Physics 101H

General Physics 1 – Honors

Lecture 40 – 11/16/22

Simple Harmonic Motion

Summary

Topics

Monday: Fluid dynamics [chapter 14]

- Continuity equation
- Bernoulli equation

Today: Oscillations [chapter 15]

- Mass on a spring
- Simple harmonic motion

Announcements

Today: **Problem Set 8 posted**
Wednesday November 23: **Problem Set 8 due**

Quick quiz

Instructions: This quiz is for your own learning. There are three questions and each question has two columns. Write your own solution, without reference to your notes, the textbook, or your neighbour, **in the first column**. Once you have tried to answer all the questions, discuss the questions with a neighbour and fill in any incomplete answers **in the second column**. Keep your sheet for future reference.

Mass on a spring

Oscillations are **periodic** motions of a system (they repeat with constant **period**)

- For example, going around a circle at a constant speed

Everything that we need to know about oscillations is actually captured by the motion of a mass on a spring – mathematically they are entirely equivalent

The more general name for this behaviour is **simple harmonic motion**

Simple harmonic motion

Key quantities:

- Amplitude
- Frequency
- Period
- Phase
- Velocity
- Maximum velocity
- Acceleration
- Maximum acceleration

Problem Set in Colab

For Problem Set 8

1. Navigate to https://colab.research.google.com/drive/1rlQNhtKPlmuIEVW69whPHjD1bx7w6NPv?usp=sharing
2. Make a copy and save it to your own Google Drive
3. Use your own copy to make changes until you are happy with your answers
4. Upload a pdf of your notebook as your submission to Blackboard and along with a link to your colab document **as a comment** (you must make it accessible to anyone with a link)

Python

Python [see www.python.org] is a widely-used free scripting language

- Web development
- Mathematical applications and data analysis
- General-use system scripts

Excellent introduction and tutorials at https://www.w3schools.com/python/default.asp

Commonly used with Jupyter notebooks [see https://jupyter.org/]

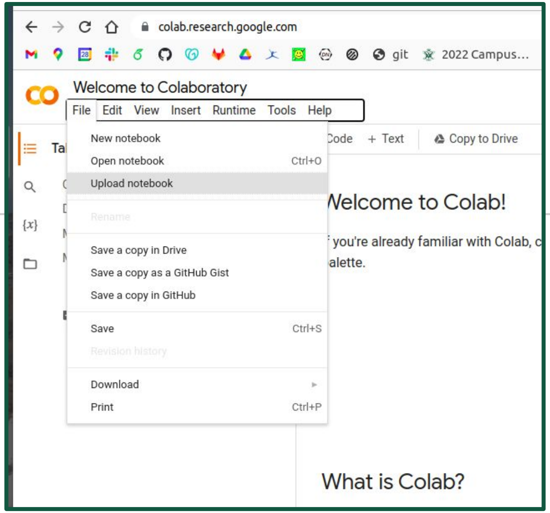
- Interactive environment
- Runs in a browser

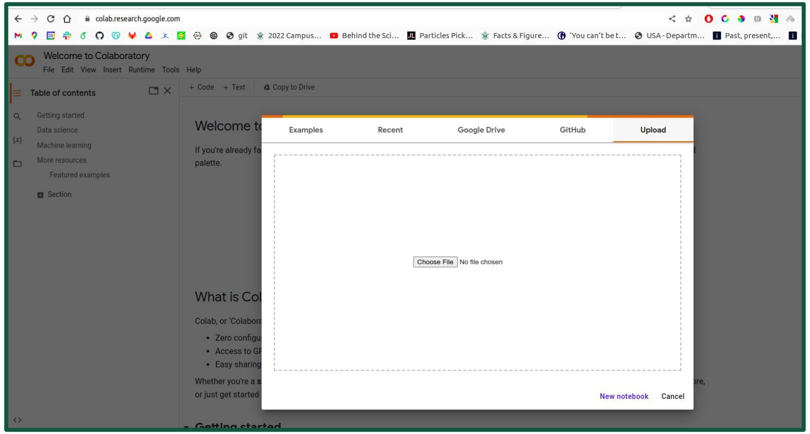
Easy to install (typically already installed on many systems), but there's no need!

