Physics 101H

General Physics 1 – Honors

Lecture 48 – 12/05/22

Special Relativity II

Summary

See https://openstax.org/books/college-physics-2e/ chapter 28 and https://openstax.org/details/books/university-physics-volume-3 chapter 5

Topics

Friday: Special relativity

- Galilean relativity
- Special relativity
- Time dilation

Today: Special relativity II

- Length contraction
- Lorentz transformations
- Relativistic momentum and energy

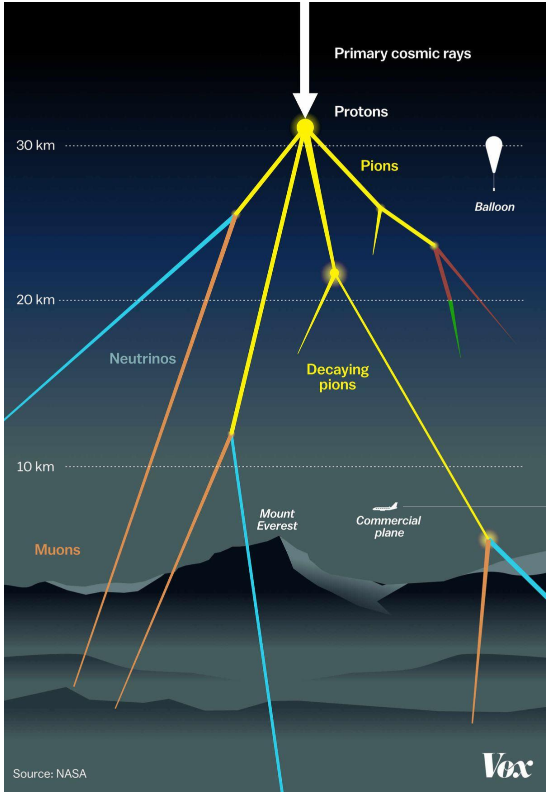
Announcements

Wednesday December 6: **Problem Set 9 due**

Tuesday December 13: **Final exam 9 am to 12 midday**

Cosmic muons

Muons are created by high energy cosmic rays interacting with nuclei in the upper atmosphere



Lorentz transformations

Time dilation leads to another strange phenomenon – **length contraction**

Taken together, time dilation and length contraction tell us that both time and space coordinates must change – according to a **Lorentz transformation** – when we transform from one reference frame to another moving relative to the first

Now that both time and space transform under Lorentz transformations, and this means that velocities **don't add**

Relativistic momentum and energy

Special relativity insists that the laws of physics don't change under Lorentz transformations, that is, when we move from one inertial reference frame to another.

But our old definition of momentum is changed under a Lorentz transformation

Therefore we need to redefine momentum and energy!

Example: Show that relativistic definition of momentum leads to (probably) the most famous formula in physics.

Example: Show that the usual nonrelativistic kinetic energy is the low-speed limiting case of the relativistic kinetic energy.

Summary

Physics is the same in all inertial reference frames

- speed of light in a vacuum is constant in all reference frames

Time dilates and lengths contract in frames moving relative to each other

- explains why we can observe muons created in the upper atmosphere

Lorentz transformations govern the relationship between different frames

Mass is another form of energy

Summary

Topics

Today: Special relativity II

- Length contraction
- Lorentz transformations
- Relativistic momentum and energy

Wednesday: Quantum mechanics

- Heisenberg uncertainty equation
- Particles and waves
- Schrödinger equation

Announcements

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PHYSICS 101 - HONORS

Lecture 48 12/5/22

Muons (slide 3)

Muons (μ) are created at 10 - 20 km above the Earth's surface

μ lifetime is 2.2 μs (2.2×10^{-6} s)

μ typically travel at $v \simeq 0.9997c$

$$\Rightarrow \quad d = v \Delta t_p$$
$$= 0.9997 \cdot 3 \times 10^{8} \cdot 2.2 \times 10^{-6}$$
$$\simeq 660 \text{ m} \quad \text{or} \quad 0.66 \text{ km}$$

← so how can we detect them at the Earth's surface?

The answer: time dilation!

According to the μ, it travels for $\Delta t_p = 2.2\,\mu s$.

But according to __me__ (moving relative to the μ), it travels for

$$\Delta t = \gamma \Delta t_p$$
$$= \frac{1}{\sqrt{1 - v^2/c^2}} \Delta t_p = \frac{1}{\sqrt{1 - 0.9997^2}} \cdot 2.2 \times 10^{-6}$$
$$\simeq 8.4 \times 10^{-5} \text{ s}$$

Therfore, according to me, the μ travels

$$d = v \Delta t$$
$$= 0.9997 \cdot 3 \times 10^8 \cdot 8.4 \times 10^{-5}$$
$$\simeq 25{,}000 \text{ m} \quad (!)$$

↖ so it can definitely reach the Earth's surface

How can this make sense, because the clock ticks at the "right" speed for the μ in <u>its</u> frame ?!

