General Physics 1–Honors (PHYS 101H): Midterm 2 Wednesday, November 16, 2022

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Overview and instructions

In this midterm you will apply your understanding of conservation of energy, elastic and inelastic collisions, and rotational motion.

Read the following instructions carefully.

There are **five questions**, for a total of **100 points**. **Attempt all questions**. The exam will finish at 11:45 am. Please write your name **on every sheet of paper you submit**. It is helpful if you include page numbers at the bottom of each page, too.

You may use:

- an electronic calculator;
- your own formula sheet, written or printed on two sides of letter paper.

You may **not** use:

- electronic devices (except a calculator), including phones, tablets and laptops (unless previously arranged);
- textbooks or other reference resources;
- course notes or slides.

The first three questions are multiple choice. Your answer to these multiple choice questions should be written out and submitted as part of the rest of your solutions. For example, you could write, "Problem 1: my answer is **(a)**." Do **not** circle the options on the exam itself; I will not collect the exams and you will not receive credit for your answer.

The remaining questions require written solutions. You should show all your working and include important intermediate steps, equations, and results. You can receive partial credit for these problems, even if you don't complete the problem or provide a correct final answer. Please ensure that you highlight or emphasise your final answer (for example, by circling or underlining the final answer).

You are responsible for ensuring your solutions are legible. Present your solutions legibly and as logically as you can. What this means in practice is the following:

1. Try to identify what "kind" or "type" of question is being asked, for example "projectile motion", "conservation of energy", or "two dimensional collision".
2. Draw a labelled diagram.
3. Write down what quantities you know.
4. Write down the relevant equations.

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6. Write down the relevant equations.
7. When carrying out manipulations or substituting values into equations, try to write each equality on separate lines or otherwise distinguish them clearly.
8. Circle or underline your final answers to identify them clearly.

Some hints for tackling problems in general:

1. Only substitute values at the end of your calculation and try to carry out all manipulations symbolically.
2. Double check the order of magnitude of your answer.
3. Double check the units of your answer.
4. Double check the number of significant figures of your answer. Remember that I am only looking for approximately the correct number of significant digits. If quantities are given to two or three significant digits, quote your answer to two or three (not one or five). Similarly if quantities are given to eight significant digits, do not quote your answer to two.

You do not have to tackle the questions in order. Briefly read through them all and then start on one!

Short questions

Remember, your answer to these multiple choice questions should be written out and submitted as part of the rest of your solutions.

Question 1 — 15pts

A massless spring with spring constant $k = 12.0$ N/m hangs vertically from a ceiling, initially at its relaxed length. You attach a mass 0.1 kg to the end and bring it down to a position that is 10 cm below the initial position. You then let go. What is the upward acceleration of the mass right after you let go?

(a) -2.2 $\mathrm{m/s^2}$ (b) 0 $\mathrm{m/s^2}$ (c) 2.2 $\mathrm{m/s^2}$ (d) 11 $\mathrm{m/s^2}$ (e) 22 $\mathrm{m/s^2}$

Solution 1

The answer is (c). The upward force due to the spring is $k \cdot \Delta y$. The downward force due to gravity is mg. Therefore the net force is

$$F_{\text{net}} = k \cdot \Delta y - mg,$$

and the acceleration caused by this net force is

$$a = \frac{F_{\text{net}}}{m} = \frac{k \cdot \Delta y - mg}{m} = \frac{12.0 \cdot 0.1 - 0.1 \cdot 9.81}{0.1} = 2.19\,\mathrm{m/s^2}.$$

Question 2 — 15pts

Two cylinders **of the same mass and radius** roll without slipping down an incline plane. One cylinder is solid and of uniform density. The other cylinder is hollow and the mass is uniformly distributed in a cylindrical shell (that is, like a hollow pipe). Both cylinders are released simultaneously from the same height and are initially at rest. Which cylinder reaches the bottom of the slope first, and why?

(a) The solid cylinder reaches the bottom first, because the moment of inertia is larger and therefore the cylinder rolls faster.

(b) The solid cylinder reaches the bottom first, because the moment of inertia is smaller and therefore the translational speed is larger.

(c) The hollow cylinder reaches the bottom first, because the moment of inertia is smaller and therefore the cylinder rolls faster.

(d) The hollow cylinder reaches the bottom first, because the moment of inertia is larger and therefore the translational speed is larger.

(e) Both cylinders reach the bottom at the same time, because they have the same mass and the acceleration due to gravity is independent of mass.

Solution 2

The answer is (b). The solid cylinder has a smaller moment of inertia, because the mass is distributed closer to the axis of rotation. The total kinetic energy is composed of rotational,

$I\omega^2/2$, and translational kinetic energy, $mv^2/2$. At the bottom of the motion, conservation of energy ensures that the total kinetic energy is the same for both cylinders. The rotational kinetic energy is smaller for the solid cylinder (because I is smaller), so the translational kinetic energy must be larger, and therefore the solid cylinder has a larger translational speed.

