Physics 101H

General Physics 1 – Honors

Lecture 5 – 9/8/22

1D motion

Summary

Topics

Today: vectors [chapter 2]

- Products

Kinematics in 1D [chapter 3]

- Describing motion in 1D
- Position, velocity, acceleration

Announcements

Yesterday: **Problem set 1 assigned**

In on dimensional kinematics, vectors are distinguished from scalars by having a direction that is denoted by positive or negative values.

The boiling point of nitrogen is -195.795 C.
Is temperature a vector or a scalar quantity?

Cartesian and other coordinates

In **two dimensions**

- **Cartesian coordinates** defined by (x,y) axes
- **Polar coordinates** defined by magnitude and angle (r,θ)

In **three dimensions**

- **Cartesian coordinates** defined by (x,y,z) axes
- **Spherical coordinates** defined by magnitude and two angles (r,θ,φ)
- **Cylindrical coordinates** defined by radius, one angle and one height (r,θ,z)

Products of vectors

Dot product (or **scalar product**) of two vectors gives a number

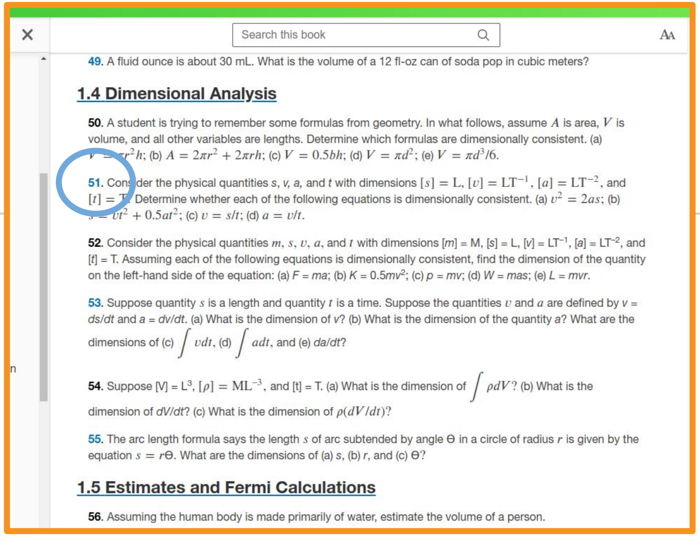
Cross product (**vector product**) of two vectors can be thought of as giving another vector

Want more practice?

Check out the following problems in the textbook

In Chapter 2:

- Conceptual questions: 5, 13, 21
- Problems: 25, 31, 45, 53, 65, **91**



Search this book

49. A fluid ounce is about 30 mL. What is the volume of a 12 fl-oz can of soda pop in cubic meters?

1.4 Dimensional Analysis

50. A student is trying to remember some formulas from geometry. In what follows, assume A is area, V is volume, and all other variables are lengths. Determine which formulas are dimensionally consistent. (a) $V = \pi r^2 h$; (b) $A = 2\pi r^2 + 2\pi rh$; (c) $V = 0.5bh$; (d) $V = \pi d^2$; (e) $V = \pi d^3/6$.

51. Consider the physical quantities s, v, a, and t with dimensions $[s] = \mathrm{L}$, $[v] = \mathrm{LT}^{-1}$, $[a] = \mathrm{LT}^{-2}$, and $[t] = \mathrm{T}$. Determine whether each of the following equations is dimensionally consistent. (a) $v^2 = 2as$; (b) $s = vt^2 + 0.5at^2$; (c) $v = s/t$; (d) $a = v/t$.

52. Consider the physical quantities m, s, v, a, and t with dimensions $[m] = \mathrm{M}$, $[s] = \mathrm{L}$, $[v] = \mathrm{LT}^{-1}$, $[a] = \mathrm{LT}^{-2}$, and $[t] = \mathrm{T}$. Assuming each of the following equations is dimensionally consistent, find the dimension of the quantity on the left-hand side of the equation: (a) $F = ma$; (b) $K = 0.5mv^2$; (c) $p = mv$; (d) $W = mas$; (e) $L = mvr$.

53. Suppose quantity s is a length and quantity t is a time. Suppose the quantities v and a are defined by $v = ds/dt$ and $a = dv/dt$. (a) What is the dimension of v? (b) What is the dimension of the quantity a? What are the dimensions of (c) $\int v\,dt$, (d) $\int a\,dt$, and (e) da/dt?

54. Suppose $[V] = \mathrm{L}^3$, $[\rho] = \mathrm{ML}^{-3}$, and $[t] = \mathrm{T}$. (a) What is the dimension of $\int \rho\,dV$? (b) What is the dimension of dV/dt? (c) What is the dimension of $\rho(dV/dt)$?

