General Physics 1–Honors (PHYS 101H): Final Exam–Solutions Tuesday, December 13, 2022

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Overview and instructions

In this final you will apply your understanding of the topics covered in Physics 101–Honors. These topics include: kinematics and dynamics in one and two dimensions; Newton's laws of motion; conservation of energy; elastic and inelastic collisions; rotational kinematics and dynamics; gravitation; fluid statics and dynamics; simple harmonic motion and oscillations; and wave motion.

Read the following instructions carefully.

There are **ten questions**, for a total of **200 points**. **Attempt all questions**. The exam will start at 9:00 am and finish at 12:00 midday. Please write your name **on every sheet of paper you submit**. It is helpful if you include page numbers at the bottom of each page, too.

You may use:

- an electronic calculator;
- your own formula sheet, written or printed on **two sides** of letter paper.

You may **not** use:

- electronic devices (except a calculator), including phones, tablets and laptops (unless previously arranged);
- textbooks or other reference resources;
- course notes or slides.

The first five questions are multiple choice. Your answer to these multiple choice questions should be written out and submitted as part of the rest of your solutions. For example, you could write, "Problem 1: my answer is **(a)**." Do **not** circle the options on the exam itself; I will not collect the exams and you will not receive credit for your answer.

The remaining five questions require written solutions. You should show all your working and include important intermediate steps, equations, and results. You can receive partial credit for these problems, even if you don't complete the problem or provide a correct final answer. Please ensure that you highlight or emphasise your final answer (for example, by circling or underlining the final answer).

You are responsible for ensuring your solutions are legible. Present your solutions legibly and as logically as you can.

In practice, this means:

1. Try to identify what "kind" or "type" of question is being asked, for example "projectile motion", "conservation of energy", or "two dimensional collision".
2. Draw a labelled diagram.
3. Write down what quantities you know.
4. Write down the relevant equations.
5. Write down what quantities you know.
6. Write down the relevant equations.
7. When carrying out manipulations or substituting values into equations, try to write each equality on separate lines or otherwise distinguish them clearly.
8. Circle or underline your final answers to identify them clearly.

Some hints for tackling problems in general:

1. Only substitute values at the end of your calculation and try to carry out all manipulations symbolically.
2. Double check the order of magnitude of your answer.
3. Double check the units of your answer.
4. Double check the number of significant figures of your answer. Remember that I am only looking for approximately the correct number of significant digits. If quantities are given to two or three significant digits, quote your answer to two or three (not one or five). Similarly if quantities are given to eight significant digits, do not quote your answer to two.

You do not have to tackle the questions in order. Briefly read through them all and then start on one!

Short questions

Remember, your answer to these multiple choice questions should be written out and submitted as part of the rest of your solutions.

Question 1 **15pts**

When you stand at rest on a floor, you exert a downward normal force on the floor. Does this force cause the Earth to accelerate in the downward direction?

(a) Yes, but the Earth is very massive, so you don't notice the motion.

(b) Yes, but you accelerate along with the Earth, so you don't notice the motion.

(c) No, because the normal force is a fictitious (pseudo) force.

(d) No, because you are also pulling on the earth gravitationally.

(e) No, because there is also friction at your feet.

Solution 1

The answer is (d). The normal force from you on the earth is equal and opposite to the gravitational force from you on the earth. So the net force on the earth is zero, and it therefore doesn't accelerate. This is exactly the same logic as that used to explain why **you** don't accelerate.

Question 2 **15pts**

Clifford likes to drop his ball in the river and watch the ball float downstream before swimming after it. The river flows at a constant rate of $u = 1$ m/s, and Clifford watches the ball for 3 s before he starts swimming. Assuming that Clifford can swim with a uniform acceleration of 1.5 m/s^2, how long does it take for Clifford to catch up with his ball? Assume that both the ball and Clifford travel in a straight line downstream, and that both start their motion from rest.

(a) 1.4 s (b) 2.8 s (c) 3.4 s (d) 5.6 s (e) 6.8 s

Solution 2

The answer is (b).

