General Physics 1–Honors (PHYS 101H): Midterm 1 Monday October 3, 2022

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Overview and instructions

In this midterm you will apply your understanding of kinematics, and your knowledge of Newton's laws of motion, to problems in one and two dimensions.

Read the following instructions carefully.

There are **five questions**, for a total of **100 points**. **Attempt all questions**. The exam will finish at 11:45 am. Please write your name **on every sheet of paper you submit**. It is helpful if you include page numbers at the bottom of each page, too.

You may use:

- an electronic calculator;
- your own formula sheet, written or printed on one side of letter paper.

You may **not** use:

- electronic devices (except a calculator), including phones, tablets and laptops (unless previously arranged);
- textbooks or other reference resources;
- course notes or slides.

The first three questions are multiple choice. Your answer to these multiple choice questions should be written out and submitted as part of the rest of your solutions. For example, you could write, "Problem 1: my answer is **(a)**." Do **not** circle the options on the exam itself; I will not collect the exams and you will not receive credit for your answer.

The remaining questions require written solutions. You should show all your working and include important intermediate steps, equations, and results. You can receive partial credit for these problems, even if you don't complete the problem or provide a correct final answer. Please ensure that you highlight or emphasise your final answer (for example, by circling or underlining the final answer).

You are responsible for ensuring your solutions are legible. Present your solutions legibly and as logically as you can. What this means in practice is the following:

1. Write down what quantities you know.
2. Write down the relevant equations.
3. When carrying out manipulations or substituting values into equations, try to write each equality on separate lines or otherwise distinguish them clearly.
4. Circle or underline your final answers to identify them clearly.

Some hints for tackling problems in general:

1. Only substitute values at the end of your calculation and try to carry out all manipulations symbolically.
2. Double check the order of magnitude of your answer.
3. Double check the units of your answer.
4. Double check the number of significant figures of your answer. Remember that I am only looking for approximately the correct number of significant digits. If quantities are given to two or three significant digits, quote your answer to two or three (not one or five). Similarly if quantities are given to eight significant digits, do not quote your answer to two.

You do not have to tackle the questions in order. Briefly read through them all and then start on one!

Short questions

Remember, your answer to these multiple choice questions should be written out and submitted as part of the rest of your solutions.

Question 1 **10pts**

There are three fundamental physical constants that are associated with the three theories that form the pillars of modern physics. These are: Planck's constant, $\hbar = 1.05 \times 10^{-34}$ kg m^2/s, which characterizes quantum mechanics; the gravitational constant, $G = 6.67 \times 10^{-11}$ m^3 /(kg s^2), which characterizes general relativity, the theory of gravity; and the speed of light, $c = 3.0 \times 10^8$ m/s, which is associated with special relativity. These constants can be combined to yield quantities with dimensions of length and mass. These quantities are known as the Planck length (λ_P) and the Planck mass (m_P), respectively. Which of the following pairs of combinations of fundamental constants give the correct expressions for the Planck length and Planck mass, respectively?

(a) $\sqrt{\hbar G/c^6}$, $\sqrt{\hbar c/G}$.

(b) $\sqrt{\hbar G/c^5}$, $\sqrt{\hbar c/G}$.

(c) $\sqrt{\hbar G/c^4}$, $\sqrt{\hbar c^2/G}$.

(d) $\sqrt{\hbar G/c^3}$, $\sqrt{\hbar c/G}$.

(e) $\sqrt{\hbar G/c^2}$, $\sqrt{\hbar c^2/G}$.

Solution 1

The answer is (d). Note that $\sqrt{\hbar G/c^5}$ is the Planck time!

Question 2 **10pts**

The three plots in Figure 1 show three **independent** graphs of three independent motions. The first figure shows the acceleration versus time plot for one-dimensional motion in a certain setup. The second figure shows the velocity versus time plot for the one-dimensional motion of a **different** setup. The third figure shows the displacement versus time plot for **yet another** setup. Which of the twelve labeled points correspond to zero acceleration? (To repeat, the three setups have nothing to do with each other. That is, the velocity plot is not the velocity curve associated with the position in the position plot, and so on.)

