General Physics 1–Honors (PHYS 101H): Problem Set 1–Solutions

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Overview

The written Problem Sets will help you gain experience with how to present your solutions to university-level physics problems. This will be necessary for your midterm and final exams, as well as future courses thoughout your undergraduate career. Present your solutions legibly and as logically as you can. What this means in practice is the following:

1. Write down what quantities you know.
2. Write down the relevant equations.
3. When carrying out manipulations or substituting values into equations, try to write each equality on separate lines. For example:

$$\begin{aligned} x(t_0) &= \frac{1}{2}at_0^2 + v_0t_0 + x_0 \\ &= \frac{1}{2} \cdot 9.8 \cdot 1.0^2 + 1.5 \cdot 1.0 + 0.7 \\ &= \boxed{7.1\,\mathrm{m}}. \end{aligned}$$

4. Circle or underline your final answers to identify them clearly (see the equation above)..

I will post solutions that will also provide one possible model for how to present solutions.

Some hints:

1. Only substitute values at the end of your calculation and try to carry out all manipulations symbolically.
2. Double check the order of magnitude of your answer.
3. Double check the units of your answer.
4. Double check the number of significant figures of your answer (do not give more significant figures than the question provides for physical quantities).

This Problem Set, in particular, will provide practice in applying the equations for kinematics to problems in one dimension. You will also practice applying a Taylor series and deriving a new relation from the kinematic equations.

This Problem Set is worth 50 points; there are five questions in this Problem Set.

Instructions

Read these instructions carefully. You must submit your Problem Set as a **single PDF** file (it is best to use an app like Adobe Scan to make your solutions legible), with the file name `lastname_hwXX.pdf` (replace `lastname` with your last name and `XX` with the problem set number).

Figure 1: My dog. Very floofy.

If you do not submit your Problem Set according to these instructions, you will be deducted five points.

Question 1 **10pts**

My dog (see Figure 1!) is very floofy, but can be a little stubborn. When he encounters an interesting scent at some spot on one of our walks, he usually wants to explore it in detail, but sadly sometimes we don't have time. If I pull him with a force $\mathbf{F} = (90.0\hat{\imath} + 137.0\hat{\jmath} + 23.0\hat{k})$ N along the leash:

(a) What is the magnitude of the pulling force (in N)?

(b) What angle does the leash make with the vertical?

Solution 1

(a) The magnitude of the force is given by

$$
\begin{aligned}
|\mathbf{F}| &= \sqrt{90.0^2 + 137.0^2 + 23.0^2} \\
&\boxed{= 166\,\mathrm{N}}
\end{aligned}
$$

(b) The angle with respect to the vertical can be found in several ways. One method is to calculate the magnitude of the vector in the (x, y) plane, which is

$$F_{xy} = \sqrt{90.0^2 + 137.0^2},$$

and then to use this as the base of the triangle of height $F_z = 23.0$. Then the angle is

$$
\begin{aligned}
\theta &= \arctan\left(\frac{F_{xy}}{F_z}\right) \\
&= \arctan\left(\frac{\sqrt{90.0^2 + 137.0^2}}{23.0}\right) \\
&= \boxed{82.0^\circ}
\end{aligned}
$$

The alternative is to use the magnitude of the force as the size of the hypotenuse of the triangle created by the force vector and the vertical, in which case the formula is

$$\theta = \arccos\left(\frac{23.0}{165.5}\right) \qquad\qquad = \boxed{82.0^\circ}$$

Question 2 **10pts**

Suppose that a person takes 0.51 s to react and move their hand to catch an object they have dropped.

(a) How far (in m) does the object fall near Earth's surface, where $g = 9.8$ m/s^2?

(b) How far (in m) does the object fall near the Moon's surface, where the acceleration due to gravity is one sixth (1/6) of that on Earth?

Solution 2

We assume that the initial velocity of the object is zero (so that the object is dropped and not thrown downwards), so $v_0 = 0$ m/s^2. All motion occurs in the vertical direction, so we can use the equations of one dimensional kinematics. We define vertically upwards as the positive y direction, and define the initial position as $y_0 = 0$ m.

(a) The distance is given by

$$\begin{aligned} s &= \frac{1}{2}at^2 + v_0 t + y_0 \\ &= \frac{1}{2}(-9.8)0.51^2 + 0 \cdot 0.51 + 0 \\ &= -1.27\,\mathrm{m} \end{aligned}$$

Taking account of the sign, which means that the object has moved vertically down, the distance dropped is $\boxed{s = 1.3\,\mathrm{m}}$.[**Note that the result should be given to two significant figures, because the quantities given in the problem are only given to two significant figures.**]

(b) Here one can repeat this calculation using this formula, but with a new value for the acceleration. Or one can notice that this formula is linear in the acceleration, which means that if we divide the acceleration by six, we can simply divide the result of part (a) by six, too. Thus the result is $\boxed{s_{\mathrm{moon}} = 0.21\,\mathrm{m}}$.

Question 3 **10pts**

A hot-air balloon rises from ground level at a constant velocity of 5.3 m/s. Three minutes after liftoff, a sandbag is dropped accidentally from the balloon. (Assume the upward direction is positive.)

(a) Calculate the time it takes for the sandbag to reach the ground (in s).

(b) Calculate the velocity (in m/s) of the sandbag when it hits the ground. (Indicate the direction with the sign of your answer.)

(c) What assumptions did you make in this calculation? Suppose that we actually recreated this question in an actual experiment and measured the actual time it took to reach the ground. Do you think that your result in part (a) would be greater or smaller than the actual time taken in our real experiment, and why?

