Physics 101H

General Physics 1 – Honors

Lecture 32 – 11/2/22

Kepler's Laws

Summary

Topics

Monday: Gravity [chapter 13]
- Newton’s laws of gravitation
- Gravitational fields and potential energy

Today: Kepler’s laws [chapter 13]
- Planetary motion and orbits
- Kepler’s laws

Announcements

Today:	**No problem set** **Practice exam posted**
Wednesday November 9:	**Midterm 2**

Planetary and celestial motion

Astronomy is one of the founding branches of physics

- Tends to be treated as distinct these days

Observational astronomy known since 1800 BC (at least)

- Cultures across the world have observed the stars and planets and used models to predict astronomical events

Modern Western understanding typically dated to around 1700s

- Encoded in Kepler's laws

Kepler’s laws

Formulated based on observational data (primarily by Tycho Brahe)

Derivable from Newton’s laws of motion and of gravity

1. Planets move in **elliptical orbits** with the Sun at one focus
2. Line between the Sun and a planet sweeps out **equal areas in equal times**
3. Square of the orbital period of a planet is proportional to the cube of the **semimajor axis** of the orbit

Practice in pairs

Instructions: Discuss the following question with a neighbour. Your answers will not be graded; your discussion is for your own learning.

Question: Two satellites, *A* and *B*, of the same mass orbit the Earth in concentric orbits. The distance of satellite *B* from the Earth's centre is twice the distance of *A* from Earth's centre. What is the ratio of the tangential speed of *B* to that of *A*?

(a) $\frac{1}{2}$ (b) $1/\sqrt{2}$ (c) 1 (d) $\sqrt{2}$ (e) 2

NEXT WEEK:
THE SECOND MIDTERM IS ON WEDNESDAY NOVEMBER 9

Midterm 2

Good News: No problem set assigned today!

Bad News: Second midterm will take place on **Wednesday November 9**!

You will have 45 minutes to complete the exam

- 3 multiple choice questions
- 2 handwritten solution problems

Bring paper and something(s) to write with! (Spare paper will be available)

Topics cover Chapters 1 to 11 and include:

- Vectors
- 1D and 2D kinematics
- Newton's laws of motion
- Conservation of energy
- Conservation of momentum and collisions
- Rotational motion
- Angular momentum and torque

You may prepare your own formula sheet – **two sides** of **letter paper (215.9 x 279.4 mm)**

You may bring a calculator, but phones, tablets and laptops are not allowed

Remember you are here to learn and understand the physics!

Studying for midterm 2

Studying for the midterm:

- Work through Problem Sets
- Work through examples from class and in the textbook

When working through problems (especially someone else's solution):

- Cover up the solution and try to work out the next step in the solution
- If you can't figure that out, uncover just the first step and then try to figure out the next steps
- Try to *self-explain*, that is - write down your thought process and what principles, concepts or equations are being applied at each step.

The Society of Physics Students offers free student tutoring **Thursday 6-8pm** in Small 122

https://www.wm.edu/as/physics/undergrad/bor-undergrad-resources/sps/index.php

Remember that you are here to learn and understand the physics!

[But also remember there are two methods for calculating your final grade]

Summary

Topics

Today: Kepler's laws [chapter 13]

- Planetary motion and orbits
- Kepler's laws

Wednesday: Escape velocity [chapter 13]

- Conservation of energy
- Escape velocity
- General relativity (briefly)

Announcements

Today:	**No problem set** **Practice exam posted**
Wednesday November 9:	**Midterm 2**

PHYSICS 101 - HONORS

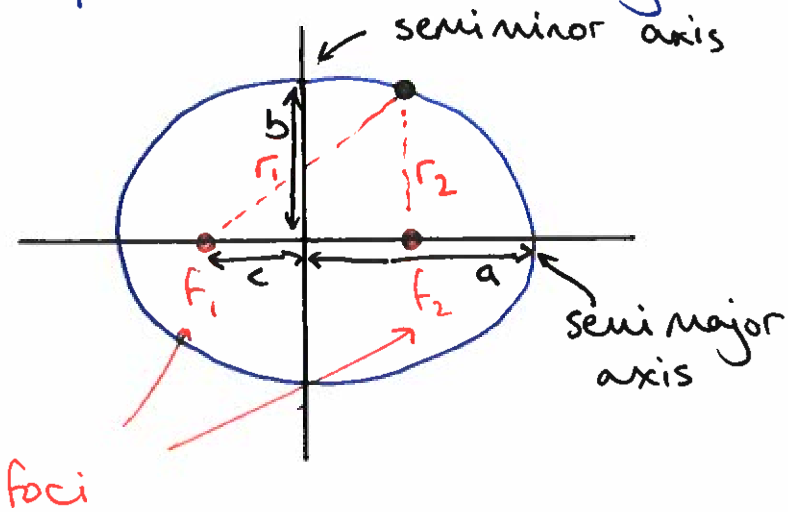
Lecture 32 11/2/22

Kepler's laws (slide 4)