Run in Google Colab [see https://colab.research.google.com/]

Colab

1. Open your browser
2. Navigate to https://colab.research.google.com/
3. Upload notebook to work on it
   a. Appears as a splash screen, or
   b. Navigate to “File > Upload notebook”
4. Execute cells by pressing the “play” button or hitting “Shift-Enter”
5. Enter “playground mode” if you want to make changes that won’t be saved





Air resistance in Colab

Python programs have a typical structure

1. Load modules (external routines that provide more functionality)

   [You can basically ignore this part for now]

2. Set up ("**declare**") **variables**

   [Generally important to pick useful names for things]

3. Manipulate variables by **calling functions**, modules or other routines

   [In other words, do stuff and calculate things]

4. Display results in some helpful format or form

   [Basically – make and display plots]

Summary

Topics

Today: Oscillations [chapter 15]

- Mass on a spring
- Simple harmonic motion

Tomorrow: Oscillations [chapter 15]

- SHM and uniform circular motion
- Damped oscillations

Announcements

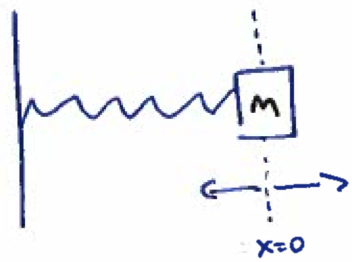
Today: **Problem Set 8 posted**
Wednesday November 23: **Problem Set 8 due**

PHYSICS 101 - HONORS

Lecture 40 11/16/22

Mass on a spring (slide 4)

Let's look at a spring lying on a frictionless table



The only force acting on the mass is the spring force $\bar{F}_s = -k\bar{x}$

Working in 1D, and applying Newton's second law:

$$F_{net} = ma \Rightarrow -kx = ma$$

But $a = \frac{d^2x}{dt^2} \; (= \ddot{x})$

$$\Rightarrow a = -\frac{k}{m}x \quad \text{or} \quad \ddot{x} = -\frac{k}{m}x \quad \left(\frac{d^2x}{dt^2} = -\frac{k}{m}x\right)$$

Introducing $\omega^2 \equiv \frac{k}{m}$ we can write this as

$$\ddot{x} = -\omega^2 x$$

← second order ordinary differential equation

ie two time derivatives

To solve this, let's guess

$$x(t) = e^{\alpha t}$$

$$\Rightarrow \dot{x} = \frac{dx}{dt} = \alpha e^{\alpha t}$$

$$\Rightarrow \ddot{x} = \frac{d^2x}{dt^2} = \alpha^2 e^{\alpha t}$$

Plugging this into our ODE gives

$$\underbrace{\alpha^2 e^{\alpha t}}_{\ddot{x}} = -\omega^2 \underbrace{e^{\alpha t}}_{x}$$

$$\Rightarrow -\omega^2 = \alpha^2 \quad \text{or} \quad \omega^2 = -\alpha^2$$

$$\Rightarrow \omega = \pm i\alpha \quad \leftarrow \quad i^2 = -1$$

So $x(t) = e^{\pm i\omega t}$ satisfies $\ddot{x} = -\omega^2 x$!

But what does it mean to have a complex exponential?

We can understand this through the Taylor series

$$e^x = 1 + x + \frac{1}{2!}x^2 + \frac{1}{3!}x^3 + \frac{1}{4}x^4 + \ldots$$

$$\Rightarrow e^{i\omega t} = 1 + i\omega t + \frac{1}{2!}(i\omega t)^2 + \frac{1}{3!}(i\omega t)^3 + \frac{1}{4!}(i\omega t)^4 + \ldots$$

$$= 1 + i\omega t - \frac{1}{2!}\omega^2 t^2 - \frac{i}{3!}\omega^3 t^3 + \frac{1}{4!}\omega^4 t^4$$

$$= 1 - \frac{1}{2!}\omega^2 t^2 + \frac{1}{4!}\omega^4 t^4 + \ldots$$

$$+ i\omega t - \frac{i}{3!}\omega^3 t^3 + \ldots$$

} separate odd and even powers of (ωt)

$$= 1 - \frac{1}{2!}\omega^2 t^2 + \frac{1}{4!}\omega^4 t^4 + \ldots + i\left(\omega t - \frac{1}{3!}\omega^3 t^3 + \ldots\right)$$

