General Physics 1–Honors (PHYS 101H): Practice Final Exam–Solutions Tuesday, December 13, 2022

Chris Monahan
Department of Physics, William & Mary

Overview and instructions

In this final you will apply your understanding of the topics covered in Physics 101–Honors. These topics include: kinematics and dynamics in one and two dimensions; Newton's laws of motion; conservation of energy; elastic and inelastic collisions; rotational kinematics and dynamics; gravitation; fluid statics and dynamics; simple harmonic motion and oscillations; and wave motion.

Read the following instructions carefully.

There are **ten questions**, for a total of **200 points**. **Attempt all questions**. The exam will start at 9:00 am and finish at 12:00 am. Please write your name **on every sheet of paper you submit**. It is helpful if you include page numbers at the bottom of each page, too.

You may use:

- an electronic calculator;
- your own formula sheet, written or printed on two sides of letter paper.

You may **not** use:

- electronic devices (except a calculator), including phones, tablets and laptops (unless previously arranged);
- textbooks or other reference resources;
- course notes or slides.

The first five questions are multiple choice. Your answer to these multiple choice questions should be written out and submitted as part of the rest of your solutions. For example, you could write, "Problem 1: my answer is **(a)**." Do **not** circle the options on the exam itself; I will not collect the exams and you will not receive credit for your answer.

The remaining five questions require written solutions. You should show all your working and include important intermediate steps, equations, and results. You can receive partial credit for these problems, even if you don't complete the problem or provide a correct final answer. Please ensure that you highlight or emphasise your final answer (for example, by circling or underlining the final answer).

You are responsible for ensuring your solutions are legible. Present your solutions legibly and as logically as you can. What this means in practice is the following:

1. Try to identify what "kind" or "type" of question is being asked, for example "projectile motion", "conservation of energy", or "two dimensional collision".

2. Draw a labelled diagram.
3. Write down what quantities you know.
4. Write down the relevant equations.
5. Write down what quantities you know.
6. Write down the relevant equations.
7. When carrying out manipulations or substituting values into equations, try to write each equality on separate lines or otherwise distinguish them clearly.
8. Circle or underline your final answers to identify them clearly.

Some hints for tackling problems in general:

1. Only substitute values at the end of your calculation and try to carry out all manipulations symbolically.
2. Double check the order of magnitude of your answer.
3. Double check the units of your answer.
4. Double check the number of significant figures of your answer. Remember that I am only looking for approximately the correct number of significant digits. If quantities are given to two or three significant digits, quote your answer to two or three (not one or five). Similarly if quantities are given to eight significant digits, do not quote your answer to two.

You do not have to tackle the questions in order. Briefly read through them all and then start on one!

Short questions

Remember, your answer to these multiple choice questions should be written out and submitted as part of the rest of your solutions.

Question 1 **15pts**

A cart on a frictionless air track is moving at 0.5 m/s when the air is suddenly turned off. The cart comes to rest after travelling 1 m. The experiment is repeated, but now the cart is moving at 1 m/s when the air is turned off. How far does the cart travel before coming to rest, assuming the coefficient of kinetic friction is constant for both cases?

(a) 1 m (b) 2 m (c) 3 m (d) 4 m (e) 5 m

Solution 1

The answer is (d). The cart comes to rest when the kinetic energy is completely lost to friction. In other words, when the work done by friction equals the original kinetic energy, which is given by

$$E_K^{(1)} = \frac{mv_1^2}{2} = \frac{m}{2} \cdot \frac{1}{2^2} = \frac{m}{8}.$$

In the second case, the kinetic energy is

$$E_K^{(2)} = \frac{mv_2^2}{2} = \frac{m}{2} \cdot 1^2 = \frac{m}{2}.$$

So

$$E_K^{(2)} = 4E_K^{(2)}.$$

The work done by friction is the product of the force times the distance. The frictional force is the same in both cases, so the distance in the second case must be four times as large as in the first. Therefore $d = 4 \cdot 1 = 4$ m.

