General Physics 1–Honors (PHYS 101H): Problem Set 3–Solutions

Chris Monahan
Department of Physics, William & Mary

Overview

The written Problem Sets will help you gain experience with how to present your solutions to university-level physics problems. This will be necessary for your midterm and final exams, as well as future courses thoughout your undergraduate career. Present your solutions legibly and as logically as you can. What this means in practice is the following:

1. Write down what quantities you know.
2. Write down the relevant equations.
3. When carrying out manipulations or substituting values into equations, try to write each equality on separate lines. For example:

$$\begin{aligned} x(t_0) &= \frac{1}{2}at_0^2 + v_0t_0 + x_0 \\ &= \frac{1}{2}\cdot 9.8\cdot 1.0^2 + 1.5\cdot 1.0 + 0.7 \\ &= \boxed{7.1\,\mathrm{m}}. \end{aligned}$$

4. Circle or underline your final answers to identify them clearly (see the equation above)..

I will post solutions that will also provide one possible model for how to present solutions.

Some hints:

1. Only substitute values at the end of your calculation and try to carry out all manipulations symbolically.
2. Double check the order of magnitude of your answer.
3. Double check the units of your answer.
4. Double check the number of significant figures of your answer (do not give more significant figures than the question provides for physical quantities).

This Problem Set, in particular, will provide practice in applying Newton's second law of motion with the equations for kinematics to problems in two dimensions and allow you to practice analysing circular motion.

This Problem Set is worth 50 points; there are three questions in this Problem Set.

Instructions

Read these instructions carefully. You must submit your Problem Set as a **single PDF** file (it is best to use an app like Adobe Scan to make your solutions legible), with the file name `lastname_hwXX.pdf` (replace `lastname` with your last name and `XX` with the problem set number).

If you do not submit your Problem Set according to these instructions, you will be deducted five points.

Question 1 **15pts**

A mass m is attached via a massless string of length ℓ to the tip of a frictionless cone. The half-angle at the vertex of the cone is θ. If the mass moves around in a horizontal circle at speed v on the cone, find:

(a) The tension in the string;

(b) The normal force from the cone;

(c) The maximum speed for which the mass stays on the cone.

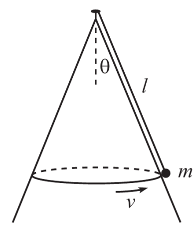

Figure 1: Diagram for Question 1.

Solution 1

(a) Let's use a reference frame that is oriented parallel and perpendicular to the cone (see Figure 2). For the mass to circle around the cone, there must be a centripetal acceleration acting

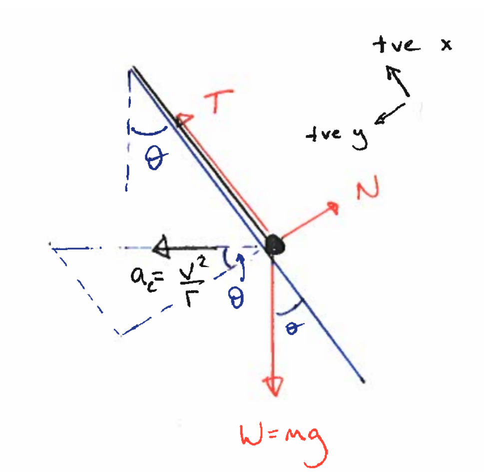

Figure 2: Diagram for Solution 1(a).

horizontally,

$$a_c = \frac{v^2}{r} = \frac{v^2}{l \sin\theta}.$$

This acceleration has a component $a_c \sin\theta$ that acts parallel to the cone. For the direction parallel to the cone, Newton's second law therefore tells us that

$$\begin{aligned} F_{\text{net}} &= T - mg\cos\theta \\ &= m(a_c \sin\theta) \\ &= m\frac{v^2}{l\sin\theta}\sin\theta \\ &= \frac{mv^2}{l}. \end{aligned}$$

Thus the tension is

$$\boxed{T = mg\cos\theta + \frac{mv^2}{l}}.$$

(b) The centripetal acceleration has a component $a_c \cos\theta$ that acts perpendicular to the cone. For the direction perpendicular to the cone, Newton's second law therefore tells us that

$$\begin{aligned} F_{\text{net}} &= mg\sin\theta - N \\ &= m(a_c \cos\theta) \\ &= m\frac{v^2}{l\sin\theta}\cos\theta \\ &= \frac{mv^2}{l\tan\theta}. \end{aligned}$$

