General Physics 1–Honors (PHYS 101H):
Problem Set 5

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Overview

The written Problem Sets will help you gain experience with how to present your solutions to university-level physics problems. This will be necessary for your midterm and final exams, as well as future courses thoughout your undergraduate career. Present your solutions legibly and as logically as you can. What this means in practice is the following:

1. Write down what quantities you know.
2. Write down the relevant equations.
3. When carrying out manipulations or substituting values into equations, try to write each equality on separate lines. For example:

$$\begin{aligned} x(t_0) &= \frac{1}{2}at_0^2 + v_0t_0 + x_0 \\ &= \frac{1}{2} \cdot 9.8 \cdot 1.0^2 + 1.5 \cdot 1.0 + 0.7 \\ &= \boxed{7.1\,\mathrm{m}}. \end{aligned}$$

4. Circle or underline your final answers to identify them clearly (see the equation above)..

I will post solutions that will also provide one possible model for how to present solutions.

Some hints:

1. Only substitute values at the end of your calculation and try to carry out all manipulations symbolically.
2. Double check the order of magnitude of your answer.
3. Double check the units of your answer.
4. Double check the number of significant figures of your answer (do not give more significant figures than the question provides for physical quantities).

This Problem Set, in particular, will provide practice in applying conservation of energy to analyse motion in one and two dimensions and studying elastic collisions in one and two dimensions.

This Problem Set is worth 50 points; there are three questions in this Problem Set.

Instructions

Read these instructions carefully. You must submit your Problem Set as a **single PDF** file (it is best to use an app like Adobe Scan to make your solutions legible), with the file name `lastname_hwXX.pdf` (replace `lastname` with your last name and `XX` with the problem set number). If you do not submit your Problem Set according to these instructions, you will be deducted five points.

Question 1 **15pts**

A massless spring with spring constant k hangs vertically from a ceiling, initially at its relaxed length. A mass m is then attached to the bottom and released.

(a) Calculate the total potential energy (gravitational and spring) of the system, as a function of the height y (which is negative), relative to the initial position.

(b) Find y_0, the point at which the potential energy is at a minimum. Make a rough plot of the potential energy as a function of y. Label the plot to indicate the coordinates of the minimum and the intersection of the curve with the axes.

(c) Rewrite the potential energy as a function of $z = y - y_0$ and make a labelled rough plot of this new form of the potential, as a function of z. Explain why your result shows that a hanging spring can be considered to be considered to be a spring in a world without gravity, provided that the equilibrium point, y_0, is now called the "relaxed" length of the spring.

Question 2 **15pts**

A bead is initially at rest at the top of a fixed frictionless hoop with radius R that lies in a vertical plane. The bead is then given an infinitesimal push so that it slides down and around the hoop.

(a) What is the speed of the bead after it has fallen through an angle θ (measured relative to the vertical).

(b) Take the time derivative of your result (don't forget to use the chain rule) to verify that the tangential acceleration $\mathrm{d}v/\mathrm{d}t$ equals the tangential component of the acceleration due to gravity.

Question 3 **20pts**

In a one-dimensional collision, a mass M moving with velocity V collides with a mass m that is at rest. Assume that all collisions are elastic.

(a) Show that the resulting velocities are given by

$$V_M = \frac{(M-m)}{M+m}V, \qquad v_m = \frac{2M}{M+m}V.$$

(b) What is the relationship between these velocities if $m \ll M$? And what about if $M \ll m$?

Imagine that you now add a third stationary mass, μ, between M (still initially moving with velocity V) and m (which is still at rest).

(c) Find an expression for μ, in terms of m and M, that maximises the final velocity of m after all collisions.