Question 2 **15pts**

Clifford likes to drop his ball in the river and watch it float downstream before swimming after it. The river flows at a constant rate of $u = 1$ m/s, and Clifford watches the ball for 3 s before he starts swimming. Assuming that Clifford can swim with a uniform velocity of 2 m/s^2, how long does it take for Clifford to catch up with his ball? Assume that both the ball and Clifford travel in a straight line downstream.

(a) 1.5 s (b) 3.0 s (c) 4.5 s (d) 6.0 s (e) 7.5 s

Solution 2

The answer is (b).

The ball travels a total distance of

$$x_{\text{ball}} = 3 \cdot u + t \cdot u = (3+t)u \,\text{m}$$

until Clifford catches up with it at time $3+t$. He then starts swimming with an initial velocity $v_0 = 2$ m/s, starting at initial position $x_0 = 0$ m, and has acceleration $a = 0$ m/s^2. Clifford travels a total distance

$$x_{\text{Clifford}} = v_0 t.$$

These distances must be equal, so

$$(3+t)u = v_0 t,$$

which can be rewritten as

$$t = \frac{3}{v_0 - u}.$$

This gives

$$t = \frac{3}{2-1} = 3\,\text{s}.$$

Question 3 **15pts**

A siren has a sound intensity level of 100 dB at a distance of 100 m from the siren itself. What is the intensity of the sound at a distance of 1 km from the siren?

(a) 1 W/m^2 (b) 10^{-1} W/m^2 (c) 10^{-2} W/m^2 (d) 10^{-3} W/m^2 (e) 10^{-4} W/m^2

Solution 3

The answer is (e).

The sound intensity level is given by

$$\beta = 10 \log_{10} \frac{I}{I_0},$$

so in this case we have

$$\frac{I}{I_0} = 10^{\beta/10} = 10^{10}.$$

Thus

$$I = 10^{10} I_0 = 10^{10} \cdot 10^{-12}\,\text{W/m}^2 = 10^{-2}\,\text{W/m}^2.$$

For a spherical sound wave, emitted from a point source (like the siren), we have

$$I_2 = \frac{R_1^2}{R_2^2} I_1,$$

so at $R_2 = 1\,\text{km}$, this becomes

$$I_2 = \frac{100^2}{1000^2} I_1 = 10^{-2} I_1 = 10^{-2} \cdot 10^{-2}\,\text{W/m}^2 = 10^{-4}\,\text{W/m}^2.$$

Question 4 **15pts**

Blood flows through a coronary artery that is partially blocked by deposits along the artery wall. Assuming ideal fluid flow, which statements about this blood flow are correct?

(a) The flow speed of the fluid is larger in the wide part of the artery, because the flux is larger in the wide part of the artery.

(b) The flow speed of the fluid is larger in the wide part of the artery, because the flux is the same in the wide part of the artery.

(c) The flow speed of the fluid is the same in the wide part of the artery, because the flux is the same in all parts of the artery.

(d) The flow speed of the fluid is smaller in the wide part of the artery, because the flux is the same in all parts of the artery.

(e) The flow speed of the fluid is smaller in the wide part of the artery, because the flux is smaller in the wide part of the artery.

Solution 4

The answer is (d).

The assumption of ideal fluid flow means that the blood is treated as incompressible, so the flux is the same throughout the artery. The flux is the product of the fluid speed and the cross-sectional area, so if the area increases (i.e. the artery is wider) then the speed must go down (to ensure the flux remains constant).

Question 5 **15pts**

What is the weight of an 70 kg astronaut on the Moon? The mass of the Moon is $0.0123M_E$, where M_E is the mass of the Earth, and the radius of the Moon is $0.2727R_E$, where R_E is the radius of the Earth.

