General Physics 1–Honors (PHYS 101H): Problem Set 9–Solutions

Chris Monahan
Department of Physics, William & Mary

Overview

The written Problem Sets will help you gain experience with how to present your solutions to university-level physics problems. This will be necessary for your midterm and final exams, as well as future courses thoughout your undergraduate career. Present your solutions legibly and as logically as you can. What this means in practice is the following:

1. Write down what quantities you know.
2. Write down the relevant equations.
3. When carrying out manipulations or substituting values into equations, try to write each equality on separate lines. For example:

$$\begin{aligned} x(t_0) &= \frac{1}{2}at_0^2 + v_0t_0 + x_0 \\ &= \frac{1}{2} \cdot 9.8 \cdot 1.0^2 + 1.5 \cdot 1.0 + 0.7 \\ &= \boxed{7.1\,\mathrm{m}}. \end{aligned}$$

4. Circle or underline your final answers to identify them clearly (see the equation above)..

I will post solutions that will also provide one possible model for how to present solutions.

Some hints:

1. Only substitute values at the end of your calculation and try to carry out all manipulations symbolically.
2. Double check the order of magnitude of your answer.
3. Double check the units of your answer.
4. Double check the number of significant figures of your answer (do not give more significant figures than the question provides for physical quantities).

This Problem Set, in particular, will provide practice in analysing simple harmonic motion and oscillatory motion.

This Problem Set is worth 50 points; there are two questions in this Problem Set.

Instructions

Read these instructions carefully. You must submit your Problem Set as a **single PDF** file (it is best to use an app like Adobe Scan to make your solutions legible), with the file name `lastname_hwXX.pdf` (replace `lastname` with your last name and `XX` with the problem set number). If you do not submit your Problem Set according to these instructions, you will be deducted five points.

Question 1 **20pts**

A spring connects a stationary wall to a mass m. A second spring connects that mass to a second mass m. The system lies on a frictionless horizontal table. The springs have the same relaxed length and the spring constant of the first spring is nk, while the spring constant of the second spring is k. The system is illustrated in figure **??**. Assuming that it is possible to set up initial

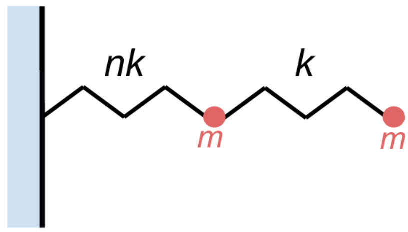

Figure 1: Diagram for Question 1.

conditions so that the masses oscillate back and forth, with the two springs always having equal lengths at any given instant, what is n?

Solution 1

At a given instant, each of the springs will be stretched by the same amount, x. Then, relative to the equilibrium positions, the two masses will be at x and $2x$. Thus, the forces due to the springs have magnitude $(nk)x$ and kx.

Applying Newton's second law to the left mass gives

$$-nkx + kx = m\frac{\mathrm{d}^2x}{\mathrm{d}t^2},$$

while the equation for the second mass is

$$-kx = m \cdot 2\frac{\mathrm{d}^2x}{\mathrm{d}t^2}.$$

Rearranging these differential equations gives

$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = -\frac{(n-1)k}{m}x,$$
$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = -\frac{k}{2m}x.$$

The frequency of these two oscillations must be the same for the two springs to maintain the same length. This means we must have

$$\frac{(n-1)k}{m} = \frac{k}{2m},$$

or

$$n - 1 = \frac{1}{2}.$$

Thus the solution is

$$\boxed{n = \frac{3}{2}}.$$

Question 2 **30pts**

A lobster person's buoy is a solid wooden cylinder of radius r and mass m. It is weighted at one end so that it floats upright in calm seawater, with density ρ. A passing shark tugs on the slack rope mooring the buoy to the lobster trap, pulling the buoy down a distance x from its equilibrium position and releasing it.

(a) Show that the buoy undergoes simple harmonic motion if the resistive effects of the water are ignored.

(b) Determine the period of the oscillations.

Solution 2

(a) The buoyant force provides a restoring force, causing the buoy to return to equilibrium. Archimedes' principle tells us that the buoyant force is

$$\begin{aligned} B &= \rho_{H20} V_{\text{displaced}} g \\ &= \rho_{H20} \pi r^2 g. \end{aligned}$$

This buoyant force generates an acceleration, so, applying Newton's second law, we have

$$B = ma,$$

or

$$\rho_{H20}(\pi r^2 x) g = -ma.$$

Using $a = \ddot{x}$, this can be written as

$$\rho_{H20} \pi r^2 g x = -m\ddot{x},$$

or as

$$\ddot{x} = -\frac{\rho_{H20} \pi r^2 g}{m} x.$$

Defining

$$\omega^2 = \frac{\rho_{H20} \pi r^2 g}{m},$$

this can be expressed as the simple harmonic oscillator differential equation

$$\boxed{\ddot{x} = -\omega^2 x},$$

as required.

(b) The angular frequency of oscillations is given by

$$\omega = \sqrt{\frac{\rho_{H20}\pi r^2 g}{m}}.$$

This is related to the linear frequency and the period via

$$\omega = 2\pi f = \frac{2\pi}{T}.$$

Therefore

$$\frac{2\pi}{T} = \sqrt{\frac{\rho_{H20}\pi r^2 g}{m}},$$

so that the frequency is

$$\frac{2\pi}{\sqrt{\rho_{H20}\pi r^2 g/m}} = T,$$

or

$$\boxed{T = \frac{2}{r}\sqrt{\frac{\pi m}{\rho g}}}.$$