55. The arc length formula says the length s of arc subtended by angle Θ in a circle of radius r is given by the equation $s = r\Theta$. What are the dimensions of (a) s, (b) r, and (c) Θ?

1.5 Estimates and Fermi Calculations

56. Assuming the human body is made primarily of water, estimate the volume of a person.

Note that answers are provided for questions with blue numbers (odd numbered)

Click on the number to be taken to the answer.

But make sure you at least **try** the problem first!

Kinematics in 1D

Chapter 3

Kinematics

Kinematics is the description of the motion of objects

Dynamics is the explanation of the cause of the motion of objects

We will start with kinematics in one dimension (1D)

We are chiefly concerned with the **position** of an object **as a function of time**

Question: What is the difference between distance and displacement?

Position, displacement, and distance

Position – vector describing where object is relative to some reference frame

Displacement – change in position [a vector]

Distance travelled – total length of journey [a scalar]

Velocity

Velocity – a vector describing the **rate of change** of **position**

Speed – the magnitude of the velocity [a scalar]

Instantaneous velocity – the time derivative of position

Acceleration

Acceleration – a vector describing the **rate of change** of **velocity**

Instantaneous acceleration – the time derivative of velocity

Example: The position of a particle as a function of time is given by $x(t) = \sin(t)$. Find the instantaneous velocity and acceleration as a function of time. Determine the average velocity and average acceleration over the period $t = 0$ to $t = 2\pi$.

Free fall

Question: Why do we have to specify "near the Earth's surface" in our definition of free fall?

Constant acceleration is a special case that we will revisit regularly

Free fall (falling under the influence of gravity and no other forces) is a special case of the special case of constant acceleration

In the absence of air resistance, all objects dropped near the Earth's surface fall with a constant acceleration (g), directed towards the Earth

When is Problem Set 1 due? And how do you submit it?

Summary

Vectors [chapter 2]

- Scalar products determine the projection of one vector on another
- Vector products produce a vector perpendicular to the original two vectors

Kinematics in 1D [chapter 3]

- Quantities characterising motion can be scalars or vectors
- Distance and speed are scalars
- Displacement, velocity, and acceleration are vectors
- Instantaneous and average velocity and acceleration are different!

Tomorrow: kinematics in 2D

- Describing motion in 2D
- Generalising position, velocity, and acceleration to 2D

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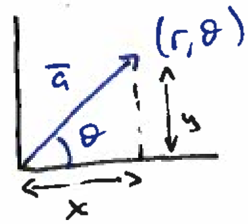
Lecture 5 9/8/22

Polar coordinates



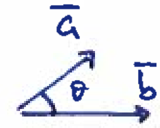
$r = |\bar{a}| = \sqrt{x^2+y^2}$

$\tan\theta = y/x$

$x = r\cos\theta \quad y = r\sin\theta$

Scalar product (slide 4)

Denoted $\bar{a}\cdot\bar{b}$ and given by $\bar{a}\cdot\bar{b} = |\bar{a}||\bar{b}|\cos\theta$



In coordinate/component form $\bar{a}\cdot\bar{b} = a_x b_x + a_y b_y + a_z b_z$

$\bar{a} = (a_x\hat{i} + a_y\hat{j} + a_z\hat{k})$

Maps two vectors to a scalar $\quad \bar{b} = (b_x\hat{i} + b_y\hat{j} + b_z\hat{k})$

If $\bar{a}$ is perpendicular to $\bar{b}$ then $\bar{a}\cdot\bar{b} = |\bar{a}||\bar{b}|\cos 90^\circ = 0$

Gives us the component of $\bar{a}$ pointed in the direction of $\bar{b}$

Vector product (slide 5)

Denoted $\bar{a}\times\bar{b}$ – gives another vector with magnitude

$|\bar{a}\times\bar{b}| = |\bar{a}||\bar{b}|\sin\theta$

and direction perpendicular to both

Not commutative $\quad \bar{a}\times\bar{b} \neq \bar{b}\times\bar{a}$

N.B. cross product of parallel vectors is zero!