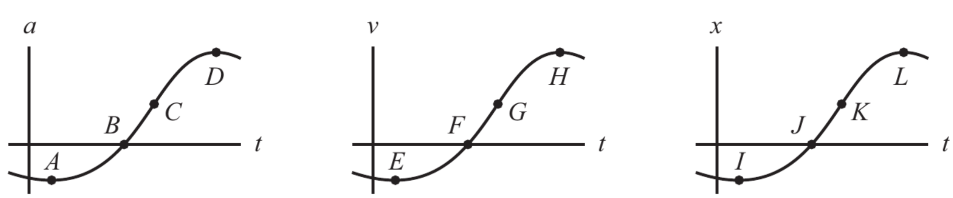

Figure 1: Diagram for Question 2.

(a) A, B, E, H
(b) A, E, H, J
(c) B, E, H, K
(d) B, F, H, L
(e) B, F, I, L

Solution 2

The answer is (c). On the acceleration plot, the acceleration is zero when it intersects the horizontal axis (point B). On the velocity plot, the acceleration is zero when the gradient of the velocity curve is zero, which occurs at points E and H. On the displacement plot, the acceleration is zero when the velocity is at a maximum (in which case the slope of the velocity is zero). The maximum velocity occurs when then displacement curve has a maximum slope, which is point K.

Question 3 **10pts**

A block slides down a plane inclined at angle θ. What should the coefficient of kinetic friction be so that the block slides with constant velocity?

(a) 1
(b) $\sin\theta$
(c) $\cos\theta$
(d) $\tan\theta$
(e) $\cot\theta$

Solution 3

The answer is (d). Resolving components parallel to the surface, we have

$$F_K = mg\sin\theta,$$

and resolving components perpendicular to the surface, we have

$$N = mg\cos\theta.$$

But the force due to friction is related to the normal force by

$$F_K = \mu_K N,$$

or

$$mg\sin\theta = \mu_K mg\cos\theta.$$

Solving this gives answer (d).

Longer questions

Remember, present your solutions legibly and as logically as you can. Highlight your final answer by underlining or circling it.

Question 4 **30pts**

You wish to throw a ball to a friend who is a distance 8.50 m away, and you want the ball to just barely clear a wall of height $h = 3.00$ m that is located halfway to your friend (that is, at a distance 4.25 m from you).

(a) At what angle should you throw the ball?

(b) What is the minimum speed at which you need to throw the ball?

(c) How long does your friend have to react, between seeing the ball as it passes over the wall and when they catch it, if you through it at the minimum required speed? Assume your friend sees the ball at the instant that it is at the top of its motion.

Solution 4

(a) The maximum height for a projectile is given by

$$h_{\text{max}} = \frac{v_0^2 \sin^2 \theta}{2g},$$

and the range is

$$R = \frac{2v_0^2 \sin \theta \cos \theta}{g}.$$

We are given the height and the range, but not the speed, so we need to rearrange these expressions to obtain an equation that relates the angle to the height and the range. The easiest way to do this is divide them, which gives

$$\frac{h}{R} = \frac{\sin \theta}{4 \cos \theta}.$$

In other words,

$$\tan \theta = \frac{h}{4R},$$

or

$$\theta = \arctan \frac{4h}{R}.$$

Plugging in our numbers we obtain

$$\theta = \arctan \frac{4 \cdot 3.00}{8.50} = \boxed{54.7^\circ}.$$

(b) We can rearrange the formula for the height to find

$$v_0 = \frac{\sqrt{2gh_{\text{max}}}}{\sin \theta} = \frac{\sqrt{2 \cdot 9.81 \cdot 3.00}}{\sin 54.7^\circ} = \boxed{9.4\,\text{m/s}}.$$

(c) The simplest way to solve this is to notice that there is no acceleration in the horizontal direction. In this direction, the velocity is constant and given by

$$v_x = v_0 \cos \theta.$$

The time taken is therefore (note that the distance in this case is half the range)

$$t = \frac{\Delta x}{v_x} = \frac{4.25}{9.4 \cos 54.7^\circ} = \boxed{0.78\,\text{s}}.$$

Question 5 **40pts**

A car is on a race track with banked curves (i.e. the surface of the race track surface is at an angle with respect to the horizontal ground), illustrated in figure 2. The radius of curvature of the track is $R = 50.0$ m.

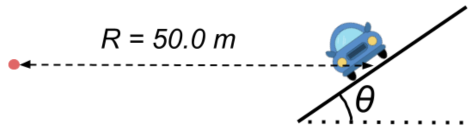

Figure 2: Diagram for Question 5.

(a) What is the angle of the banked curve that is needed for the car to travel at a speed of $v = 60$ mph (26.82 m/s) around the curve, even when it is icy?

