Physics 101H

General Physics 1 – Honors

Lecture 34 – 11/4/22

Part II review

Midterm 2

Good News: No problem set assigned today!

Bad News: Second midterm will take place on **Wednesday November 9**!

You will have 45 minutes to complete the exam

- 3 multiple choice questions
- 2 handwritten solution problems

Bring paper and something(s) to write with! (Spare paper will be available)

Topics cover Chapters 1 to 11 and include:

- Vectors
- 1D and 2D kinematics
- Newton's laws of motion
- Conservation of energy
- Conservation of momentum and collisions
- Rotational motion
- Angular momentum and torque

You may prepare your own formula sheet – **two sides** of **letter paper (215.9 x 279.4 mm)**

You may bring a calculator, but phones, tablets and laptops are not allowed

Remember you are here to learn and understand the physics!

Topic overview

Work and energy conservation:

- Work and work-energy theorem
- Conservative forces
- Kinetic energy and potential energy
- Conservation of energy
- Power

Momentum and collisions:

- Conservation of momentum
- Elastic and inelastic collisions
- Collisions in 1D and 2D

Rotational motion:

- Rotational kinematics
- Relating angular and linear kinematics
- Moment of inertia
- Conservation of energy

Rotational dynamics:

- Torque
- Newton's second law for rotational motion
- Angular momentum
- Conservation of angular momentum
- Rolling motion

Example: Two marbles, one twice as heavy as the other, are dropped to the ground from the roof of a building. Just before hitting the ground, the heavier marble has kinetic energy

(a) equal to the lighter marble

(b) twice as much as the lighter marble

(c) half as much as the lighter marble

(d) four times as much as the lighter marble

Example: A block of mass m is attached to a ceiling by a spring with spring constant k and relaxed length l. Initially the spring is compressed to a length of $l/2$. If the block is released, at what distance below the ceiling will the block be brought to instantaneous rest by the spring?

Example: What is the work done over a distance L by a force that depends on the distance as $F(x) = a\,x + b\,x^3$?

Example: A line of N balls of mass m lie at rest on a frictionless table. The first ball is given a kick and acquires a speed v. It collides and sticks to the second ball, and the resulting blob collides and sticks to the third ball. This process continues until all balls have joined the blob. What is the resulting speed of the blob?

Example: A mass *m* moving to the east with speed v_0 collides elastically, but not head on, with a mass *2m* at rest. The smaller mass moves northward (perpendicular to the original direction of motion), what angle does the resulting velocity of the larger mass make with the east-west direction?

Example: The left end of a massless stick with length l is placed on the corner of a table. A point mass m is attached to the centre of the stick, which is initially held horizontal and then released. What is the normal force of the table on stick, immediately after release?

Example: A disc of mass m and radius r spins clockwise with angular speed w at a distance d to the right of a point P. What is the angular momentum of the disc, relative to the point P?

Example: An ice skater in training spins on the spot, initially with their arms outstretched and holding 5 kg dumbbells (one in each hand), with angular speed w_0. The skater then draws their hands in and holds the dumbbells close to their body. What is their new angular speed? Assume that the skater can be modelled as a cylinder of 70 kg and radius 0.15 m, and that their arms are 0.75 m long.

Studying for midterm 2

Good luck!

Studying for the midterm:

- Work through Problem Sets
- Work through examples from class and in the textbook

When working through problems (especially someone else's solution):

- Cover up the solution and try to work out the next step in the solution
- If you can't figure that out, uncover just the first step and then try to figure out the next steps
- Try to *self-explain*, that is - write down your thought process and what principles, concepts or equations are being applied at each step.

The Society of Physics Students offers free student tutoring **Thursday 6-8pm** in Small 122

https://www.wm.edu/as/physics/undergrad/bor-undergrad-resources/sps/index.php

Remember that you are here to learn and understand the physics!

[But also remember there are two methods for calculating your final grade]

PHYSICS 101 - HONORS

Lecture 34 11/4/22

> Warning: I cannot cover all question types! So make sure you look at the practice exam.