The μ "thinks" it has travelled

$$L_\mu = v \Delta t_p$$
$$= v \frac{\Delta t}{\gamma}$$
$$= \frac{1}{\gamma}(v \Delta t)$$

whereas I think it has travelled

$$L_p = v \Delta t$$
$$= \gamma \cdot L_\mu$$

← $L_p > L_\mu$ because $\gamma > 1$

⇒ According to the μ, it has travelled less distance!

↑ length contraction

Lorentz transformations (slide 4)

Clearly both space and time must change (or at least their coordinates do) when we move from one reference frame to another moving relative to the first

$$x' = \gamma(x - vt)$$
$$y' = y$$
$$z' = z$$
$$t' = \gamma\left(t - \frac{v}{c^2}x\right)$$

} Lorentz transformations relate two frames, moving in the x direction relative to each other

↖ Compare Galilean transformations

$$x_B = v_{AB}t + x_A$$
$$y_B = y_A$$
$$z_B = z_A$$
$$t_B = t_A = t$$

Speeds and velocities no longer add

In one frame $u = \frac{dx}{dt}$, but in the other $u' = \frac{dx'}{dt'}$

Now $x' = \gamma(x - vt)$ (v: relative speed) $\Rightarrow dx' = \gamma(dx - v\,dt)$

$t' = \gamma\left(t - \frac{v}{c^2}x\right) \Rightarrow dt' = \gamma\left(dt - \frac{v}{c^2}dx\right)$

$$\Rightarrow u' = \frac{dx'}{dt'} = \frac{\gamma(dx - v\,dt)}{\gamma\left(dt - \frac{v}{c^2}dx\right)} = \frac{dt\left(\frac{dx}{dt} - v\right)}{dt\left(1 - \frac{v}{c^2}\frac{dx}{dt}\right)} = \frac{u - v}{1 - \frac{uv}{c^2}}$$

compare to $\bar{u}_B = \bar{u}_{AB} + \bar{u}_A = \bar{v} + \bar{u}_A$

Relativistic momentum and energy (slide 5)

It turns out we need to define

$\bar{p} = \gamma m \bar{v}$ ← not $\bar{p} = m\bar{v}$

and

$E_K = (\gamma - 1) mc^2$ ← not $E_K = \frac{1}{2} mv^2$

The energy can be expressed as

$$E_K = \gamma mc^2 - mc^2$$

or

$$\gamma mc^2 = E_K + mc^2 \equiv E$$

γmc^2 ↑ total energy

E_K ↑ energy due to motion

mc^2 ↖ rest energy ⇒ mass is a form of energy

Momentum example (slide 6)

Start from $p = \gamma m v$

$$\Rightarrow p^2c^2 = (\gamma m v)^2 c^2$$

$$= \frac{1}{1 - v^2/c^2} \cdot m^2 v^2 c^2$$

$$= \frac{c^2}{c^2 - v^2} m^2 v^2 c^2$$

$$= \frac{v^2}{c^2 - v^2} m^2 c^4 = \frac{v^2}{c^2 - v^2} (mc^2)^2$$

Now let's add $(mc^2)^2$

$$\Rightarrow p^2c^2 + (mc^2)^2 = \frac{v^2}{c^2-v^2}(mc^2)^2 + (mc^2)^2$$

$$= \left(\frac{v^2}{c^2-v^2} + 1\right)(mc^2)^2$$

$$= \left(\frac{v^2 + c^2 - v^2}{c^2-v^2}\right)(mc^2)^2$$

$$= \frac{c^2}{c^2-v^2}(mc^2)^2$$

$$= \frac{1}{1-v^2/c^2}(mc^2)^2$$

$$= \gamma^2(mc^2)^2$$

$$= E^2$$

$$\Rightarrow \boxed{E^2 = m^2c^4 + p^2c^2}$$

and for $v = 0 \Rightarrow p = 0$

$$\Rightarrow \boxed{E = mc^2} \quad \#$$

Kinetic energy example (slide 7)

$$E_K = (\gamma - 1)mc^2$$

$$= \left(\frac{1}{\sqrt{1-v^2/c^2}} - 1\right)mc^2$$

$$= \left[\left(1 - v^2/c^2\right)^{-1/2} - 1\right]mc^2$$

Use the Taylor series

$$(1+x)^n \simeq 1 + nx + \frac{(n-1)n}{2}x^2 + \dots$$

$$\Rightarrow \left(1 - v^2/c^2\right)^{-1/2} \simeq 1 - \frac{1}{2}\left(-v^2/c^2\right) + \dots$$

$$= 1 + \frac{v^2}{2c^2} + \dots$$

$$\Rightarrow E_K \simeq \left[\left(1 + v^2/2c^2 + \dots\right) - 1\right] mc^2$$

$$= \frac{v^2}{2c^2} mc^2$$

$$= \frac{mv^2}{2}$$

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