first!

Summary

Topics

Today: kinematics in 2D [chapter 4]

- Circular motion
- Relative velocity
- Galilean transformations
- Lorentz transformations

Monday: Forces [chapter 5]

- Types of forces
- Field forces and contact forces

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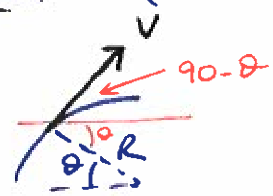
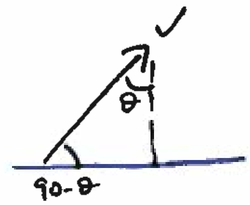
Wheel example: (slide 3)

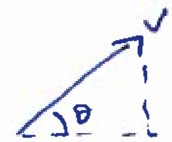
First note:





$v_x = v\sin\theta$
$v_y = v\cos\theta$

Recall $h = \dfrac{v_i^2 \sin^2\theta}{2g}$ but for θ defined by



So here $h = \dfrac{v^2\sin^2(90-\theta)}{2g} = \dfrac{v^2}{2g}\cos^2\theta$

$\Rightarrow$ total height is initial height + height gained

$$H = (R + R\sin\theta) + \frac{v^2\cos^2\theta}{2g}$$

To maximise, take $\dfrac{dH}{d\theta} = 0$

$$\Rightarrow \frac{dH}{d\theta} = R\cos\theta - \frac{v^2}{g}\sin\theta\cos\theta = 0$$

$$\Rightarrow \left(R - \frac{v^2}{g}\sin\theta\right)\cos\theta = 0$$

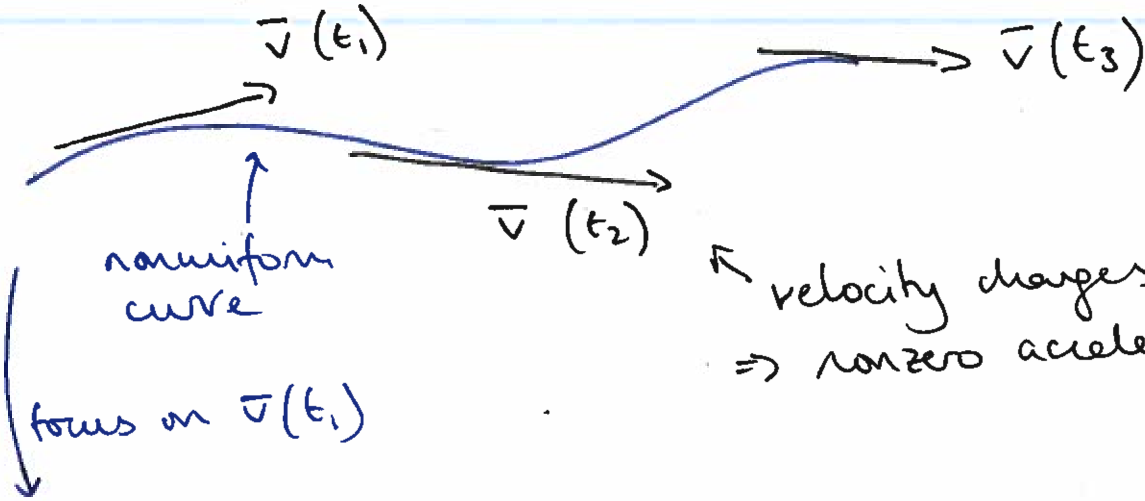
$\sin\theta = \dfrac{gR}{v^2}$ ← this is <1 if $v^2 > gR$ ✓

$\theta = \pi/2$ ($-\pi/2$ unphysical)