(b) What is the angle of the banked curve that would be needed for the car to travel at the same speed if there is no ice and the coefficient of static friction of the race track is 1.2? Explain whether your answer makes sense conceptually.

Solution 5

(a) Let's start by looking at the horizontal direction. Applying Newton's second law in this direction, we have

$$N \sin\theta = ma_c. \tag{1}$$

In the vertical direction, we have

$$N \cos\theta - mg = 0.$$

We can rearrange this second equation to obtain an expression for the normal force

$$N = \frac{mg}{\cos\theta},$$

which we plug into our first equation (equation (1)) to give

$$\frac{mg}{\cos\theta} \cdot \sin\theta = ma_c.$$

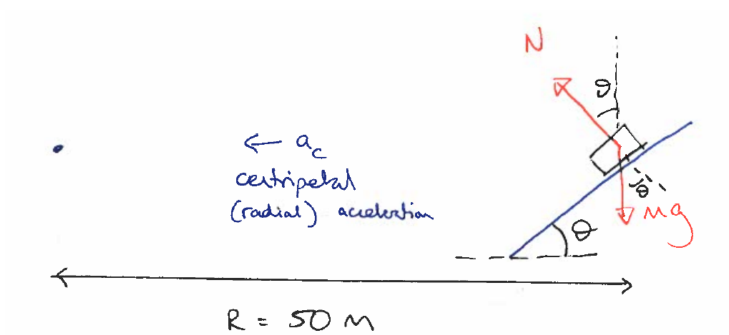

Figure 3: Diagram for question 5(a).

We need to find the angle, so let's rearrange this as

$$\tan\theta = \frac{a_c}{g}.$$

The centripetal acceleration is not given, but we can relate to the speed, which is given, via

$$a_c = \frac{v^2}{R}.$$

We obtain

$$\tan\theta = \frac{v^2}{gR},$$

or

$$\theta = \arctan\frac{v^2}{gR} = \arctan\frac{26.82^2}{9.81\cdot 35.0} = \boxed{55.7°}$$

(b) In the presence of friction, applying Newton's second law to the horizontal and vertical directions, respectively, we have

$$\begin{aligned} N\sin\theta + F_S\cos\theta &= ma_c \\ N\cos\theta - F_S\sin\theta - mg &= 0. \end{aligned}$$

We know that the frictional force and the normal are related by

$$F_S = \mu_S N,$$

so these equations become

$$\begin{aligned} N\sin\theta + \mu_S N\cos\theta &= ma_c \\ N\cos\theta - \mu_S N\sin\theta &= mg. \end{aligned}$$

We can write these as

$$\begin{aligned} N(\sin\theta + \mu_S\cos\theta) &= ma_c \\ N(\cos\theta - \mu_S\sin\theta) &= mg, \end{aligned}$$

and dividing them gives

$$\frac{\sin\theta + \mu_S\cos\theta}{\cos\theta - \mu_S\sin\theta} = \frac{a_c}{g}.$$

To solve this for θ, we write

$$\sin\theta + \mu_S\cos\theta = \frac{a_c}{g}(\cos\theta - \mu_S\sin\theta),$$

and then collect the $\sin\theta$ terms on one side and the $\cos\theta$ terms on the other:

$$\sin\theta + \frac{a_c\mu_S}{g}\sin\theta = \frac{a_c}{g}\cos\theta - \mu_S\cos\theta,$$

or

$$\sin\theta\left(1 + \frac{a_c\mu_S}{g}\right) = \left(\frac{a_c}{g} - \mu_S\right)\cos\theta.$$

This we can write as

$$\tan\theta = \frac{a_c/g - \mu_S}{1 + a_c\mu_S/g},$$

or

$$\theta = \arctan\frac{a_c/g - \mu_S}{1 + a_c\mu_S/g}.$$

To complete the solution, we use $a_c = v^2/R$ again and we have

$$\theta = \arctan\frac{v^2/(gR) - \mu_S}{1 + v^2\mu_S/(gR)} = \arctan\frac{26.82^2/(9.81 \cdot 50.0) - 1.25}{1 + 26.82^2 \cdot 1.25/(9.81 \cdot 50.0)} = \boxed{5.5^\circ}.$$

This answer makes sense! We would expect the presence of friction to reduce the angle needed, because it acts against the apparent outward motion of the car. In other words, it serves as a second source of centripetal acceleration, which means that the component of the normal force that is needed to keep the car on the track is smaller, which means the angle can be smaller.