Thus the normal force is

$$\boxed{N = mg\sin\theta - \frac{mv^2}{l\tan\theta}}.$$

(c) The requirement that the mass stays in contact with the cone is

$$N \geq 0.$$

This means that

$$N = mg\sin\theta - \frac{mv^2}{l\tan\theta} \geq 0,$$

which in turn requires

$$mg\sin\theta \geq \frac{mv^2}{l\tan\theta}.$$

The maximum speed occurs for

$$mg\sin\theta = \frac{mv_{\text{max}}^2}{l\tan\theta},$$

which gives us

$$\boxed{v_{\text{max}} = \sqrt{gl\sin\theta\tan\theta}}.$$

Question 2 **15pts**

A mass m is connected to the end of a massless string of length ℓ. The top end of the string is attached to a ceiling that is a distance ℓ above the floor. Initial conditions have been set up so that the mass swings around in a horizontal circle, with the string always making an angle θ with respect to the vertical, as shown in Fig. 3. If the string is cut, what horizontal distance does the mass cover between the time the string is cut and the time the mass hits the floor?

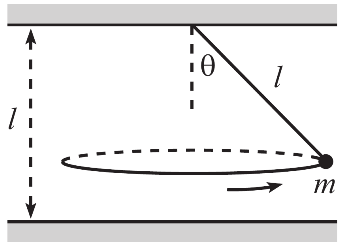

Figure 3: Diagram for Question 2.

Solution 2

Let's use a reference frame that is oriented horizontally and vertically (see Figure 4).

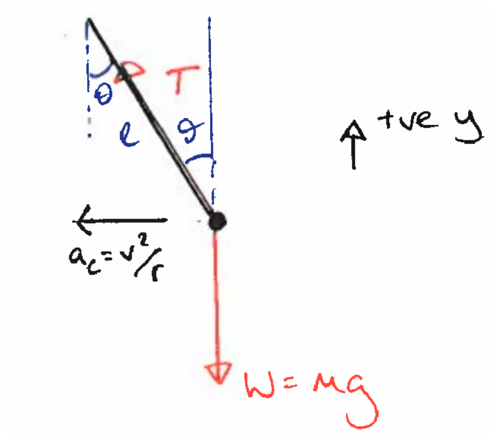

Figure 4: Diagram for Solution 2(a).

Before we can find the distance travelled when the string is cut, we need to find the initial velocity, which is given by the velocity of the circular motion. The mass does not accelerate vertically, so we must have

$$T\cos\theta = mg$$

in the vertical direction. This means the tension is

$$T = \frac{mg}{\cos\theta}.$$

For the mass to undergo uniform circular motion, there must be a centripetal acceleration acting horizontally,

$$a_c = \frac{v^2}{r} = \frac{v^2}{l\sin\theta}.$$

In the horizontal direction, the only force acting is the component of the tension, so Newton's second law tells us that

$$T\sin\theta = ma_c = \frac{mv^2}{l\sin\theta}.$$

But we can substitute in our expression for the tension from our horizontal equation, which tells us that

$$\frac{mg}{\cos\theta} \cdot \sin\theta = \frac{mv^2}{l \sin\theta}.$$

Rearranging this, we have

$$v = \sqrt{gl \sin\theta \tan\theta}.$$

This is the initial velocity for the resulting free fall motion when the string is cut.

We now have parabolic motion with an initial velocity that is strictly horizontal. The starting point of this motion is a height $h = l - l\cos\theta$. Our 1D kinematic equation in the vertical direction

$$y = \frac{at^2}{2} + v_0 t + y_0,$$

becomes

$$0 = -\frac{gt^2}{2} + 0 + l(1 - \cos\theta).$$

Here I am using the reference frame in which the upward vertical direction is the positive y direction.

We can solve this for the time of flight, which is

$$t = \sqrt{\frac{2l(1-\cos\theta)}{g}}.$$

We now apply this to the motion in the horizontal direction, to find the distance travelled in the x direction. This is given by

$$\begin{aligned} x &= v_0 t \\ &= \sqrt{gl \sin\theta \tan\theta} \cdot \sqrt{\frac{2l(1-\cos\theta)}{g}} \\ &= \boxed{l\sqrt{2\sin\theta\tan\theta(1-\cos\theta)}}. \end{aligned}$$

Question 3 **20pts**

A block with mass m is projected up along the surface of a plane inclined at an angle θ of the horizontal. The initial speed is v_0, and the coefficients of both static and kinetic friction are $\mu_S = \mu_K = 1$. The block reaches a high point and then slides back down to its starting point.

(a) Show that θ must be greater than 45° for the block to slide back down from its high point (and not just remain at rest at the high point).

