Physics 101H

General Physics 1 – Honors

Lecture 44 – 11/28/22

Standing Waves

Summary

Topics

Last week: Reflection and transmission [chapter 16]

- Solutions to the wave equation
- Reflection and transmission
- Wave energy

Today: Superposition [chapter 16]

- Superposition
- Standing waves
- Boundary conditions

Announcements

Wednesday November 30:	**Problem Set 9 posted**
Wednesday December 6:	**Problem Set 9 due**

Forced oscillations

Remember **damped oscillations**?

And **forced oscillations**?

Quick quiz

Instructions: This quiz is for your own learning. There are four questions and each question has two columns. Write your own solution, without reference to your notes, the textbook, or your neighbour, **in the first column**. Once you have tried to answer all the questions, discuss the questions with a neighbour and fill in any incomplete answers **in the second column**. Keep your sheet for future reference.

Superposition

We can **add** (and **subtract**) waves! This is the **principle of superposition**

If two or more waves move through a medium, the resultant wave function at any point is the algebraic sum of the values of the individual wave functions.

Leads to **interference**

- **constructive** interference
- **destructive** interference

And to **standing waves**

Boundary conditions

Infinite media can support any (and all) possible standing waves

But **boundary conditions** impose restrictions on the possible standing waves, causing

- normal modes
- quantisation

The **fundamental frequency** (lowest normal mode) dictated by the system's properties

- higher allowed frequencies are called **harmonics**

Multiple choice

Instructions: Consider the following question. After you have had a chance to think, I will ask you to raise your hands to indicate your answer.

Question: A person swings on a swing. When the person sits still, the swing oscillates back and forth at its natural frequency. If, instead the person stands on the swing, the natural frequency:

(a) increases

(b) decreases

(c) stays the same

Summary

Topics

Today: Superposition [chapter 16]

- Superposition
- Standing waves
- Boundary conditions

Wednesday: Sound [chapter 17]

- Sound
- Intensity
- Sound perception

Announcements

Wednesday November 30:	**Problem Set 9 posted**
Wednesday December 6:	**Problem Set 9 due**

PHYSICS 101 - HONORS

Lecture 44 11/28/22

Superposition (slide 5)

Constructive interference – two waves superpose to cause displacement in the same direction

Destructive interference – two waves with displacement in opposite directions superpose to give zero (or reduced) displacement

We can apply this to our known travelling wave solutions

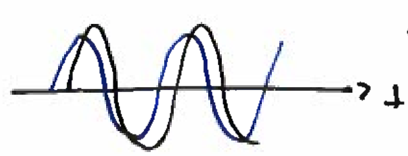
[recall $y(x,t) = A \sin[k(x-vt)]$]

Let $y_1 = A \sin[k(x-vt)]$ $\qquad y_2 = A \sin[k(x-vt)+\phi]$

a phase shift

Then consider

$$y_{tot} = y_1 + y_2$$



Recall that

$$\sin a + \sin b = 2\cos\left(\frac{a-b}{2}\right)\sin\left(\frac{a+b}{2}\right)$$

$$\Rightarrow y_{tot} = A\sin a + A\sin b$$

$a = k(x-vt)$ $b = k(x-vt) + \phi$

$$= 2A\cos\left\{\frac{k(x-vt) - [k(x-vt)+\phi]}{2}\right\}$$

$$\cdot \sin\left\{\frac{k(x-vt) + [k(x-vt)+\phi]}{2}\right\}$$

$$= \underbrace{2A\cos\left(\frac{\phi}{2}\right)}_{\substack{\text{new amplitude} \\ \text{[does not depend on } x \text{ or } t\text{!]}}}\ \underbrace{\sin\left[k(x-vt) + \phi/2\right]}_{\substack{\text{new travelling wave with same} \\ \text{wavelength and frequency, but a different phase}}}$$

So if $\phi = 0$ or $2\pi, 4\pi, 6\pi \ldots$ then $\cos\left(\frac{\phi}{2}\right) = 1$

and we have

$$y_{tot} = 2A\sin\left[k(x-vt)\right]$$

← • constructive interference
• same wave, twice the amplitude

If $\phi = \pi$ or $3\pi, 5\pi, \ldots$ then $\cos\left(\frac{\phi}{2}\right) = 0$

and we have

$$y_{tot} = 0$$

← • complete destructive interference

Standing waves can be formed from the superposition of left- and right-moving waves

$y_1 = A \sin[k(x-vt)]$ $y_2 = A \sin[k(x+vt)]$

travelling right   travelling left

In this case

$$y_{tot} = y_1 + y_2$$

$$= 2A \cos\left\{\frac{k(x-vt) - k(x+vt)}{2}\right\} \cdot \sin\left\{\frac{k(x-vt) + k(x+vt)}{2}\right\}$$

$$= 2A \cos(-kvt) \sin(kx)$$

$\cos(-x) = \cos(x)$
$\omega = kv$

$$= 2A \underbrace{\cos(\omega t)}_{\text{separate } t \text{ dependence}} \underbrace{\sin(kx)}_{\text{separate } x \text{ dependence}}$$

this is a standing wave

Note that: at $kx = 0, \pi, 2\pi, 3\pi, \ldots$ we have $y_{tot} = 0$

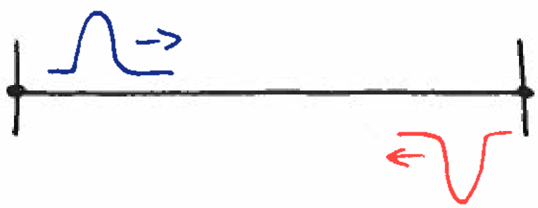
$\Rightarrow$ these are nodes

at $kx = \pi/2, 3\pi/2, 5\pi/2, \ldots$ we have $y_{tot} = 2A$ (at $t=0$, say)

$\Rightarrow$ these are antinodes

Boundary conditions (slide 5)

Let's consider a wave on a string fixed at both ends



Reflected waves are inverted!

$\Rightarrow$ travels back and forth with same λ, f, A

$\Rightarrow$ standing waves created!

We know standing waves are given by

$$y(x,t) = 2A\cos(\omega t)\sin(kx)$$

But now we have <u>boundary conditions</u>

At $x = 0$ we <u>must</u> have $y(0,t) = 0$

and similarly at $x = L$ we have $y(L,t) = 0$

$$\Rightarrow y(L,t) = 2A\cos(\omega t)\sin(kL) = 0 \quad \text{for any } t$$

Thus we must have $\sin(kL) = 0$ or $kL = 0, \pi, 2\pi, 3\pi, \ldots$

$$\Rightarrow k = 0, \frac{\pi}{L}, \frac{2\pi}{L}, \frac{3\pi}{L}, \ldots \quad \text{or} \quad \boxed{k_n = \frac{n\pi}{L}}$$

n is an <u>integer</u>

But $k = \frac{2\pi}{\lambda} \Rightarrow \frac{n\pi}{L} = \frac{2\pi}{\lambda}$ or $\lambda = \frac{2L}{n}$

only certain wavelengths are allowed!!

Also $v = \lambda f = \sqrt{\frac{T}{\mu}} \Rightarrow f = \frac{n}{2L}\sqrt{\frac{T}{\mu}}$

the waves are quantised

$n = 1$ is the fundamental frequency $n \geq 2$ are harmonics