The ball travels a total distance of

$$x_{\text{ball}} = 3 \cdot u + t \cdot u = (3 + t)u\,\text{m}$$

until Clifford catches up with it at time $3 + t$, after swimming for a time t. He starts swimming with an initial velocity $v_0 = 0$, starting at initial position $x_0 = 0$ m, and has acceleration $a = 1.5$ m/s^2. Clifford therefore travels a total distance

$$x_{\text{Clifford}} = \frac{a}{2}t^2.$$

These distances must be equal, so

$$(3+t)u = \frac{a}{2}t^2,$$

which can be rewritten as

$$\frac{a}{2}t^2 - ut - 3u = 0.$$

The quadratic equation gives

$$\begin{aligned} t &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-(-u) \pm \sqrt{u^2 - 4(a/2)(-3u)}}{2(a/2)} \\ &= \frac{u \pm \sqrt{u^2 + 6au}}{a} \\ &= \frac{1 \pm \sqrt{1 + 6 \cdot 1.5}}{1.5}. \end{aligned}$$

Only one of these solutions is physical, and gives $t = 2.77$ s.

Question 3 **15pts**

A race car passes a stationary fan watching the race from a point in the middle of a straightaway. The race car takes 2 s to travel 110 m along the straightaway, and emits sound at a constant frequency of 600 Hz. What frequency does the fan hear as the car approaches?

(a) 428 Hz (b) 514 Hz (c) 600 Hz (d) 720 Hz (e) 840 Hz

Solution 3

The answer is (d). The observed frequency due to the Doppler effect is

$$\begin{aligned} f_O &= f_s \frac{v}{v - v_s} \\ &= 600 \frac{330}{330 - 55} \\ &= 720\,\text{Hz}. \end{aligned}$$

Question 4 **15pts**

A boat carrying a large boulder is floating on a lake. The boulder is thrown overboard and sinks. The water level in the lake (with respect to the shore):

(a) drops, (b) rises, (c) stays the same.

Solution 4

The answer is (a). In the boat, the boulder displaces its weight in water (which occupies a volume larger than the boulder, because the boulder is more dense than the water). Once dropped in the water, the boulder displaces its volume in water. Therefore the water level goes down.

Question 5 **15pts**

Gliese 667Cc is an exoplanet (a planet outside our solar system) orbiting a red dwarf about 22 light-years from Earth, and is about 4.5 times as massive as the Earth. What is the acceleration due to gravity on Gliese 667Cc, relative to the acceleration due to gravity on Earth g? The radius of Gliese 667Cc is approximately 1.5 times larger than the Earth's radius, 6.37×10^3 km.

(a) $g/4$ (b) $g/3$ (c) $3g/2$ (d) $2g$ (e) $3g$

Solution 5

The answer is (d). The acceleration due to gravity is given by

$$g = \frac{GM}{R^2},$$

so

$$\begin{aligned}
\frac{g_G}{g} &= \frac{GM_G/R_G^2}{GM_E/R_E^2} \\
&= \frac{M_G R_E^2}{M_E R_G^2} \\
&= \frac{4.5M_E \cdot R_E^2}{M_E \cdot (1.5R_E)^2} \\
&= 2.
\end{aligned}$$

Longer questions

Remember, present your solutions legibly and as logically as you can. Highlight your final answer by underlining or circling it.

Question 6 **25pts**

A uniform ladder leans against a smooth (frictionless) wall. The ladder has length 4 m and mass 10 kg. The base of the ladder is at rest on a rough horizontal floor, and the coefficient of static friction between the floor and the ladder is 0.5.

(a) Show that the largest angle that the ladder can lean, with respect to the floor, and remain in equilibrium is 45°. **[10pts]**

(b) A person, of mass 65 kg, now climbs a distance of 3 m up the ladder, with the ladder leaning at 45° to the floor. A second person puts their foot against the base of the ladder and provides a force directed towards the wall to keep the ladder in place. What force must the person provide at the base of the ladder to ensure that the ladder does not slip? **[15pts]**

Solution 6

The forces acting on the ladder are illustrated in figure 1.