(b) Assuming that $\theta > 45°$, find the times of the up and down motions.

Solution 3

(a) Let's use a reference frame that is oriented parallel and perpendicular to the slope (see Figure 5). In this frame, there is no acceleration perpendicular to the slope, so $N = mg\cos\theta$. We know the static friction force satisfies

$$F_S \leq \mu_s N = mg\cos\theta,$$

where $\mu_s = 1$ is the coefficient of static friction (note – we need the static case, because we are considering the very top of the motion, where the block is momentarily static).

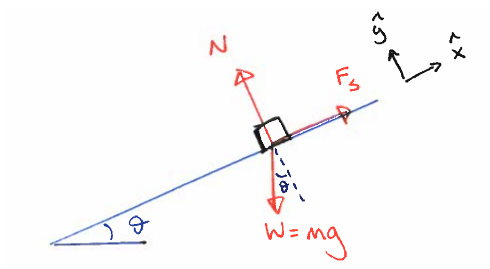

Figure 5: Diagram for Solution 3(a).

The block will slide down if the component of gravity that acts parallel to the surface is greater than the maximum value of the static friction. Thus,

$$mg\sin\theta > mg\cos\theta,$$

which we can rearrange to obtain

$$\tan\theta > 1.$$

The solution of this is

$$\boxed{\theta > 45^\circ},$$

as required.

(b) On the way up, there are two forces acting parallel to the surface and they both point downwards: the force due to gravity and the force due to kinetic friction. These are

$$F_{\text{gravity}} = -mg\sin\theta, \qquad F_k = -\mu_k N = -\mu_k mg\cos\theta = -mg\cos\theta.$$

Here μ_k is the coefficient of kinetic friction and the question tells us that $\mu_k = 1$. I use a minus sign to indicate these act in the negative x direction (down the surface). The total force acting back down the plane is therefore

$$F_{\text{gravity}} + F_k = -mg(\sin\theta + \cos\theta).$$

This means the acceleration, which acts down the surface, is

$$a_{\text{up}} = -g(\sin\theta + \cos\theta).$$

Thus the total time for the upward motion is

$$t_{\text{up}} = \frac{v_f - v_i}{a_{\text{up}}} = \frac{0 - v_0}{(-g(\sin\theta + \cos\theta))} = \boxed{\frac{v_0}{g(\sin\theta + \cos\theta)}}.$$

To find the total time down, we need to find the maximum distance travelled up the surface. For this we can use

$$v_f^2 - v_i^2 = 2a_{\rm up}\Delta x_{\rm up},$$

in the direction parallel to the surface (here we consider the upward leg of the motion, so $v_i^{\rm up} = v_0$ and $v_f^{\rm up} = 0$). Thus

$$\Delta x_{\rm up} = -\frac{v_i^2}{a_{\rm up}} = \frac{v_0^2}{2g(\sin\theta + \cos\theta)}.$$

Note this is a positive quantity! This makes sense – the block moves up the surface until it stops, and I have defined this direction as the positive x direction.

For the downward journey, the total force acting on the block (parallel to the surface) is

$$F_{\rm gravity} - F_k = -mg(\sin\theta - \cos\theta),$$

so the acceleration downwards is

$$a_{\rm down} = g(\cos\theta - \sin\theta).$$

The initial velocity in the downward direction is zero, $v_0^{\rm down} = 0$, so our kinematic equation,

$$x_{\rm down} = \frac{a_{\rm down}t_{\rm down}^2}{2} + v_0^{\rm down}t + x_0$$

becomes

$$\Delta x_{\rm down} = \frac{a_{\rm down}t_{\rm down}^2}{2},$$

or

$$t_{\rm down}^2 = \frac{2\Delta x_{\rm down}}{a_{\rm down}}.$$

Now we substitute in our expression for Δx. But, be careful! Note that $\Delta x_{\rm down} = -\Delta x_{\rm up}$! Using this, and plugging in the acceleration, we find

$$\begin{aligned} t_{\rm down}^2 &= \frac{2}{g(\cos\theta - \sin\theta)}\left(-\frac{v_0^2}{2g(\sin\theta + \cos\theta)}\right) \\ &= \frac{v_0^2}{g^2(\sin\theta - \cos\theta)(\sin\theta + \cos\theta)}. \end{aligned}$$

Thus we find the time taken to go down is

$$\boxed{t_{\rm down} = \frac{v_0}{g}\frac{1}{\sqrt{(\sin\theta - \cos\theta)(\sin\theta + \cos\theta)}}}$$