Key equations

W ·

Work and energy conservation:

- $W = \int \bar{F} \cdot d\bar{r}$
- $W = \Delta E_K = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$ — and $E_K = \frac{1}{2}mv^2$ — work-energy theorem for a single particle
- $W = \Delta E_p = \frac{k}{2}(x_2^2 - x_1^2)$ — and $E_p = \frac{k}{2}x^2$ — work done compressing a spring
- $W = \Delta E_g = -mg(y_2 - y_1)$ — and $E_g = mgh$ — work done by gravity going from y_1 to y_2
- $P = \frac{dW}{dt} = \bar{F} \cdot \bar{v}$
- $E^i_{TOT} = E^f_{TOT}$ — conservation of energy

Momentum and collisions

- $\bar{p} = m\bar{v}$
- $\bar{I} = \int \bar{F}\, dt = \Delta \bar{p}$
- $\bar{F} = \frac{d\bar{p}}{dt}$ — Newton II

- $\bar{p}_i = \bar{p}_f$ conservation of momentum
- $\bar{r}_{cm} = \frac{1}{M}\int \bar{r}\, dm$
- $\bar{v}_{cm} = \frac{d}{dt}\bar{r}_{cm}$ and $\bar{a}_{cm} = \frac{d\bar{v}_{cm}}{dt}$
- $\Delta v = u \ln\left(\frac{M_i}{M_f}\right)$ rocket equation

Rotational motion

cf.

- $\bar{\omega} = \frac{d\bar{\theta}}{dt}$ — $\bar{v} = \frac{d\bar{x}}{dt}$
- $\bar{\alpha} = \frac{d\bar{\omega}}{dt} = \frac{d^2\bar{\theta}}{dt^2}$ — $\bar{a} = \frac{d\bar{v}}{dt} = \frac{d^2\bar{x}}{dt^2}$
- $\bar{\omega} = \bar{\alpha}t + \bar{\omega}_0$ — $\bar{v} = \bar{a}t + \bar{v}_0$
- $\bar{\theta} = \frac{\bar{\alpha}t^2}{2} + \bar{\omega}_0 t + \bar{\theta}_0$ — $\bar{x} = \frac{\bar{a}t^2}{2} + \bar{v}_0 t + \bar{x}_0$
- $I = \int r^2\, dm$
- $E_K^{rot} = \frac{1}{2} I\omega^2$ — $E_K^{lin} = \frac{1}{2}mv^2$
- $\bar{L} = \bar{r}\times\bar{p} = I\bar{\omega}$ — $\bar{p} = m\bar{v}$
- $\bar{\tau} = \frac{d\bar{L}}{dt} = \bar{r}\times\bar{F}$ — $\bar{F} = \frac{d\bar{p}}{dt}$
- $\bar{\tau}_{net} = I\bar{\alpha}$ — $\bar{F}_{net} = m\bar{a}$

$s = r\theta$

$v_t = \omega r$

$\alpha = a_t/r$

$a_c = v_t^2/r$

Kinetic energy example (slide 4)

Both masses fall with same acceleration

$\Rightarrow$ both have the same speed

$$\Rightarrow E_K^{heavy} = \frac{1}{2}(2m)v^2 = mv^2$$

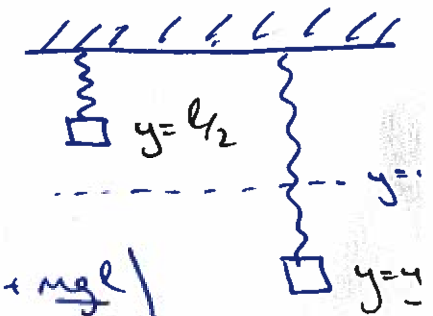
$$E_K^{light} = \frac{1}{2}mv^2 = \frac{1}{2}E_K^{heavy} \Rightarrow \underline{\underline{(b)}}$$

Spring example (slide 5)