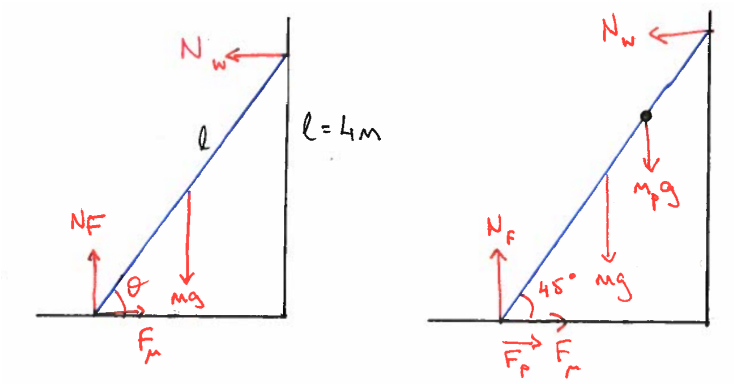

Figure 1: Diagram for Solution 6. Lefthand diagram represents the forces for part (a) and the righthand diagram the forces for part (b).

(a) Applying Newton's second law in the horizontal direction, and requiring that the ladder is in static equilibrium, gives

$$N_W - \mu N_F = 0,$$

while vertically we have

$$N_F - mg = 0.$$

To find the maximum angle, we need to use the rotational equilibrium condition and we choose the centre of the ladder as our axis of rotation. This gives

$$N_F \cos\theta \cdot \frac{\ell}{2} - \mu N_F \sin\theta \frac{\ell}{2} - N_W \sin\theta \frac{\ell}{2} = 0,$$

where ℓ is the length of the ladder.

Substituting $N_W = \mu N_F$, we obtain

$$N_F \cos\theta - 2\mu N_F \sin\theta = 0,$$

or

$$\tan\theta = \frac{1}{2\mu}.$$

Plugging in our values, we find

$$\theta = \arctan\left[\frac{1}{2 \cdot 0.5}\right] = \boxed{45^\circ}.$$

(b) We cannot assume all forces remain the same. So let's start with the equilibrium condition for the vertical forces, which says that

$$N_F - m_L g - m_P g = 0,$$

or

$$N_F = (m_L + m_P)g.$$

The condition for equilibrium for the horizontal forces is

$$N_W - F_P - F_\mu = 0,$$

or

$$F_P = N_W - F_\mu.$$

We can connect these by using $F_\mu = \mu N_F$, so

$$F_P = N_W - \mu \cdot (m_L + m_P)g.$$

To find N_W, we next apply the condition for rotational equilibrium, around the base of the ladder. This gives us

$$N_W \ell \sin\theta - m_P g \frac{3\ell}{4} \sin\theta - m_L g \frac{\ell}{2} = 0.$$

In other words,

$$N_W \ell \sin\theta = m_P g \frac{3\ell}{4} \sin\theta + m_L g \frac{\ell}{2} \sin\theta,$$

or

$$N_W = \left(\frac{3m_P}{4} + \frac{m_L}{2}\right) g.$$

Putting this together, we have

$$\begin{aligned} F_P &= \left(\frac{3m_P}{4} + \frac{m_L}{2}\right) g - \mu(m_L + m_P)g \\ &= \left[\left(\frac{3}{4} - \mu\right) m_P + \left(\frac{1}{2} - \mu\right) m_L\right] g \\ &= \left[\left(\frac{3}{4} - \frac{1}{2}\right) m_P + \left(\frac{1}{2} - \frac{1}{2}\right) m_L\right] g \\ &= \frac{65 \cdot 9.81}{4} \\ &= \boxed{159\,\mathrm{N}}. \end{aligned}$$

Question 7 **25pts**

A balloon is filled with helium until the whole balloon occupies a volume of 2 litres (there are 1000 litres in 1 m^3). The balloon is tied down with a string of negligible mass. In the following, you can neglect the volume of the rubber of the balloon itself.

