Physics 101H

General Physics 1 – Honors

Lecture 49 – 12/07/22

Quantum Mechanics

Summary

See https://openstax.org/books/college-physics-2e/ chapter 29 and https://openstax.org/details/books/university-physics-volume-3 chapters 6 & 7

Topics

Monday: Special relativity II

- Length contraction
- Lorentz transformations
- Relativistic momentum and energy

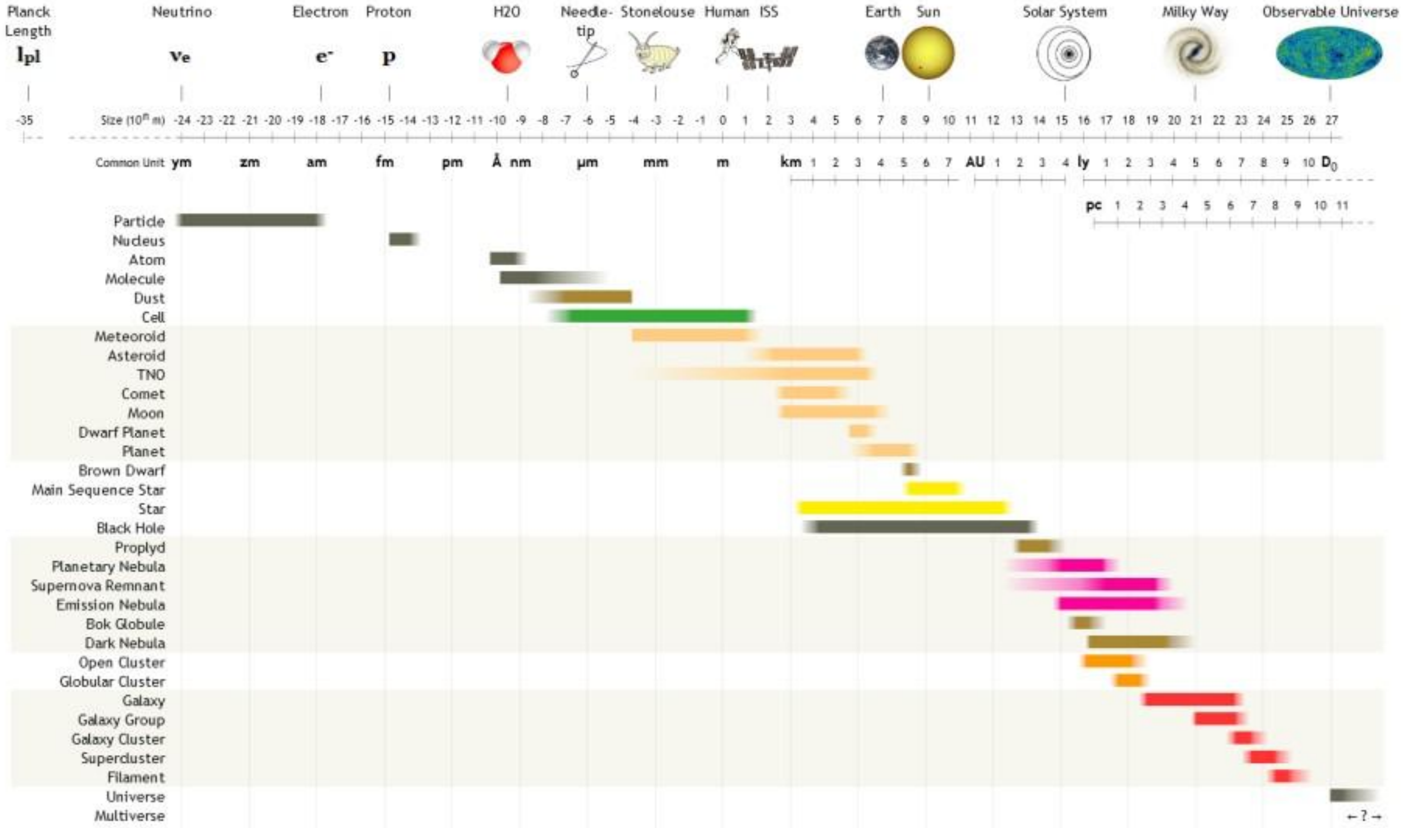
Today: Quantum Mechanics

- Heisenberg uncertainty equation
- Particles and waves
- Schrödinger equation

Announcements

Wednesday December 6:	**Problem Set 9 due**
Tuesday December 13:	**Final exam 9 am to 12 midday**