Conservation of energy: $E_s^i + E_g^i + E_K^i = E_s^f + E_g^f + E_K^f$

$$\Rightarrow \frac{1}{2}k\left(\frac{\ell}{2}\right)^2 + mg\left(\frac{\ell}{2}\right) + 0 = \frac{1}{2}ky^2 + mgy + 0$$

$$\Rightarrow \frac{1}{2}ky^2 + mgy - \left(\frac{k\ell^2}{8} + \frac{mg\ell}{2}\right) = 0$$



$$y^2 + \frac{2mg}{k}y - \alpha = 0 \quad \nwarrow \quad \alpha = \frac{2}{k}\left(\frac{k\ell^2}{8} + \frac{mg\ell}{2}\right)$$

$$y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-\frac{2mg}{k} \pm \sqrt{\frac{4m^2g^2}{k^2} + 4\alpha}}{2}$$

$$= -\frac{mg}{k} \pm \frac{2}{k}\sqrt{m^2g^2 + \frac{\ell^2}{4} + \frac{mg\ell}{k}}$$

or: $\frac{1}{2}ky^2 - \frac{k\ell^2}{8} = \frac{mg\ell}{2} - mgy$

$$\Rightarrow \frac{1}{2}k\left(y - \frac{\ell}{2}\right)\left(y + \frac{\ell}{2}\right) = -mg\left(y - \ell/2\right)$$

$$\Rightarrow \frac{1}{2}k\left(y + \ell/2\right) = -mg$$

$$y + \ell/2 = -\frac{2mg}{k}$$

$$y = -\frac{2mg}{k} - \ell/2$$

distance below ceiling is

$$d = \ell + \left(\frac{2mg}{k} + \ell/2\right) = \frac{2mg}{k} + 3\ell/2 \Leftarrow$$

Force example (slide 6)

Work done is given by

$$W = \int \bar{F} \cdot d\bar{r} \quad \text{in general}$$

In one dimension we can write this as

$$\bar{F}(\bar{r}) = F(x)\,\hat{x} \quad \text{and} \quad d\bar{r} = dx\,\hat{x}$$

$$\Rightarrow \bar{F}(\bar{r}) \cdot d\bar{r} = F(x)\,\hat{x} \cdot dx\,\hat{x} = F(x)\,dx\;\hat{x}\cdot\hat{x} = F(x)\,dx$$

Thus in one dimension

$$W = \int F(x)\,dx$$

$$= \int_0^L (ax + bx^3)\,dx$$

$$= a\int_0^L x\,dx + b\int_0^L x^3\,dx$$

$$= a\left.\frac{x^2}{2}\right|_0^L + b\left.\frac{x^4}{4}\right|$$

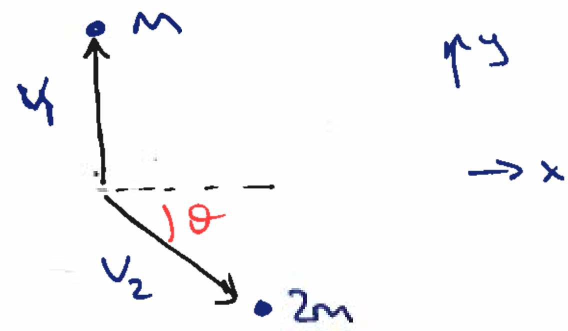
$$= \frac{aL^2}{2} + \frac{bL^4}{4}$$

Blob example (slide 7)

Use conservation of momentum!

$$mv = (Nm)v_2 \Rightarrow v_2 = v/N$$

2D collision example (slide 8)



Conservation of momentum:

y direction : $0 = mv_1 - 2m v_2 \sin\theta \Rightarrow v_2 \sin\theta = \frac{v_1}{2}$

x direction : $mv_0 = 0 + 2m v_2 \cos\theta \Rightarrow v_2 \cos\theta = \frac{v_0}{2}$