(a) Assuming that the density of air is 1.3 $\mathrm{kg/m}^3$, find the bouyant force on the balloon. **[5pts]**

(b) Assuming that the mass of the rubber in the balloon is 1 g, and the balloon floats in equilibrium at the end of the string, what is the tension in the string? The density of helium is 0.16 kg/m^3. **[10pts]**

(c) What is the upward acceleration of the balloon, if the string is untied and the balloon released? **[10pts]**

Solution 7

(a) The bouyant force is given by Archimedes' principle, which tells us that the force is equal to the weight of fluid (in this case, air) displaced.

The weight of air displaced is

$$F_B = 2 \cdot \frac{1}{1000} \cdot 1.3 \cdot 9.81 = \boxed{0.0255\,\text{N}}.$$

(b) The total mass of the balloon is

$$m_b = 0.001 + 2 \cdot \frac{1}{1000} \cdot 0.16 = 0.00132\,\text{kg}.$$

Thus the net force on the balloon, which is required to be zero, is

$$T + m_b g - F_B = 0.$$

Therefore the tension is given by

$$\begin{aligned} T &= F_B - m_b g \\ &= 0.026 - 0.00132 * 9.81 \\ &= \boxed{0.0126\,\text{N}}. \end{aligned}$$

(c) Now the net force on the balloon is

$$m_b g - F_B = m_b a.$$

The resulting acceleration is

$$\begin{aligned} a &= g - \frac{F_B}{m_b} \\ &= 9.81 - \frac{0.0255}{0.00132} \\ &= -9.51. \end{aligned}$$

Note that the sign convention here is that the force due to gravity (downwards) is positive. In other words, the acceleration is

$$\boxed{a = 9.51\,\text{m/s}^2 \text{ upwards.}}$$

Question 8 **25pts**

A sphere of mass m can move across a horizontal, frictionless surface. The sphere is attached to a spring. At time $t = 0$ the sphere is pulled aside from the equilibrium position, $x = 0$, to a distance d in the positive direction and released from rest.

(a) What is the (angular) frequency with which the spring–mass system oscillates after being released? **[5pts]**

(b) What is the period of oscillation of the mass? **[10pts]**

(c) Describe qualitatively (in words) what would happen to the motion of the ball, after it is released, if it instead moves across a surface with friction. Include in your description what will happen to the frequency, period, and amplitude of the oscillatory motion in the presence of friction. **[10pts]**

(d) The full solution for underdamped oscillatory motion (oscillatory motion in the presence of friction) is

$$x(t) = Ae^{-bt/(2m)} \cos(\omega t + \phi).$$

Here b is a coefficient that characterises the size of the frictional force. Show that if this frictional force is very small (which means that $b/2m$ is very small), the resulting motion is approximately the same as simple harmonic motion without damping. **[5pts]**

Solution 8

(a) Hooke's law tells us that the force is proportional to the displacement,

$$F_S = -kx,$$

where k is the spring constant. Thus

$$k = \boxed{\frac{F_S}{d}}.$$

The spring-mass system behaves a simple harmonic oscillator, so the frequency is just

$$\omega_0^2 = \frac{k}{m}.$$

Thus

$$\omega_0 = \sqrt{\frac{k}{m}} = \boxed{\sqrt{\frac{F_S}{md}}}.$$

(b) The period is

$$T = \frac{2\pi}{\omega_0},$$

or

$$\boxed{T = 2\pi\sqrt{\frac{md}{F_S}}}$$

(c) Key points:

(a) Friction leads to **damped oscillations**.
(b) Exact behaviour depends on size of friction relative to spring constant.
(c) Behaviour can be overdamped (a lot of friction), underdamped (a little friction), or critically damped.
(d) The angular frequency is given by

$$\omega = \sqrt{\frac{k}{m} - \frac{b^2}{2m}}.$$

(e) In the case of a small amount of damping, the system will oscillate with almost the same frequency and period, but with exponentially decreasing amplitude.
(f) Note that the frequency and period do **not** decrease with time (but may decrease relative to the natural frequency ω_0).

