# Physics 101H

## General Physics 1 – Honors

Lecture 8 – 9/14/22

Uniform circular motion

# Problem sets

Problem Set 2 has been posted

Due by the **start of class** on **Wednesday 21 September**

Remember

- I will **drop the lowest grade** on your weekly Problem Sets

MAKING SURE YOUR WORK IS LEGIBLE IS *YOUR* RESPONSIBILITY

# Summary

**Topics**

**Yesterday: kinematics in 2D**

- Constant acceleration
- Projectile motion

**Today: kinematics in 2D [chapter 4]**

- Examples in 2D
- Uniform circular motion

**Announcements**

**Today: Problem set 1 due**
**Problem set 2 assigned**

**Example:** A mountain lions can jump a height of 4 m, when leaving the ground at an angle of 45 degrees. At what speed does it leave the ground to make this jump?

**Example:** A park ranger wishes to tranquilise a Virginia Northern Flying Squirrel that is sitting on the edge of a branch of a tree. The sound of the tranquiliser gun will scare the Flying Squirrel and it will jump from the branch the instant the ranger fires their tranquiliser gun. Where should the ranger aim?

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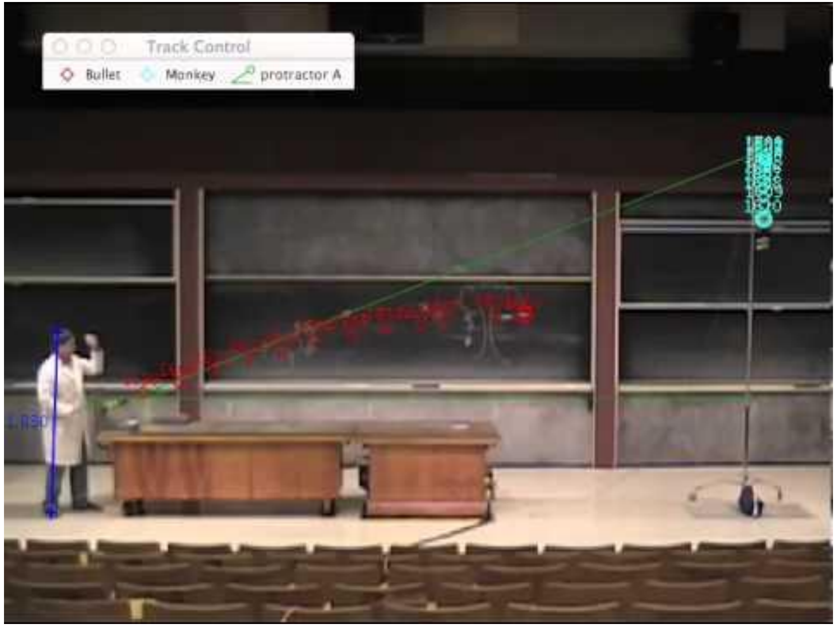
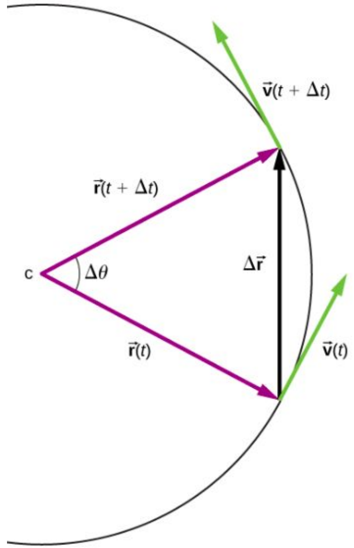


**Is an object that is moving in a circle with constant speed accelerating?**

# Uniform circular motion

Uniform circular motion is motion in a circle at a constant speed

Special case of motion in 2D with constant acceleration




**Period** – time taken to complete one revolution

**Angular speed** – rate at which the angle changes

# Summary

**Topics**

**Today: kinematics in 2D [chapter 4]**

- Examples in 2D
- Uniform circular motion

**Tomorrow: kinematics in 2D [chapter 4]**

- Circular motion
- Relative velocity
- Galilean transformations

**Announcements**

**Today:** **Problem set 1 due**
**Problem set 2 assigned**

# NEXT WEEK:
# NO CLASS ON THURSDAY OR FRIDAY

# PHYSICS 101 - HONORS

## Lecture 8 9/14/22

## Mountain Lion example (but better)

Note 1D $\rightarrow$ 2D

$$x = \frac{at^2}{2} + v_0 t + x_0 \qquad \left\{ \begin{array}{l} \bar{r} = \bar{a}\frac{t^2}{2} + \bar{v}_0 t + \bar{r}_0 \\ \bar{v} = \bar{v}_0 + \bar{a}t \end{array} \right.$$

$$v = v_0 + at$$

vector equations apply to each component separately

$$v_f^2 = v_i^2 + 2a\Delta x \rightarrow ?$$

need scalar (dot) product

$$\bar{v}_f \cdot \bar{v}_f = \bar{v}_i \cdot \bar{v}_i + 2\bar{a} \cdot (\bar{x} - \bar{x}_i)$$

