Physics 101H

General Physics 1 – Honors

Lecture 43 – 11/21/22

Reflection and Transmission

Summary

Topics

Friday: Waves [chapter 16]
- Forced oscillations
- Types of waves
- Wave equation

Today: Reflection and transmission [chapter 16]
- Solutions to the wave equation
- Reflection and transmission
- Wave energy

Monday: Superposition [chapter 16]
- Superposition
- Standing waves
- Boundary conditions

Announcements

Wednesday November 23: Problem Set 8 due

PHYSICS 101 - HONORS

Lecture 43 11/21/22

Solutions to the wave equation

Recall the wave equation $\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$

$v = \sqrt{\frac{T}{\mu}}$ linear mass density $\mu = \frac{dm}{dl}$

This equation tells us that the solution must have (up to a factor $1/v^2$) the same result when we differentiate it twice with respect to x and to t

Ansatz : $y(x,t) = A \sin(kx - \omega t)$

recall - "guess"

$$\Rightarrow \frac{\partial y}{\partial x} = Ak \cos(kx - \omega t)$$

$$\frac{\partial^2 y}{\partial x^2} = -Ak^2 \sin(kx - \omega t)$$

$$\frac{\partial y}{\partial t} = A(-\omega)\cos(kx - \omega t)$$

$$\frac{\partial^2 y}{\partial t^2} = -A(-\omega)^2 \sin(kx - \omega t)$$

Check : $-Ak^2 \sin(kx - \omega t) = -\frac{A\omega^2}{v^2}\sin(kx - \omega t)$

$$\Rightarrow A\sin(kx - \omega t) = \frac{\omega^2}{v^2k^2} A\sin(kx - \omega t)$$

For this equation to be consistent, we must have

$$\frac{\omega^2}{v^2 k^2} = 1 \quad \Rightarrow \quad \omega^2 = v^2 k^2 \quad \text{or} \quad \omega = vk$$

note $2\pi f = \frac{2\pi}{\lambda} \cdot v$ or $v = f\lambda$ ↑velocity

wave number → $k = \frac{2\pi}{\lambda}$ ← wavelength

Thus:

- wave equation

$$\boxed{\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 y}{\partial t^2}}$$

- wave function

$$y(x,t) = A \sin\left[k(x - vt)\right]$$

with $\omega = vk$, $\omega = 2\pi f$

$v = f\lambda$, $k = 2\pi/\lambda$

Consider $t = 0$:

$$y(x_0, t=0) = A \sin\left[k(x_0 - 0)\right] = A \sin(k x_0)$$

Consider $t = t_1$ — define $x_1 = x_0 - vt_1$

$$y(x_0, t=t_1) = A \sin\left[k(x_0 - vt_1)\right] = A \sin(k x_1)$$

These have the same <u>waveforms</u>, but at shifted positions, with $\Delta x = x_1 - x_0 = (x_0 - vt_1) - x_0 = -vt_1$

$\Rightarrow$ v is indeed the wave velocity!

This solution is a <u>travelling wave</u>, moving to the <u>right</u>

Left-moving travelling wave would be $A \sin\left[k(x + vt)\right]$

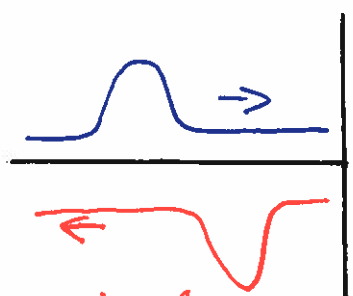
Reflection and transmission

When waves encounter a change in the medium they change their behaviour - they are **reflected** or **transmitted**

↑ bounce backwards

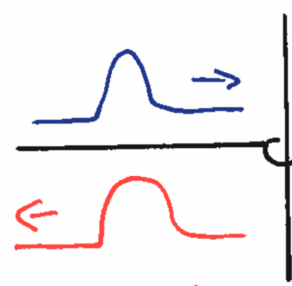
← carry on, but perhaps with different properties

Reflection



inverted

immovable support



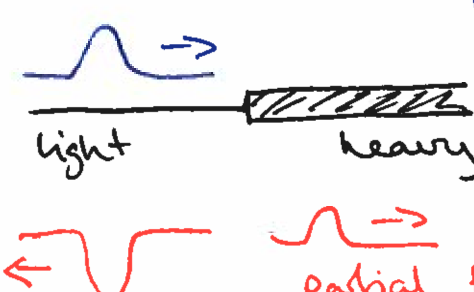
movable support

not inverted

string exerts upward force on wall ⇒ wall exerts equal and opposite downward force on the string