Scales



Colors/Categories

- Microscopic Objects/Black Holes/Universe
- Macroscopic Substellar Objects
- Stars
- Nebulae/Diffuse Objects
- Star Clusters
- Galaxy-/Groupings
- Dust/Transitional Objects
- Biomatter

Conversion Table

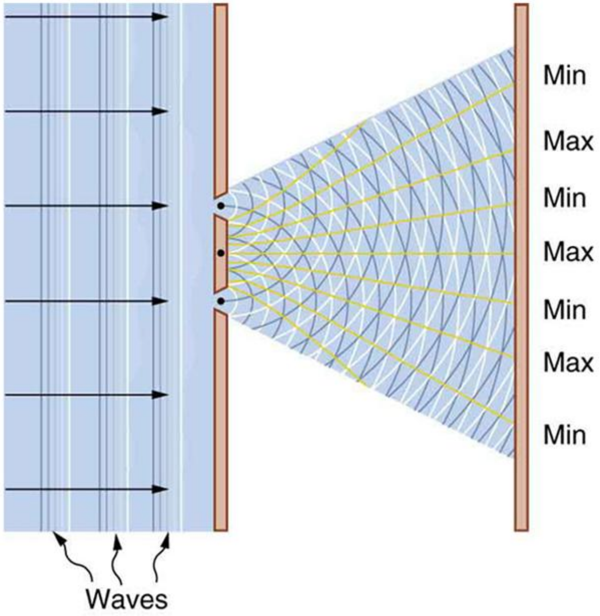
	l_{pl}	m	AU	ly	pc	D_0
Planck Length l_{pl}	1	1.6×10^{-35}	1.1×10^{-46}	1.7×10^{-51}	5.2×10^{-52}	1.2×10^{-61}
Meter m	6.3×10^{34}	1	6.7×10^{-12}	1.1×10^{-16}	3.2×10^{-17}	7.7×10^{-27}
Astronomical Unit AU	9.4×10^{45}	1.5×10^{11}	1	1.6×10^{-5}	4.8×10^{-6}	1.2×10^{-15}
Light Year ly	5.9×10^{50}	9.5×10^{15}	6.3×10^{4}	1	0.3	7.3×10^{-11}
Parsec pc	1.9×10^{51}	3.1×10^{16}	2.1×10^{5}	3.26	1	2.4×10^{-10}
Observable Universe D_0	8.1×10^{60}	1.3×10^{26}	8.7×10^{14}	1.4×10^{10}	4.2×10^{9}	1