Solution 3

(a) We don't have quite enough information to solve this problem immediately, so we must calculate one more quantity. In this case, we can calculate the distance from which the sandbag is dropped, because we know the upward velocity and time travelled for the balloon itself. This is given by

$$y = v_0 t = 5.3 \cdot 180 = 954.0\,\mathrm{m}.$$

Now we have the distance travelled by the sandbag, its acceleration, and its initial velocity. This means we can use

$$y = \frac{at^2}{2} + v_0 t + y_0,$$

or (with $a = -g$)

$$\frac{-gt^2}{2} + v_0 t + y_0 - y = 0.$$

This is a quadratic equation for the time, and the solutions are

$$\begin{aligned} t &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= -\frac{v_0}{2(-g/2)} \pm \frac{1}{2(-g/2)}\sqrt{v_0^2 - 4(-g/2)(y_0 - y)} \\ &= \frac{v_0}{g} \mp \frac{1}{g}\sqrt{v_0^2 + 2g(y_0 - y)}. \end{aligned}$$

Putting in some numbers, we have

$$t = \frac{5.3}{9.8} \mp \frac{1}{9.8}\sqrt{5.3^2 + 2 \cdot 9.8(954 - 0)} = -13.4\,\mathrm{s} \qquad \text{or} \qquad 14.5\,\mathrm{s}.$$

The negative solution is unphysical, so our result is

$$\boxed{t = 14.5\,\mathrm{s}}.$$

(b) Now that we have the time taken, we can use

$$\begin{aligned} v_f &= v_0 + at \\ &= 5.3 + (-9.8) \cdot 14.5 \\ &= \boxed{-137\,\mathrm{m/s}}. \end{aligned}$$

(c) We assumed that we can neglect air resistance [**This assumption in particular required to be mentioned, although others could be included.**]. The actual time would be **longer** than calculated in part (a), because air resistance would decrease the speed of the sandbag (although, probably not by much).

Question 4 **10pts**

If a ball is dropped from rest at height h, and if the drag force from the air takes the form $F_d = bv$, then it can be shown that the ball's height as a function of time equals

$$y(t) = h - \frac{mg}{b}\left(t - \frac{m}{b}\left(1 - e^{-bt/m}\right)\right)$$

Expand the exponential as a Taylor Series to find an approximate expression for $y(t)$ in the limit where t is very small.

Solution 4

The formula for the Taylor expansion, in x, of an arbitrary function $f(x)$ around $x = a$ is

$$f(x) = f'(a) + f'(a)(x-a) + \frac{f''(a)}{2}(x-a)^2 + \ldots$$

Here the prime indicates a derivative of $f(x)$ with respect to x. So, for example, $f''(x)$ is the second derivative of $f(x)$ with respect to x, and $f''(a)$ is the second derivative of $f(x)$ evaluated at $x = a$.

We could calculate the derivatives of the whole function $y(t)$, but it is easiest just to look up the expansion of the exponential function, which is the only nontrivial part of the $y(t)$ function. We have

$$\exp(x) \sim 1 + x + \frac{x^2}{2} + \ldots$$

In our case, $x = (-bt)/m$, and our function becomes

$$\begin{aligned} y(t) &\sim h - \frac{mg}{b}\left\{t - \frac{m}{b}\left[1 - \left(1 + \left(\frac{-bt}{m}\right) + \frac{1}{2}\left(\frac{-bt}{m}\right)^2 + \ldots\right)\right]\right\} \\ &= h - \frac{mg}{b}\left\{t - \frac{m}{b}\left[\frac{bt}{m} - \frac{b^2t^2}{2m^2} + \ldots\right]\right\} \\ &= h - \frac{mg}{b}\left\{t - t + \frac{bt^2}{2m} + \ldots\right\} \\ &= \boxed{h - \frac{gt^2}{2} + \ldots} \end{aligned}$$

This is exactly form you would expect from the one dimensional kinematics equation, $y = at^2/2 + v_0 t + y_0$, for freefall ($a = -g$) from rest ($v_0 = 0$) from postion $y_0 = 0$! In other words, for very short times, the friction does not really have an effect on the kinematics of the fall.

Question 5 **10pts**

Use the equations describing the motion of an object experiencing constant acceleration to show that the relation between the initial (v_i) and final velocities (v_f), the acceleration (a), and the distance an object travels (Δx) is $v_f^2 + v_i^2 = 2a\Delta x$.

Solution 5

Let's start from two equations we know well:

$$x(t) = \frac{1}{2}at^2 + v_i t + x_0,$$

and

$$v(t) = at + v_i.$$

This second equation tells us the time elapsed is

$$t = \frac{v_f - v_i}{a}.$$

From the first equation, the displacement is

$$\Delta x = x - x_0 = \frac{a}{2}t^2 + v_i t,$$

and let's plug in the expression for the time obtained from our second equation into this expression, to obtain

$$\begin{aligned}\Delta x &= \frac{a}{2}\left(\frac{v_f - v_i}{a}\right)^2 + v_i\left(\frac{v_f - v_i}{a}\right) \\ &= \frac{1}{2a}(v_f - v_i)^2 + \frac{v_i}{a}(v_f - v_i).\end{aligned}$$

Multiplying by $(2a)$, this give

$$\begin{aligned}2a\Delta x &= (v_f - v_i)^2 + v_i(v_f - v_i) \\ &= (v_f - v_i)\left[(v_f - v_i) + 2v_i\right] \\ &= (v_f - v_i)\left(v_f + v_i\right) \\ &= v_f^2 - v_i^2.\end{aligned}$$

Rearranging this, we obtain

$$\boxed{v_f^2 - v_i^2 = 2a\Delta x},$$

as required!