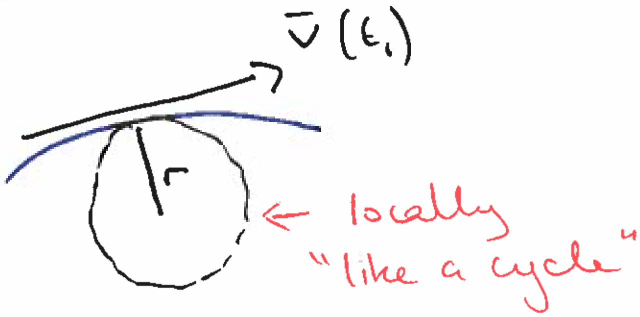
So $H_{max} = R + R\cdot\left(\dfrac{gR}{v^2}\right) + \dfrac{v^2}{2g}\left(1 - \dfrac{g^2R^2}{v^4}\right) = R + \dfrac{gR^2}{v^2} + \dfrac{v^2}{2g} - \dfrac{gR^2}{2v^2}$

$$= \boxed{R + \frac{gR^2}{2v^2} + \frac{v^2}{2g}}$$

Nonuniform motion (slide 5)



velocity changes
$\Rightarrow$ nonzero acceleration!

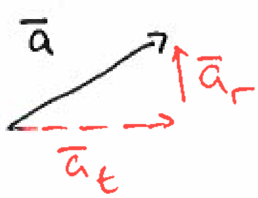


Key points:

- velocity tangent to path
- acceleration is not necessarily perpendicular to velocity

Write acceleration as $\bar{a} = \bar{a}_r + \bar{a}_t$

radial $\perp$ to $\bar{v}$ — tangential, $\parallel$ to $\bar{v}$



$$|\bar{a}| = \sqrt{|\bar{a}_r|^2 + |\bar{a}_t|^2}$$

Acceleration changes with time, so decomposition changes

$$|\bar{a}_t| = \left|\frac{d\bar{v}}{dt}\right| \rightarrow \text{changing speed}$$

$$|\bar{a}_r| = a_c = v^2/r \rightarrow \text{changing direction}$$

Velocities in frames moving at constant velocity with respect to each other add!

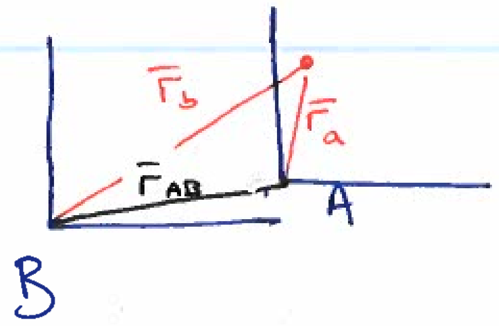
But note that accelerations don't change if the frames are moving at constant velocity with respect to each other

$$\bar{a}_b = \frac{d\bar{u}_b}{dt} = \frac{d\bar{u}_a}{dt} + \cancel{\frac{d\bar{v}_{AB}}{dt}}^{\,0} = \frac{d\bar{u}_a}{dt} = \bar{a}_a$$

Equation summary

Relative velocity $\bar{v}_{rel} = \bar{u}_b - \bar{u}_a$ for frames moving at constant velocity

Relative velocity (slide 7)



$\bar{r}_b = \bar{r}_a + \bar{r}_{AB}$

If object is moving

$\bar{u}_a = \frac{d\bar{r}_a}{dt}$ ← velocity in frame A

If A and B are not moving with respect to each other $\Rightarrow \frac{d\bar{r}_{AB}}{dt} = 0$

$$\Rightarrow \bar{u}_b = \frac{d}{dt}\bar{r}_b = \frac{d}{dt}(\bar{r}_a + \bar{r}_{AB}) = \frac{d\bar{r}_a}{dt} + \cancel{\frac{d\bar{r}_{AB}}{dt}}^{\,0} = \frac{d\bar{r}_a}{dt} = \bar{u}_a$$

velocity in frame B

If A and B are moving then

$$\bar{u}_b = \bar{u}_a + \frac{d}{dt}\bar{r}_{AB}$$

Assume $(0,0)_A = (0,0)_B$ at $t = 0$ and $\bar{v}_{AB}$ is constant

$\Rightarrow \bar{r}_{AB} = \bar{v}_{AB}\, t \Rightarrow \frac{d\bar{r}_{AB}}{dt} = \bar{v}_{AB}$

this defines a Galilean transform

$$\Rightarrow \boxed{\bar{u}_b = \bar{u}_a + \bar{v}_{AB}}$$

rewrite as $\bar{v}_{rel} = \bar{u}_b - \bar{u}_a$