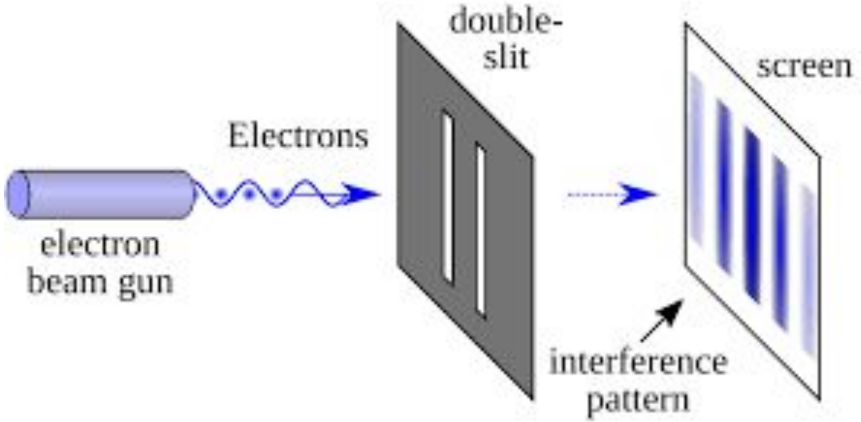
SI-Prefixes

m-	milli	10^{-3}	**k-**	kilo	10^{3}	**h-**	hecto	10^{2}
µ-	micro	10^{-6}	**M-**	mega	10^{6}	**da-**	deka	10^{1}
n-	nano	10^{-9}	**G-**	giga	10^{9}	**d-**	deci	10^{-1}
p-	pico	10^{-12}	**T-**	tera	10^{12}	**c-**	centi	10^{-2}
f-	femto	10^{-15}	**P-**	peta	10^{15}			
a-	atto	10^{-18}	**E-**	exa	10^{18}			
z-	zepto	10^{-21}	**Z-**	zetta	10^{21}			
y-	yocto	10^{-24}	**Y-**	yotta	10^{24}			

Particle-wave duality

The properties of quantum systems **depend** on **how we observe them**

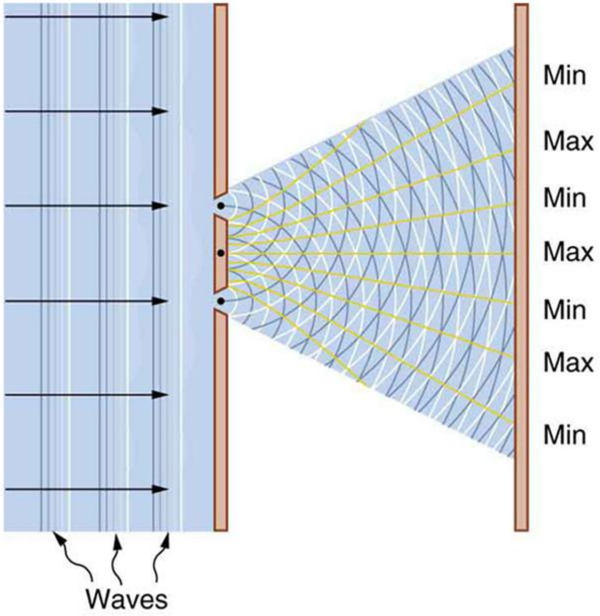


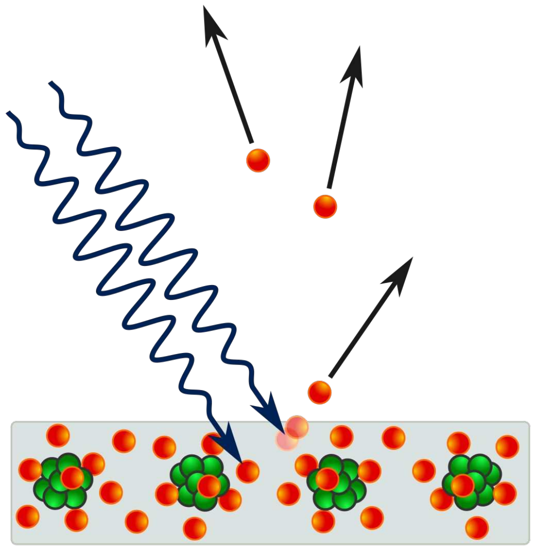


Example: Calculate your de Broglie wavelength, assuming a speed of 1 m/s.

Wave-particle duality

The properties of quantum systems **depend** on **how we observe them**





Heisenberg Uncertainty Principle

Wave nature of particles leads to inherent and irreducible uncertainties

Particle trajectories and destinations cannot be predicted precisely, even though each particle goes to a definite place

With repeated measurements, we can build up a **probability distribution**

Trying to measure a particle's position can change its momentum!