(d) If $b/2m$ is very small, then we can Taylor expand the exponential as

$$e^{-bt/(2m)} \simeq 1 - \frac{bt}{2m} + \cdots .$$

We can take just the leading term in this expansion, so that our solution becomes

$$x(t) = Ae^{-bt/(2m)} \cos(\omega t + \phi) \simeq A\,[1 + \cdots]\cos(\omega t + \phi) = \boxed{A\cos(\omega t + \phi)}.$$

This is exactly the form expected for ordinary simple harmonic motion, in the absence of friction.

Question 9 — 25pts

A large open tank, of height 10.0 m, is used to collect water for irrigation. The water can drain through a horizontal hose of diameter 5.0 cm at the base of the tank. The tank sits on a pedestal 3.5 m high. You can assume the density of water is $\rho = 1000$ kg/m^3 and atmospheric pressure is $P_{\text{atm}} = 1.01 \times 10^5$ Pa.

(a) Find the water pressure at the base of the tank. **[5pts]**
(b) Assuming that the tank is so large that the water level at the top of the tank does not drop as water leaves through the hose, what is the velocity of water as it exits the tank through the hose? **[10pts]**
(c) Find the horizontal distance the water travels before it hits the ground. Neglect friction and air resistance. **[10pts]**

Solution 9

(a) The pressure at a given depth h is given by

$$P = P_0 + \rho g h.$$

In this case, $P_0 = P_{\text{atm}}$ and $h = 10.0$ m, so

$$P = 1.013 \times 10^5 + 1000 * 9.81 * 10.0 = \boxed{1.99 \times 10^5 \text{ Pa}}.$$

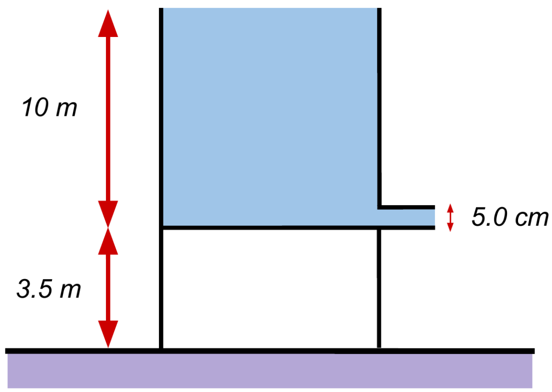

Figure 2: Diagram for Question 9.

(b) Bernoulli's equation tells us that

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2.$$

In this case, taking "1" to denote the top of the tank and "2" indicates the base of the tank, we have $P_1 = P_{\text{atm}}$, $v_1 = 0$ and $h_1 = 10$ m. We also have $P_2 = P_{\text{atm}}$ and $h_2 = 0$ m. Thus

$$P_{\text{atm}} + \rho g h_1 = P_{\text{atm}} + \frac{1}{2}\rho v_2^2.$$

Note that a common error is to take $P_2 = P^{(a)}$ (your answer to part (a)). To see why this is an error, consider some water infinitesimally outside the tank: it does not experience the pressure at the base of the tank, only air pressure, but it still has a nonzero speed.

Rearranging this equation for our desired quantity, the velocity of the water entering the hose, we obtain

$$v_2^2 = 2gh,$$

or

$$v_2 = \sqrt{2gh} = \sqrt{2 \cdot 9.81 \cdot 10.0} = \boxed{14.0\,\text{m/s}}.$$

(c) The water exits the tank with horizontal speed v_2. It undergoes parabolic motion, with acceleration in the vertical direction $g = 9.81$ m/s^2 and falls a distance $y = 3.5$ m vertically. The initial speed in the vertical direction is $v_y = 0$. Assuming that the water starts at $y_0 = 0$, and the downward vertical direction is positive, the time it takes to fall this distance is given by

$$y = \frac{at^2}{2} + v_y t + y_0,$$

or

$$y = \frac{at^2}{2}.$$

Rearranging this, we have

$$t = \sqrt{\frac{2y}{a}},$$

where we have ignored the negative square root solution, because it is unphysical.