But remember that

$$\sin(x) = x - \frac{x^3}{3!} + \dots$$

$$\cos(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} + \dots$$

$$\Rightarrow \quad e^{i\omega t} = \cos(\omega t) + i\sin(\omega t)$$

Repeating this for $e^{-i\omega t}$ gives

$$e^{-i\omega t} = \cos(\omega t) - i\sin(\omega t)$$

We can rearrange these to be

$$\cos(\omega t) = \frac{1}{2}\left(e^{i\omega t} + e^{-i\omega t}\right)$$

$$\sin(\omega t) = \frac{1}{2i}\left(e^{i\omega t} - e^{-i\omega t}\right)$$

Complex exponentials are equivalent to cosines and sines !!

Let's try solutions $x(t) = \cos(\omega t)$ and $x(t) = \sin(\omega t)$

$$\Rightarrow \ddot{x} = -\omega^2\cos(\omega t) = -\omega^2 x \quad ✓$$

$x = \cos(\omega t)$
$\dot{x} = -\omega\sin(\omega t)$
$\ddot{x} = -\omega^2\cos(\omega t)$

$$\ddot{x} = -\omega^2\sin(\omega t) = -\omega^2 x \quad ✓$$

$x = \sin(\omega t)$
$\dot{x} = \omega\cos(\omega t)$
$\ddot{x} = -\omega^2\sin(\omega t)$

Most general solution is $x(t) = A\cos(\omega t + \phi)$

↖ check it for yourself !

Simple harmonic motion (slide 5)

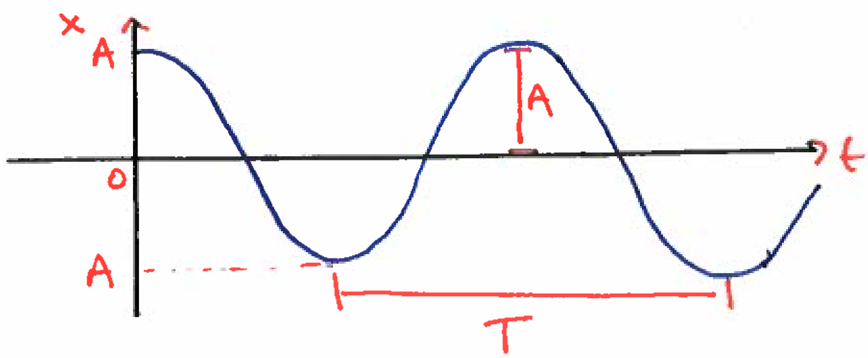
The general solution to

$\ddot{x} = -\omega^2 x$ ← this equation <u>defines</u> simple harmonic motion ↑ or simple harmonic oscillations

is

$x(t) = A\cos(\omega t + \phi)$

Solution can be drawn as



Amplitude – A – the minimum or maximum value of the position $x(t)$

Frequency – $f = \frac{\omega}{2\pi}$ (← angular frequency) – the number of oscillations per second

Period – $T = \frac{1}{f} = \frac{2\pi}{\omega}$ – time taken for one oscillation

Phase – ϕ – when the crest or trough occurs
(in the example plot $\phi = 0$ because $x = A$ at $t = 0$)

Velocity – $v = \frac{dx}{dt} = -A\omega \sin(\omega t + \phi)$ – varies periodically with time!

Maximum velocity – $|v_{max}| = A\omega = \sqrt{\frac{k}{m}}\, A$ – occurs when $\ddot{x} = 0$, equivalent to $\sin(\omega t + \phi) =$

Acceleration – $a = \frac{dv}{dt} = \frac{d^2x}{dt^2} = -A\omega^2 \cos(\omega t + \phi)$

Maximum acceleration – $|a_{max}| = A\omega^2$ – occurs when $\cos(\omega t + \phi) = 1$