(a) 8.4 N (b) 16 N (c) 31 N (d) 187 N (e) 687 N

Solution 5

The acceleration due to gravity is given by

$$g = \frac{GM}{R},$$

so

$$\begin{aligned}\frac{g_M}{g_E} &= \frac{GM_M/R_M}{GM_E/R_E} \\ &= \frac{M_M R_E}{M_E R_M} \\ &= \frac{0.0123M_E \cdot R_E}{M_E \cdot 0.2727R_E} \\ &= 0.045.\end{aligned}$$

The weight of the astronaut on the Moon is

$$\begin{aligned} W &= mg_M \\ &= m \cdot 0.045 g_E \\ &= 70 \cdot 0.045 \cdot 9.81 \\ &= 30.9\,\text{kg}. \end{aligned}$$

Longer questions

Remember, present your solutions legibly and as logically as you can. Highlight your final answer by underlining or circling it.

Question 6 — 25pts

An ice skater is spinning at 6.0 rev/s and has a moment of inertia of 0.5 $\text{kg}\,\text{m}^2$.

(a) Calculate the angular momentum of the ice skater spinning at 6.0 rev/s.

(b) The skater reduces their rate of rotation by extending their arms and increasing his moment of inertia. Find the value of his moment of inertia if the skater's rate of rotation decreases to 0.75 rev/s.

(c) Suppose instead the skater keeps their arms in and allows friction of the ice to slow the skater to 3.75 rev/s. What is the magnitude of the average torque that was exerted, if this process takes 14 s?

Solution 6

(a) The angular momentum is given by

$$L = I\omega,$$

which in this case is

$$L = 0.5 \cdot (2\pi 6) = \boxed{18.8\,\text{kg}\,\text{m}^2/\text{s}}.$$

Note that the angular velocity is related to the angular frequency given in the equation via

$$\omega = 2\pi f.$$

(b) Angular momentum is conserved, so we can use the value of the initial angular momentum and simply rearrange the equation used above:

$$I = \frac{L}{\omega} = \frac{18.8}{2\pi 0.75} = \boxed{4.00\,\text{kg}\,\text{m}^2}.$$

(c) The net torque is related to the change in the angular momentum via

$$\tau_{\text{net}} = \frac{\mathrm{d}L}{\mathrm{d}t},$$

or, in discrete form,

$$\tau_{\text{net}} = \frac{\Delta L}{\Delta t}.$$

The change in the angular momentum is

$$\Delta L = I \cdot \Delta\omega = I(\omega_f - \omega_i) = 2\pi I(f_f - f_i),$$

where f is the angular frequency.

Thus we have

$$\tau_{\text{net}} = \frac{2\pi I(f_f - f_i)}{\Delta t} = \frac{2\pi \cdot 0.5 \cdot (3.75 - 6.0)}{14} = -0.50\,\text{N}\,\text{m}.$$

We only need the magnitude, which is

$$\boxed{\tau_{\text{net}} = 0.50\,\text{N}\,\text{m}}.$$

Question 7 **25pts**

You've been given the challenge of balancing a uniform, rigid meter-stick with nonzero mass M on a pivot. Stacked on the 0-cm end of the meter stick are n identical coins, each with mass m, so that the center of mass of the coins is directly over the end of the meter stick. The pivot point is a distance d from the 0-cm end of the meter stick.

(a) Determine the distance $d = d_1$ if there are n coins on the 0-cm end of the meter stick and the system is in static equilibrium.

(b) Find the approximate value of d in the case where $m \ll M$.

(c) Is it possible to stack enough coins at the 0-cm end to achieve equilibrium with $d = 0$?

Solution 7

(a) To solve this, we need to apply the condition of rotational equilibrium. In other words, the net torque on the stick must be zero. There are two torques acting around the pivot, due to the coins at one end and the weight of the metre stick acting through the centre of mass on the other side of the pivot.