Horizontally, the water has no acceleration and therefore travels a distance

$$x = v_x t,$$

or

$$x = v_2 \cdot \sqrt{\frac{2y}{a}} = 14.007 \cdot \sqrt{\frac{2 \cdot 3.5}{9.81}} = \boxed{11.8\,\mathrm{m}}.$$

Question 10 **25pts**

A uniform rod of mass $M = 0.3\,\mathrm{kg}$ and length $L = 1.0\,\mathrm{m}$ can rotate about a hinge at its left end. A sticky ball of plasticine, of mass $m = 10.0\,\mathrm{g}$, moving with speed $v = 3.0\,\mathrm{m/s}$, strikes the rod at an angle $\theta = 90°$ and at a distance $d = \frac{2}{3}L$ from the point of rotation. The rod is initially at rest, and the ball sticks to the rod after the collision. The arrangement of the rod and the ball before the collision is shown in figure 2.

Hint: In the following, it may be helpful to know that the moment of inertia of a uniform rod of mass m and length ℓ about one end is $\frac{m\ell^2}{3}$.

(a) What kind of collision is this? And what is conserved in this collision? **[5pts]**

(b) What is the magnitude of the initial angular momentum of the ball, right before the collision, relative to the pivot point of the rod? **[5pts]**

(c) What is the total moment of inertia, with respect to the hinge, of the rod-ball-system after the collision? **[10pts]**

(d) What is the angular speed of the system immediately after the collision? **[5pts]**

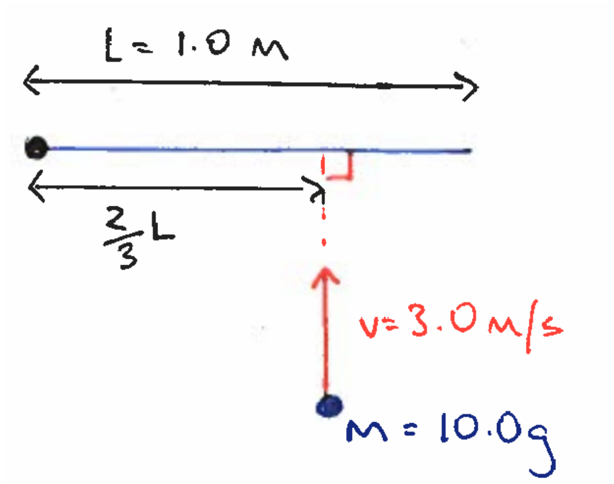

Figure 3: Diagram for Question 10.

Solution 10

(a) This is a perfectly inelastic collision. Momentum and angular momentum are **both** conserved, but kinetic energy is **not** conserved.

N.B. Full marks require both momentum and angular momentum to be mentioned. Discussion of the nonconservation of kinetic energy is not required.

(b) The magnitude of the angular momentum is given by

$$L = rp \sin\theta,$$

and in this case $\theta = 90^\circ$, so

$$L = rp.$$

This is

$$L = \frac{2L}{3} \cdot (mv) = \frac{2 \cdot 1.0}{3} \cdot (0.01 \cdot 3) = \boxed{0.02\,\mathrm{kg\,m^2/s}}$$

(c) The moment of inertia of the system is the sum of the moments of inertia of the rod and the ball

$$I = I_{\mathrm{rod}} + I_{\mathrm{ball}}.$$

Using the hint in the question,

$$I_{\mathrm{rod}} = \frac{ML^2}{3},$$

and the moment of inertia of the ball is the moment of inertia of a point particle:

$$I = mr^2.$$

Thus

$$I_{\mathrm{ball}} = md^2 = m \cdot \frac{4L^2}{9}.$$

Putting these together, we have

$$I = \frac{ML^2}{3} + \frac{4mL^2}{9} = \frac{L^2}{3}\left(M + \frac{4m}{3}\right) = \frac{1.0^2}{3}\left(0.3 + \frac{4 \cdot 0.01}{3}\right) = \boxed{0.104\,\mathrm{kg\,m^2}}.$$

(d) Angular momentum is conserved during the collision, so we can equate the initial angular momentum of the ball with the final angular momentum of the system. Then the angular speed is given by

$$L = I\omega,$$

or

$$\omega = \frac{L}{I} = \frac{0.02}{0.104} = \boxed{0.19\,\mathrm{rad/s}}.$$