Conservation of energy:

$$\frac{1}{2}mv_0^2 = \frac{1}{2}mv_1^2 + \frac{2m}{2}v_2^2$$

$$= \frac{1}{2}mv_1^2 + m\left(v_{2x}^2 + v_{2y}^2\right)$$

$$= \frac{1}{2}mv_1^2 + m\left(\frac{v_0^2}{4} + \frac{v_1^2}{4}\right)$$

$$\Rightarrow \frac{1}{2}mv_0^2 - \frac{1}{4}mv_0^2 = \frac{1}{2}mv_1^2 + \frac{1}{4}mv_1^2$$

$$\Rightarrow \quad \frac{1}{4} v_0^2 = \frac{3}{4} v_1^2 \quad \text{or} \quad v_1^2 = \frac{1}{3} v_0^2 \Rightarrow v_1 = \frac{v_0}{\sqrt{3}}$$

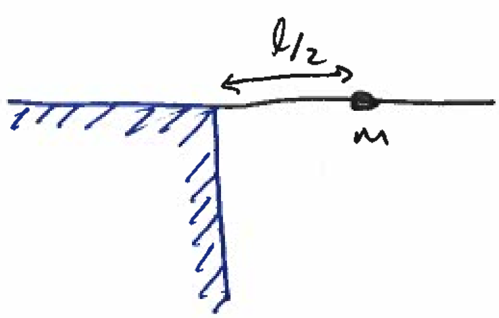
Now combine with momentum conservation

$$v_2 \sin\theta = \frac{v_1}{2} = \frac{v_0}{2\sqrt{3}}$$

$$v_2 \cos\theta = \frac{v_0}{2}$$

$$\Rightarrow \quad \tan\theta = \frac{v_0}{2\sqrt{3}} \Big/ \frac{v_0}{2} = \frac{1}{\sqrt{3}} \Rightarrow \underline{\theta = 30^\circ}$$

<u>Stick-table example</u> (slide 9)



Note I around the centre of the stick is zero!

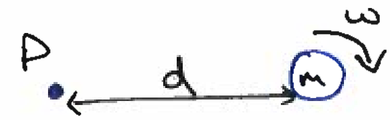
$\Rightarrow$ around the centre we have

$$\tau = I\alpha = 0$$

The only torque around the centre is due to the normal force $\Rightarrow \tau_{net} = \tau_{table} = 0 \Rightarrow \underline{N = 0}$

Angular momentum example (slide 10)

The **total** angular momentum is



$$L_{TOT} = M \bar{r}_{cm} \times \bar{v}_{cm} + L_{Acm}$$

($M \bar{r}_{cm} \times \bar{v}_{cm}$: angular momentum **of** cm; L_{Acm}: angular momentum **around** cm)

$$= 0 + I\bar{\omega}$$

$$= \frac{Mr^2}{2}\omega \quad \text{directed down}$$

Ice skater example (slide 11)

No external torques => angular momentum conserved

$$I_i \omega_0 = I_f \omega_f \quad \Rightarrow \quad \frac{\omega_f}{\omega_0} = \frac{I_i}{I_f}$$

$$I_i = I_{cyl} + I^{i}_{dumbbells}$$

$$= \frac{1}{2}Mr^2 + 2 \cdot M_{dbell}\, r^2_{dbell}$$

$$= \frac{1}{2} \cdot 70 \cdot 0.15^2 + 2 \cdot 5 \cdot 0.75^2 = 6.41 \text{ kg m}^2$$

$$I_f = I_{cyl} + I^{f}_{dumbbells}$$

$$= \frac{1}{2}Mr^2 + 2 \cdot M_{dbell} \cdot r^2 \qquad r_{dbell} = r$$

$$= \frac{1}{2} \cdot 70 \cdot 0.15^2 + 2 \cdot 5 \cdot 0.15^2 = 1.01 \text{ kg m}^2$$

$$\Rightarrow \omega_f = \frac{I_i}{I_f}\omega_0 = \frac{6.41}{1.01}\omega_0 = \underline{6.3\,\omega_0}$$