$\bar{a}\times\bar{a} = |\bar{a}||\bar{a}|\sin 0 = 0$

And $\hat{i}\times\hat{j} = \hat{k}, \quad \hat{j}\times\hat{k} = \hat{i}, \quad \hat{k}\times\hat{i} = \hat{j}$

Vector product (slide 6)

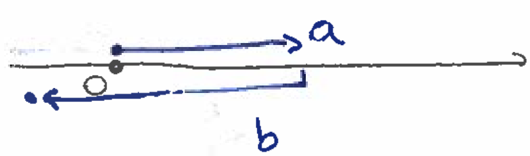
$$\bar{a} \times \bar{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix} = \hat{i}(a_y b_z - a_z b_y) - \hat{j}(a_x b_z - a_z b_x) + \hat{k}(a_x b_y - a_y b_x)$$

Position, displacement and distance (slide 9)

Positive or negative => vector

· change in position $\Delta x = x_f - x_i$

Distance is not a vector, just a magnitude (scalar)



displacement = $a - b$ (< 0)
distance = $|a - b|$ (> 0)
distance travelled = $|a| + |b|$ (> 0)

Velocity (slide 10)

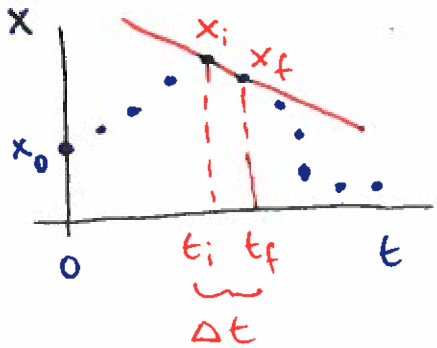
Measure an object's position at two times, subtract, then divide by time elapsed

$$V_{ave} = \frac{x_f - x_i}{t_f - t_i} = \frac{\Delta x}{\Delta t}$$

Velocity can be positive or negative => vector   units m/s
speed is magnitude of velocity => scalar

Instantaneous velocity (slide 11)

Let's say that we can measure the position of an object over shorter and shorter time intervals



Let Δt get really, really small (infinitesimal) : $\Delta t \to 0$

$$v_{inst} = \lim_{\Delta t \to 0} \frac{x_f - x_i}{t_f - t_i} = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$$

In other words $v_{inst} = \frac{dx}{dt}$!

↑ time derivative of the position function $x(t)$
slope of the position vs time graph

Acceleration (slide 12)

$$a_{ave} = \frac{v_f - v_i}{t_f - t_i} = \frac{\Delta v}{\Delta t}$$ ← units m/s^2 vector !

Let $\Delta t \to 0$ $\quad a_{inst} = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}$ ← time derivative of velocity

But $v_{inst} = \frac{dx}{dt} \Rightarrow a_{inst} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2}$ ← second time derivative of position

Acceleration example

$$v(t) = \frac{d}{dt} x(t) = \frac{d}{dt} \sin(t) = \cos(t)$$

$$a(t) = \frac{d}{dt} v(t) = \frac{d}{dt} \cos(t) = -\sin(t)$$

$$v_{ave} = \frac{x_f - x_i}{t_f - t_i} = \frac{\sin(0) - \sin(2\pi)}{2\pi - 0} = 0$$

$$a_{ave} = \frac{v_f - v_i}{t_f - t_i} = \frac{\cos(0) - \cos(2\pi)}{2\pi - 0} = 0$$

Constant acceleration (slide 14)

$$a = \frac{dv}{dt} = \text{constant} \quad (= a_0)$$

Note that $v(t) = a_0 t + v_0$ is exactly the right kind of function! ← $\frac{d}{dt} v(t) = a_0$ ✓

This can be obtained by integrating our equation $\frac{dv}{dt} = a_0$

$$\int \frac{dv}{dt}\, dt = \int a_0\, dt = a_0 \int dt \Rightarrow \int dv = a_0 \int dt$$

$$\int \frac{dv}{dt}\, dt = \int dv$$

$$v = a_0 t + v_0$$

↑ v_0: initial velocity or constant of integration

Constant acceleration (slide 15)

$$v(t) = \frac{dx}{dt} = a_0 t + v_0 \quad \Rightarrow \quad x(t) = \frac{1}{2} a_0 t^2 + v_0 t + x_0$$

(a_0 ↑ constant acceleration)

($x(t)$ ↑ obtained by integrating)

$$\int \frac{dx}{dt}\, dt = a_0 \int t\, dt + v_0 \int dt$$

If $a_0 = 0$ then

$$x(t) = v_0 t + x_0$$

$$v(t) = v_0$$

Free fall (slide 16)

$|a_0| = g$ ← acceleration due to gravity $g = 9.81\ m/s^2$ towards the Earth

$$a = -g$$

$$v(t) = -gt + v_0$$

$$y(t) = -\frac{1}{2} gt + v_0 t + y_0$$