Or separate into components and treat <u>each one</u> as 1D

For mountain lion example:

choose $y$ direction - use $v_{fy}^2 = v_{iy}^2 + 2a\Delta y$

Now $v_{fy}^2 = 0$ for $y = h$ (max. height)

$\Delta y = y - y_0 = h - 0 = h$ (max. height - initial height)

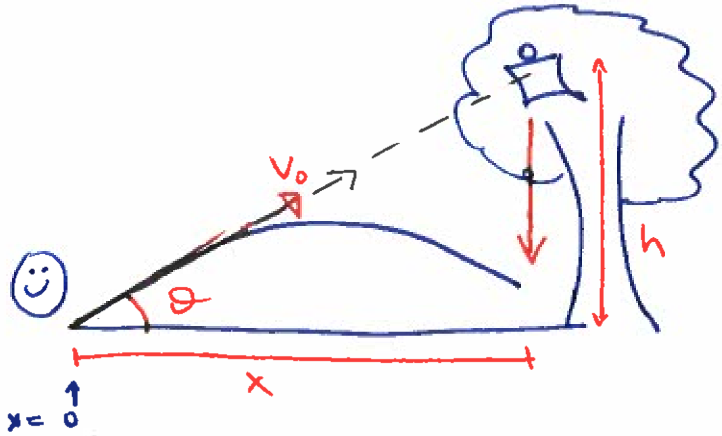
$v_{iy} = v_i \sin\theta = v_i \sin 45 = \frac{v_i}{\sqrt{2}}$

$\Rightarrow 0 = \left(\frac{v_i}{\sqrt{2}}\right)^2 + 2(-g)h$ or $\frac{v_i^2}{2} = 2gh \Rightarrow v_i = \sqrt{4gh}$

But $h = 4m$ so $v_i = \sqrt{16g} = 4\sqrt{g}$ or $\boxed{v_i \simeq 12.5\ m/s}$

# Squirrel Example

Free fall occurs independently of the horizontal motion!




In other words, the squirrel falls as fast as the projectile.

Dart: $x_d = v_0 \cos\theta \, t$

$y_d = -\frac{1}{2} g t^2 + v_0 \sin\theta \, t + 0$

Squirrel: $x_s = x$

$y_s = -\frac{1}{2} g t^2 + h$

Hit requires $x_d = x_s$ and $y_d = y_s$ at time $t$

$$t = \frac{x_d}{v_0 \cos\theta} \overset{x_d = x_s = x}{=} \frac{x}{v_0 \cos\theta} \Rightarrow y_d = -\frac{g}{2}\left(\frac{x}{v_0\cos\theta}\right)^2 + v_0 \sin\theta \frac{x}{v_0\cos\theta}$$

$$y_s = -\frac{g}{2}\left(\frac{x}{v_0\cos\theta}\right)^2 + h$$

Note that $y_d = -\frac{g}{2}\left(\frac{x}{v_0\cos\theta}\right)^2 + x\tan\theta$

$$= -\frac{g x^2}{2 v_0^2 \cos^2\theta} + \frac{x h}{x}$$

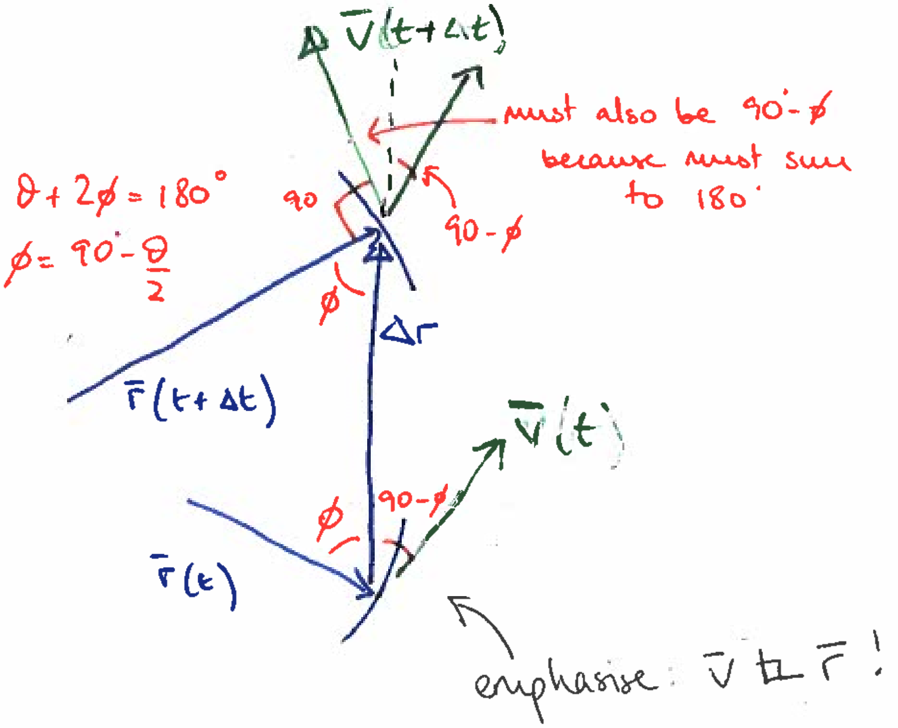
$$= -\frac{g x^2}{2 v_0^2 \cos^2\theta} + h = y_s \checkmark$$