Schrödinger equation

Describes how the **wavefunction** of a system evolves in time

Wavefunction interpreted as

probability of finding a particle at a specific position at a specific time

Can only be solved exactly for a limited number of systems

- Wikipedia lists 29
  https://en.wikipedia.org/wiki/List_of_quantum-mechanical_systems_with_analytical_solutions

Summary

Measurement affects the system being observed

Wave-particle duality occurs for quantum systems

- electrons & photons sometimes behave as waves and sometimes as particles

Energy is quantised

Simultaneous measurements of different quantities is not always possible

Wavefunctions (squared) give probability of finding a particle at a specific position

- obtained by solving the Schrödinger equation

Quick quiz

Instructions: This quiz is for your own learning. There are three questions and each question has two columns. Write your own solution, without reference to your notes, the textbook, or your neighbour, **in the first column**. Once you have tried to answer all the questions, discuss the questions with a neighbour and fill in any incomplete answers **in the second column**. Keep your sheet for future reference.

Summary

Topics

Today: Quantum mechanics

- Heisenberg uncertainty equation
- Particles and waves
- Schrödinger equation

Tomorrow and Friday: Review

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PHYSICS 101 - HONORS

Lecture 49   12/7/22

Wave-particle duality (slide 4)

Wave nature of light captured by the de Broglie wavelength

$$\lambda = \frac{h}{p}$$

- λ ← wavelength
- h ← Planck's constant 6.63×10^{-34} Js ← the clear indicator that we are dealing with quantum physics!
- p ← momentum

- e^- and γ show double slit interference, even one at a time
- trying to measure position or grating leads to single slit pattern

For ordinary matter this wavelength is negligible!

Example (slide 5)

$$\lambda_{ME} = \frac{6.63 \times 10^{-3}}{72.1} = 9.208 \times 10^{-36} \text{ m}$$

↑ a proton is $\sim 10^{-15}$ m !!

Particle - wave duality (slide 6)

The photoelectric effect shows that light can behave like a particle!

Light has energy

$$E = hf$$

energy of light

Planck's constant

frequency of light

- there is a threshold frequency below which no electrons are emitted
- electrons are emitted as soon as a photon strikes the material
- number of electrons emitted is proportional to the intensity of the light and nothing else
- maximum kinetic energy of the electrons is independent of the light intensity

Quantisation of energy also explains black body radiation - the spectrum of wavelengths emitted by a perfect radiator (an object of a given temperature that emits all wavelengths of radiation).

Heisenberg uncertainty principle (slide 7)

The wave nature of particles means there is an inherent uncertainty in a particle's position

$$\Delta x \simeq \lambda$$

To measure this position, we must interact with the particle, for example by hitting it with a photon. But this will change the momentum of the particle, but there is uncertainty in the momentum change too!

$$\Delta p \simeq \frac{h}{\lambda}$$

Decreasing the wavelength to measure x does not help - it increases the uncertainty in the momentum

$$\Rightarrow \Delta x \, \Delta p \simeq \lambda \cdot \frac{h}{\lambda} = h$$

Heisenberg showed more precisely that

$$\boxed{\Delta x \, \Delta p \geq \frac{h}{4\pi}}$$

↖ Heisenberg uncertainty principle

Another example is

$$\Delta E \, \Delta t \geqslant \frac{h}{4\pi}$$

Schrödinger equation (slide 8)

Quantum systems are described most generally by state vectors (elements of a Hilbert space), but if we are primarily interested in the position of something as a function, we can capture this through the wavefunction $\Psi(\bar{x},t)$.

The wavefunction satisfies the Schrödinger equation

$$\left[-\frac{\hbar^2}{2m}\nabla^2 + V(\bar{x})\right]\Psi(\bar{x},t) = i\hbar\frac{\partial}{\partial t}\Psi(\bar{x},t)$$

- $V(\bar{x})$: potential energy of the system
- $\nabla^2 \equiv \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$
- $\hbar \equiv \frac{h}{2\pi}$
- $i\hbar\frac{\partial}{\partial t}\Psi(\bar{x},t)$: describes how $\Psi(\bar{x},t)$ evolves in time

Interpretation of $\Psi(\bar{x},t)$:

- $|\Psi(\bar{x},t)|^2$ is the probability density of finding a particle at position $\bar{x}$ at time t
- $\int |\Psi(\bar{x},t)|^2 d^3\bar{x} = 1$ ← this says you must find it somewhere when you consider all positions

Solving the Schrödinger equation is hard!