First law

Ellipses are defined by $r_1 + r_2 = \text{constant}$

or $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$

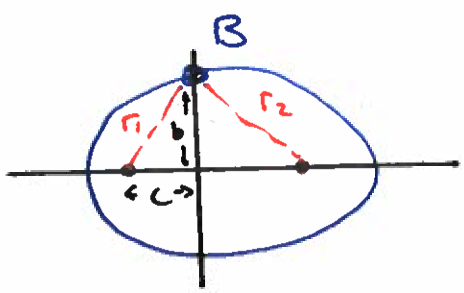


Define eccentricity

$e = \dfrac{c}{a}$ $e = 0$ circle

$e \to 1$ very squished

Consider point B

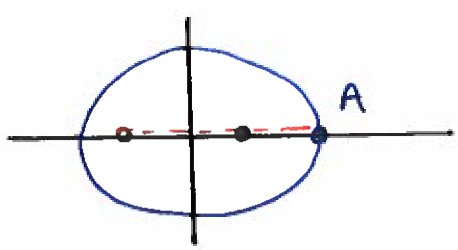


$r_1 = \sqrt{c^2 + b^2} = r_2$

$\Rightarrow r_1 + r_2 = 2r_1 = \text{constant}$

or $2\sqrt{c^2 + b^2} = \text{constant}$

Consider point A



$r_1 = a + c$

$r_2 = a - c$

$r_1 + r_2 = a + c + a - c = 2a$
$= \text{constant}$

$\Rightarrow 2\sqrt{c^2 + b^2} = 2a$ or $\underline{a^2 = b^2 + c^2}$

Second law

Force due to gravity points along $\hat{r}$

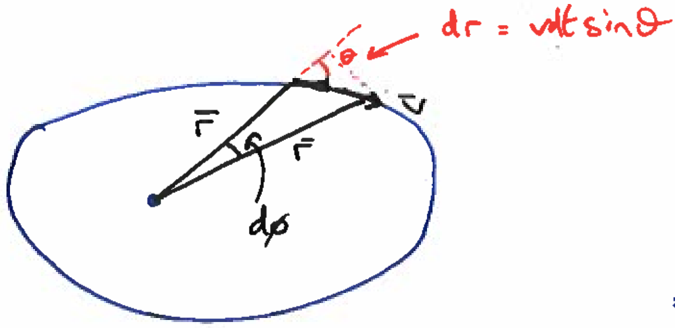
$\Rightarrow \bar{r} \times \bar{F}_g = 0$

Angular momentum is therefore conserved ← remember $\frac{d\bar{L}}{dt} = \bar{\tau}_{net}$

$\Rightarrow \bar{L}_{planet} = \bar{r} \times \bar{p} = \bar{r} \times M_{planet} \bar{v}$

or $|\bar{L}_{planet}| = M_{planet} |\bar{r}||\bar{v}| \sin\theta \quad (L_p = M_p r v \sin\theta)$

Now let's look at the area swept out by $\bar{r}$



area of the triangle is

$dA = \frac{1}{2} r \underbrace{v dt \sin\theta}_{\text{height}}$ (r = base)

$\Rightarrow \frac{dA}{dt} = \frac{1}{2} \underbrace{r v \sin\theta}_{= \frac{L_{planet}}{M_{planet}}}$

Thus $\frac{dA}{dt} = \frac{L_{planet}}{2M_{planet}}$ = a constant

$$\boxed{\frac{dA}{dt} = \text{constant}}$$

Third law

In the approximation that $e = 0$ (circular orbit)

$$F_c = F_g \quad \text{or} \quad m a_c = \frac{GmM}{r^2} \quad \Rightarrow \quad \frac{mv^2}{r} = \frac{GmM}{r^2}$$

(F_c ← centripetal force; F_g ← gravitational force)

$$\Rightarrow \frac{v^2}{r} = \frac{GM}{r^2} \quad \Rightarrow \quad v^2 = \frac{GMr}{r^2} = \frac{GM}{r}$$

But we know that $v = \frac{2\pi r}{T}$

$$\Rightarrow \left(\frac{2\pi r}{T}\right)^2 = \frac{GM}{r}$$

Rearranging for T^2:

$$\frac{GMT^2}{r} = (2\pi r)^2 \quad \text{or} \quad \boxed{T^2 = \frac{4\pi^2 r^3}{GM}}$$

This tells us that $T^2 \propto r^3$

For an ellipse this becomes $T^2 \propto a^3$ ← semimajor axis

Satellite example (slide 5)

$$F_c = F_g \Rightarrow v^2 = \frac{GM}{r} \Rightarrow v_A = \sqrt{\frac{GM}{r_A}} \quad \text{and} \quad v_B = \sqrt{\frac{GM}{r_B}}$$

$$\Rightarrow v_B / v_A = \frac{\sqrt{\frac{GM}{r_B}}}{\sqrt{\frac{GM}{r_A}}} = \sqrt{\frac{r_A}{r_B}} = \sqrt{\frac{r_A}{2r_A}} = \frac{1}{\sqrt{2}} \qquad \text{(b)}$$