# Uniform circular motion (slide 8)

Aim: 1) derive $\bar{a} \propto -\bar{r}$

2) derive $|\bar{a}|$

Translate $\bar{v}(t)$




angle between $\bar{v}(t+\Delta t)$ and $\bar{v}(t)$ is

$2(90^\circ - \phi) = 180^\circ - 2\phi = \theta$ !

as $\Delta t \to 0 \Rightarrow \theta \to 0$

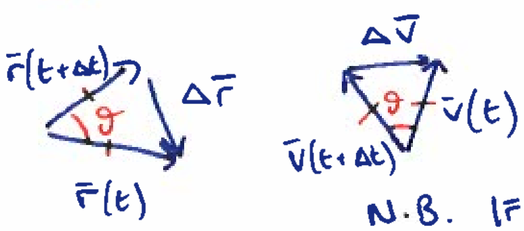
$\Rightarrow \Delta\bar{v} = \bar{v}(t+\Delta t) - \bar{v}(t)$ is more and more perpendicular to $\bar{v}(t+\Delta t)$ and $\bar{v}(t)$

$\Rightarrow \Delta\bar{v}$ starts to point inwards along $-\bar{r}$ $\Rightarrow \bar{a} \propto -\bar{r}$

Projectile motion and circular motion are two main/common types of constant acceleration motion

# Uniform circular motion

Consider




$\leftarrow$ similar triangles!

$\Rightarrow |\Delta\bar{r}| \propto |\Delta\bar{v}| \Rightarrow |\bar{r}| \propto |\bar{v}|$

N.B. $|\bar{r}(t)| = |\bar{r}(t+\Delta t)|$ by definition and $|\bar{v}(t)| = |\bar{v}(t+dt)|$ because constant speed!

So $\dfrac{|\Delta\bar{r}|}{|\bar{r}|} = \dfrac{|\Delta\bar{v}|}{|\bar{v}|} \Rightarrow \dfrac{|\Delta\bar{r}|}{|\bar{r}|\,\Delta t} = \dfrac{|\Delta\bar{v}|}{|\bar{v}|\,\Delta t}$

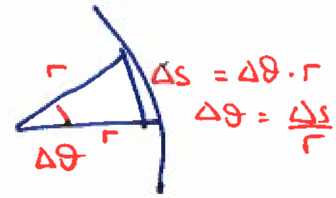
$$\Rightarrow \lim_{\Delta t \to 0} \frac{|\Delta\bar{v}|}{\Delta t} = \frac{|\bar{v}|}{|\bar{r}|} \lim_{\Delta t \to 0} \frac{|\Delta\bar{r}|}{\Delta t}$$

$$|\bar{a}| = \frac{|\bar{v}|}{|\bar{r}|} \cdot |\bar{v}| = \frac{|\bar{v}|^2}{|\bar{r}|} \Rightarrow \boxed{a = \frac{v^2}{r}}$$

# Period and speed

$$\underbrace{2\pi r}_{\text{circumference}} = \underbrace{vT}_{\substack{\text{distance}\\ \text{travelled in } T}} \Rightarrow \boxed{T = \frac{2\pi r}{v}}$$

Define $\omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \lim_{\Delta t \to 0} \frac{\Delta s}{r\Delta t}$




Note for $\Delta t \to 0$, $\Delta\theta \to 0$ and $\Delta s \to \Delta r$

$$\Rightarrow \omega = \frac{1}{r} \lim_{\Delta t \to 0} \frac{\Delta r}{\Delta t} = \frac{1}{r} v \quad \text{or} \quad \boxed{v = \omega r} \Rightarrow \boxed{T = \frac{2\pi}{\omega}}$$

and $\boxed{a_c = \frac{v^2}{r} = \omega^2 r}$