Question 3 **15pts**

Two masses, each of mass m, collide with each other on a frictionless table. Prior to the collision, one of the masses moves with speed v and the other is stationary. The masses stick together. What is the final kinetic energy of the system, assuming no mass is lost?

(a) 0 (b) $\frac{mv^2}{4}$ (c) $\frac{mv^2}{3}$ (d) $\frac{mv^2}{2}$ (e) mv^2

Solution 3

The answer is (b). This collision is perfectly inelastic, so momentum is conserved. The initial momentum is mv. After the collision, the two masses have total mass $2m$, so they must be moving with speed $v_f = v/2$. The total kinetic energy is therefore $2m \cdot (v/2)^2/2 = mv^2/4$.

Longer questions

Remember, present your solutions legibly and as logically as you can. Highlight your final answer by underlining or circling it.

Question 4 **20pts**

A mass of $2m$ moves to the east, and a mass m moves to the west, both with speed v_0. They collide elastically, but not head on, so that the mass $2m$ ends up moving northwards (that is, perpendicular to the original direction of motion). What is the speed of the mass $2m$ after the collision? The collision is illustrated in figure 1, where the dashed arrows indicate the velocities before the collision and the solid arrows indicate the velocities after the collision.

Hint: the final speed (squared) of the mass m is given in terms of its components by $v^2 = v^2_{\text{North}} + v^2_{\text{East}}$.

Solution 4

To solve this problem, we apply conservation of momentum in both the north-south (call that y) and the east-west (call that x) directions separately. In addition, we have conservation of energy for an elastic collision.

Conservation of momentum in the y direction gives us

$$0 = 2mu + mv_y.$$

Thus $v_y = -2u$. Note $v_y = v\cos\theta$, but we won't need this.

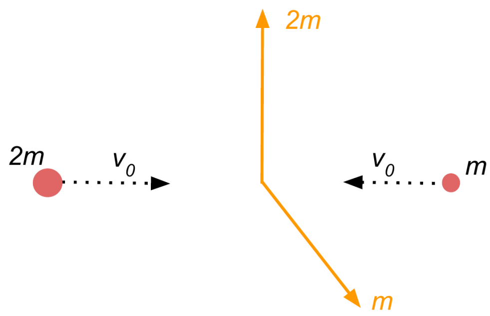

Figure 1: Diagram for Question 4.

Conservation of momentum in the x direction gives us

$$2mv_0 + m(-v_0) = mv_x.$$

Thus $v_x = v_0$.

Now we can apply conservation of energy

$$\frac{(2m)v_0^2}{2} + \frac{mv_0^2}{2} = \frac{(2m)u^2}{2} + \frac{mv^2}{2}.$$

We can substitute in our expressions for the components of v to find

$$v^2 = v_x^2 + v_y^2 = 4u^2 + v_0^2,$$

and use this to find

$$mv_0^2 + \frac{mv_0^2}{2} = mu^2 + \frac{m}{2}(4u^2 + v_0^2).$$

Rearranging this, we have

$$\frac{3mv_0^2}{2} = mu^2 + 2mu^2 + \frac{mv_0^2}{2},$$

or

$$mv_0^2 = 3mu^2.$$

Thus the final northward speed is

$$\boxed{u = \frac{v_0}{\sqrt{3}}}.$$

Question 5 **35pts**

A block of mass $m = 4.00$ kg is held against a spring with a spring constant of $k = 1500$ N/m, compressing the spring by a distance $x = 0.03$ m. The block is released and the spring extends, pushing the block along a rough horizontal surface. The coefficient of kinetic friction between the surface and the block is $\mu_K = 0.2$.

(a) Find the work done on the block by the spring as it extends from the compressed position to the equilibrium position.

(b) Find the work done by friction on the block while it moves $x = 0.03$ to the equilibrium position.

Solution 5

(a) The work done by the spring on the block is equal to the change in the potential energy of the spring, which is

$$W_S = \Delta E_P = \frac{k}{2}(\Delta x)^2.$$

Plugging in the values, this gives

$$\begin{aligned} W_S &= \frac{1}{2} \cdot 1500 \cdot (0.03)^2 \\ &= \boxed{0.675\,\mathrm{J}} \end{aligned}$$

(b) The work done by friction is given by the line integral

$$W_F = \int F_K \mathrm{d}x.$$

We know that

$$F_K = \mu_K N,$$

and applying Newton's second law in the vertical direction, we have

$$N - mg = 0.$$

Thus,

$$F_K = \mu_K mg.$$

Plugging this in to our expression for the work done, we find

$$\begin{aligned} W_F &= \int_0^{0.03} \mu_K mg \,\mathrm{d}x \\ &= mg\mu_K \int_0^{0.03} \mathrm{d}x \\ &= mg\mu_K x\Big|_0^{0.03} \\ &= mg\mu_K x, \end{aligned}$$

and putting in our numbers gives

$$W_F = 4.00 \cdot 9.81 \cdot 0.2 \cdot 0.03 = 0.235\,\mathrm{J} = \boxed{0.24\,\mathrm{J}.}$$