The torques are given by

$$\tau = Fr\sin\theta,$$

where $\theta = 90°$ for both torques in the problem. The two torques must sum to zero for equilibrium:

$$nmg \cdot d - Mg \cdot (0.5 - d) = 0.$$

Rearranging this, we have

$$d(nm + M) = \frac{M}{2},$$

so

$$\boxed{d = \frac{M}{2(nm + M)}}.$$

(b) If $m \ll M$, we need to Taylor expand our result from part (a) as follows:

$$\begin{aligned} d &= \frac{M}{2(nm+M)} \\ &= \frac{M}{2M\left(1+nm/M\right)} \\ &= \frac{1}{2}\left(1+\frac{nm}{M}\right)^{-1} \\ &\simeq \frac{1}{2}\left(1-\frac{nm}{M}+\cdots\right). \end{aligned}$$

Thus our approximate answer is

$$\boxed{\frac{1}{2}\left(1-\frac{nm}{M}\right)}.$$

(c) No! For $d = 0$, part (a) tells us that we can only have this if $M = 0$ or if $n \to \infty$ or $m \to \infty$. The first case is excluded by the statement that M is nonzero (in the qustion) and the second two are not realistic/physically feasible.

Question 8 **25pts**

A massless spring has force constant $k = 180$ N/m and connects a block of wood to a wall. The system is initially at rest, with the spring unstretched. The block has mass $M = 25$ g and is able to move without friction on a table. A gun is positioned to fire a bullet of mass $m = 9$ g into the block along the spring axis. After the gun is fired, the bullet gets embedded in the block, and the spring is compressed a maximum distance $d = 0.8$ m.

(a) What kind of collision is this? What is conserved?
(b) What is the speed of the bullet before it enters the block?
(c) What is the frequency of the resulting periodic motion of the block/bullet and spring system?

Solution 8

(a) This is a **perfectly inelastic** collision. **Momentum** is conserved, but kinetic energy is **not** conserved.

(b) Applying conservation of momentum to the collision, we have

$$mv_1 = (M+m)v_2.$$

After the collision, the spring does work to stop the motion of the block and bullet. The work done by the spring is equal to the change in kinetic energy, ΔE_K. When the block+bullet come to rest, the final speed is zero and therefore the final kinetic energy is also zero, $E_K^{(f)} = 0$. This means that the change in kinetic energy is equal to the initial kinetic energy (immediately after the collision),

$$\Delta E_K = E_K^{(f)} - E_K^{(i)} = -E_K^{(i)} = -\frac{(M+m)v_2^2}{2}.$$

We are also given the distance the block+bullet travel before coming to rest, so we can relate the work done to this distance

$$W = \int_0^d F(x)\mathrm{d}x.$$

For a spring, the magnitude of the force is

$$F = -kx,$$

so

$$W = -\int_0^d kx\,\mathrm{d}x = -\frac{kd^2}{2}.$$

Equating these two expressions for the work done, we have

$$-\frac{(M+m)v_2^2}{2} = -\frac{kd^2}{2}.$$

We can rearrange this for the velocity, to give

$$v_2^2 = \frac{kd^2}{(M+m)}.$$

We can now plug this into our equation from conservation of momentum

$$mv_1 = (M+m)v_2 = (M+m)\sqrt{\frac{kd^2}{(M+m)}} = d\sqrt{k(M+m)}.$$

Thus

$$v_1 = \frac{(M+m)v_2}{m} = \frac{d}{m}\sqrt{k(M+m)} = \frac{0.8}{0.009}\sqrt{180\cdot(0.025+0.009)} = \boxed{220\,\mathrm{m/s}}.$$

(c) The block+bullet undergo simple harmonic motion, with angular frequency given by

$$\omega^2 = \frac{k}{M+m}.$$

The frequency, f, is related to the angular frequency, ω, by

$$\omega = 2\pi f,$$

so the frequency is

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{M+m}} = \frac{1}{2\pi}\sqrt{\frac{180}{0.025+0.009}} = \boxed{11.6\,\mathrm{Hz}}.$$

Question 9 **25pts**

A 8.0 cm diameter fire hose ends with a 2.5 cm diameter nozzle. The density of water is 1.00×10^3 kg/m^3 and 1 m^3 = 1000 L.

(a) Calculate the pressure drop caused by the narrowing of the nozzle as water enters the nozzle from the hose at a rate of 40.0 L/s.

(b) If the nozzle of the hose is pointed vertically upward, what is the maximum height above the nozzle that the water can rise, neglecting air resistance?