Transmission

μ changes ⇒ v changes



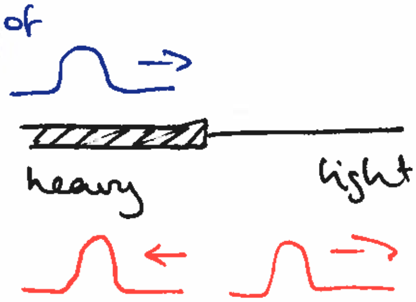
light heavy

partial transmission

partial reflection

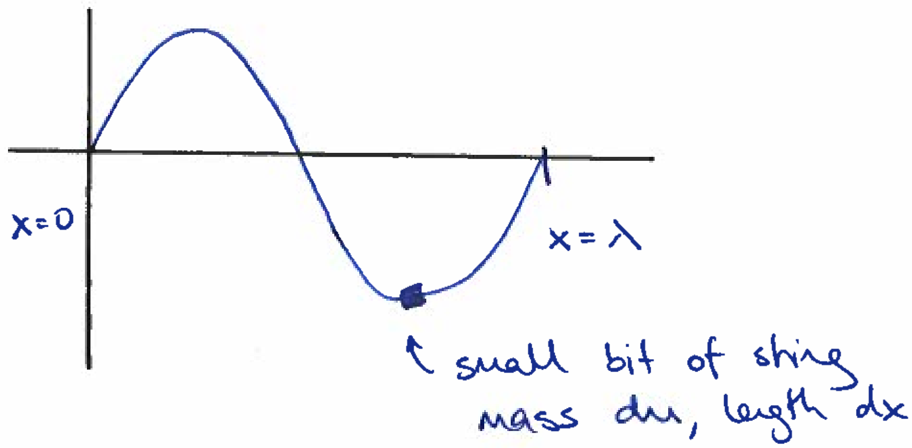
part of the energy of the wave is transmitted

rest is reflected with smaller amplitude



heavy light

Wave energy



Consider small piece of string

kinetic energy : $dE_K = \frac{1}{2} dm\, v_y^2$

speed of string not wave

$$= \frac{1}{2} dm \left(\frac{\partial y}{\partial t}\right)^2 \qquad y(x,t) = A \sin(k(x-vt))$$

$$= \frac{1}{2} dm \left(-kvA\cos(k(x-vt))\right)^2$$

$$= \frac{1}{2} dm\, k^2 v^2 A^2 \cos^2(k(x-vt))$$

Recall we introduced $\mu = \frac{dm}{dl} \Rightarrow dm = \mu\, dl = \mu\, dx$

So total kinetic energy is

$$E_K = \int_0^\lambda dE_K \Big|_{t=0} = \int_0^\lambda \frac{1}{2}\mu k^2 v^2 A^2 \cos^2(k(x-vt))\, dx \Big|_{t=0}$$

$$= \frac{\mu k^2 v^2 A^2}{2} \int_0^\lambda \cos^2(kx)\, dx = \frac{\mu k^2 v^2 A^2}{2} \int_0^\lambda \left[\frac{1}{2} + \frac{1}{2}\cos 2kx\right] dx$$

$$\Rightarrow \quad E_K = \frac{\mu k^2 v^2 A^2}{2}\left[\frac{1}{2}x\Big|_0^\lambda + \frac{1}{2}\frac{1}{2k}\sin(2kx)\Big|_0^\lambda\right]$$

$$= \frac{\mu k^2 v^2 A^2}{2}\left[\frac{\lambda}{2} - 0 + \frac{1}{4k}\sin(2k\lambda) - 0\right]$$

But $\sin(2k\lambda) = \sin\left(2\cdot\frac{2\pi}{\lambda}\cdot\lambda\right) = \sin(4\pi) = 0$!

$$\Rightarrow \quad E_K = \frac{\mu k^2 v^2 A^2 \lambda}{4}$$

We can apply the same logic to the potential energy to find

$$E_p = \frac{1}{4}\mu k^2 v^2 A^2 \lambda$$

$$\Rightarrow \quad E_{tot} = E_K + E_p = \frac{1}{4}\mu k^2 v^2 A^2 \lambda + \frac{1}{4}\mu k^2 v^2 A^2 \lambda$$

$$= \frac{1}{2}\mu k^2 v^2 A^2 \lambda \quad \text{or} \quad \boxed{\frac{1}{2}\mu \omega^2 A^2 \lambda}$$

The power is $P = \frac{E_{tot}}{T} = \frac{1}{2}\mu\omega^2 A^2 \frac{\lambda}{T} = \frac{1}{2}\mu\omega^2 A^2 v$

period

speed of wave