Solution 9

(a) Bernoulli's equation states that

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2,$$

so the pressure difference is given by

$$P_2 - P_1 = \frac{1}{2}\rho(v_1^2 - v_2^2) + \rho g(h_1 - h_2).$$

However, the hose does not change height, so $h_1 = h_2$ and we have

$$P_2 - P_1 = \frac{1}{2}\rho(v_1^2 - v_2^2).$$

To calculate the speeds, we need to use the continuity equation, which tells us that the flow rate, Q, is constant and that the fluid speed is given by

$$Q = Av.$$

Thus $v_1 = Q/A_1$ and $v_2 = Q/A_2$. The cross-sectional area in each case is given by

$$A = \pi r^2.$$

Putting this all together, we have

$$\begin{aligned} P_2 - P_1 &= \frac{\rho Q^2}{2\pi^2}\left(\frac{1}{r_1^4} - \frac{1}{r_2^4}\right) \\ &= \frac{1.00 \times 10^3 \cdot (40 \times 10^{-3})^2}{2\pi^2}\left(\frac{1}{(0.025/2)^4} - \frac{1}{(0.080/2)^4}\right) \\ &= \boxed{3.3 \times 10^6 \text{ Pa}}. \end{aligned}$$

(b) We will treat a drop of water at the end of the hose as a point particle undergoing one dimensional vertical motion under the influence of gravity. We can find the maximum height by applying conservation of energy.

The initial speed of this "point particle" of water is

$$v_2 = \frac{Q}{A_2} = \frac{Q}{\pi r_2^2},$$

and the final speed must be zero at the maximum height. Taking the initial position as $y_0 = 0$, we have

$$mgh = \frac{mv_2^2}{2},$$

from conservation of energy.

We can rearrange this for the height, to give

$$h = \frac{v_2^2}{2g} = \left(\frac{Q}{\pi r_2^2}\right)^2 \frac{1}{2g} = \frac{(40 \times 10^{-3})^2}{2\pi^2 \cdot 9.81 \cdot (0.025/2)^4} = \boxed{338\,\text{m}}.$$

Which is high!

Question 10 **25pts**

The "classical" radius of a neutron is about 0.81 fm (1 femtometre, abbreviated as fm, is 10^{-15}). The mass of a neutron is 1.675×10^{-27} kg and the gravitational constant is $G = 6.67 \times 10^{-11}$ $\text{m}^3/\text{kg}/\text{s}^2$.

(a) Assuming the neutron is spherical, calculate its density in kilograms per cubic meter.

(b) What would be the magnitude of the acceleration due to gravity, in meters per second squared, at the surface of a spherical neutron star of radius $R = 20$ km with this same density? Note that a neutron star is essentially composed of nothing but neutrons (hence the name).

(c) What would be the escape velocity for such a neutron star?

Solution 10

(a) The density is given by

$$\rho = \frac{m}{V} = \frac{m_N}{4\pi R_N^3/3} = \frac{1.675 \times 10^{-27}}{4\pi(0.81 \times 10^{-15})^3/3} = \boxed{7.52 \times 10^{17}\,\text{kg/m}^3}.$$

(b) The magnitude of the acceleration due to gravity for a spherical mass is

$$g = \frac{GM}{r^2}.$$

In this case, this mass is

$$M = \rho V = \rho \cdot \frac{4\pi r^3}{3},$$

so

$$g = \frac{GM}{r^2} \cdot \rho \cdot \frac{4\pi r^3}{3} = \frac{4\pi G\rho r}{3} = \frac{4\pi \cdot 6.67 \times 10^{-11} \cdot 7.52 \times 10^{17} \cdot 20 \times 10^3}{3} = \boxed{4.2 \times 10^{12}\,\text{m/s}}.$$

(c) The escape velocity is given by

$$v = \sqrt{\frac{2GM}{r}} = \sqrt{2gr},$$

so we have

$$v = \sqrt{2 \cdot 4.2 \times 10^{12} \cdot 20 \times 10^3} = \boxed{4.1 \times 10^8\,\text{m/s}}.$$